# Global Minimum Structures of Morse Clusters as a Function of the Range of the Potential: $81 \le N \le 160$

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We have attempted to find the putative global minimum structures of Morse clusters for cluster size N = 81-160 as a function of the range of the potential (with potential range  $\rho_0 \ge 3.0$ ). Moreover, compared to the results for  $N \le 80$  listed in the Cambridge Cluster Database, a number of new putative global minima are given. A structural and conformational analysis of  $M_{100}$  for different  $\rho_0$  was given. The sequences of the global minima as a function of  $\rho_0$  and N were studied, and the zero temperature "phase diagram" was given for an overall view of how the global minima depend upon N and  $\rho_0$ .

## 1. Introduction

Structural information is of great importance for many chemical and physical fields. In the past 20 years, developing an efficient structural optimization method to find the low-energy or ground-state structures of atomic/molecular clusters has attracted great interest.<sup>1–13</sup> Compared to bulk materials, the ground-state (or global minimum) structures of clusters are always very novel, e.g., the fullerene clusters,<sup>14</sup> the cage- and cubic-like water clusters,<sup>15,16</sup> and the hollow cage-like Au clusters.<sup>17,18</sup>

The electronic structure determines the ground-state geometric structure, but generally, it's too expensive to have a systematic global geometric optimization directly using a quantum mechanics (QM) method. Alternatively, model and empirical potentials are largely used to fit the interactions among particles, and the results are generally acceptable in a certain precision compared to the experiments or QM methods.

For various clusters, the range of the interaction is a key parameter determining the ground-state structures, e.g., at small cluster size, for sodium clusters with a long-ranged interaction, disordered and icosahedral motifs are most favored;<sup>19</sup> for the Lennard-Jones (LJ) clusters with a middle-ranged interaction, icosahedral motifs are most favored and only at some magic numbers decahedral, face-centered cubic (fcc), and tetrahedral motifs can be global minima;<sup>16</sup> and for C60 molecular clusters with a very short-ranged interaction, decahedral, tetrahedral, and close-packed motifs are predominant.<sup>20,21</sup>

Morse potential  $^{\rm 22}$  can be taken as a test system with pair interaction

$$V(r) = \epsilon \cdot e^{\rho_0 (1 - r/r_e)} [e^{\rho_0 (1 - r/r_e)} - 2]$$

where  $\epsilon$  is the pair well depth,  $r_e$  is the equilibrium distance, and the parameter  $\rho_0$  determines the potential range; larger  $\rho_0$ means more short-ranged interaction. Using the Morse potential, Doye et al.<sup>23,24</sup> have had a systematic study about how the range of the potential affects the global minimum structures. They located putative global minimum structures as a function of the range of the potential  $\rho_0$  and gave out the structural phase diagram for cluster size  $N \leq 80$ .

In this work, we try to locate the putative global minimum structures of Morse clusters as a function of  $\rho_0$  for slightly larger cluster sizes ( $81 \le N \le 160$ ) using the newly developed dynamic lattice searching (DLS)<sup>11</sup> method. Moreover, for  $N \le 80$ , a number of new global minimum structures are located.

## 2. Computational Method

DLS combines advantages of the lattice searching method and the stochastic unbiased global optimization method, and has been proved to be an efficient unbiased cluster optimization method for the optimization of LJ clusters and C60 molecular clusters.<sup>11,21</sup> Moreover, DLS can locate the lowest-energy structures of various motifs (e.g., icosahedral, decahedral, and close-packing) instead of only the global minimum one.<sup>25</sup> To have a systematic study on the global minimum structures of Morse clusters as a function of the range of the potential, at each  $\rho_0$  among 3.0, 3.5, 4.0, 4.5, 5.0, 6.0, 8.0, 10.0, and 14.0,  $10^4$  DLS runs were carried out separately, and the 20 lowestenergy minima located in the DLS runs for each  $\rho_0$  were recorded. Finally, the putative global minimum structures for potential range  $\rho_0 \geq 3.0$  are found out from the recorded minima.

#### 3. Results and Discussion

**3.1. Conformational and Structural Analysis of**  $M_{100}$ . To have an understanding of the funnel information of the energy landscape and structural information for various packing styles, a conformational and structural analysis of Morse clusters with atom number N = 100 ( $M_{100}$ ) is given, where we think that, approximately, a motif with higher hit number is more predominant in conformation and lies in a wider funnel on the energy landscape.<sup>25</sup> Figure 1 plots the conformational distribution of  $M_{100}$  during 10<sup>4</sup> DLS runs at  $\rho_0 = 3.0, 3.5, 4.5, 6.0, 8.0,$  and 14.0. Structures labeled in Figure 1 are classified by the motifs: disordered (dis), Mackay icosahedral (I), I plus anti-Mackay overlayers (I+), distorted I+ (\*I+), polyicosahedral (PI), decahedral (D), D plus anti-Mackay overlayers (D+), tetrahedral (T), face-centered cubic (fcc), and close-packed (cp).

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**Figure 1.** Structural distributions of  $M_{100}$  clusters in 10<sup>4</sup> DLS runs for  $\rho_0 = 3.0, 3.5, 4.5, 6.0, 8.0$ , and 14.0 (as labeled). The *y*-axis gives the hit numbers of various metastable local minima during the 10<sup>4</sup> DLS runs. The *x*-axis gives relative energy from the global niminum structure. The hit numbers out of range are labeled in the figure. The indexes of each motif are also labeled.



Figure 2. Various structures of  $M_{100}$  as labeled in Figure 1. Enclosed are the point groups. For some cases two sides of views are given, and the sites are given in different shades to show the topological structure more clearly.

The basic structural categories of the ordered packing are I, D, T, fcc, and cp, where the maximal coordination number of one atom is 12. fcc cluster is a fragment of fcc crystal and can be contained in an fcc tetrahedron. I is packed by 12 tetrahedrons sharing one common vertex. D is packed by 5 tetrahedrons sharing one common edge. T has a tetrahedral core and may have outer tetrahedrons sharing the four {111} faces of the core. T, fcc, and cp can be strain-free. PI is packed by I motifs sharing the five top tetrahedrons. The labeled structures are plotted in Figure 2.

At  $\rho_0 = 3.0$ , most of the located structures are disordered, and the global minimum structure (dis1) has a distorted 39atom icosahedral core and a regular tight outer shell, which is somewhat similar to the polyicosahedral core—shell clusters.<sup>26</sup> At  $\rho_0 = 3.5$ , disordered clusters are energetically in disfavor relative to distorted I+ clusters (\*I+1 and \*I+2), but still are predominant in conformation and have most hit numbers, where dis2 is similar to dis1 but its core is more like a distorted 39atom decahedral cluster. At  $\rho_0 = 4.5$ , icosahedral clusters (I, I+, PI1, and I+D) become predominant in both energy and conformation. \*I+1 is distorted I+ both in the surface and in the core to have more nearest neighbors, and can be transformed to I+ after relaxation at  $\rho_0 = 4.5$ . PI1 and PI2 can be thought as polyicosahedron,<sup>27</sup> but are based on the 55-atom Mackay icosahedron (Ih55). The core of I+D is a union of icosahedron and decahedron. At  $\rho_0 = 6.0$ , I still is the global minimum and I+ is most predominant in conformation, but the energetic gap between I and I+ increases and the energetic gaps between I and decahedral or tetrahedral clusters decrease. D1, D2, D3, and D4 are decahedral motifs; D+1, D+3, and D+4 are decahedra plus anti-layers on one side of {111} faces; D+2 is decahedron plus anti-layers on both sides of {111} faces. T1 is the 98-atom Leary tetrahedron<sup>28</sup> plus two atoms (based on the 4-atom edged tetrahedron); T2 is based on the 6-atom edged tetrahedron; T3, T4, T5, T6, T7, and T8 are based on the 5-atom edged tetrahedron. T2, fcc, cp1, cp2, cp3, and cp4 are both strain-free clusters,<sup>29</sup> but T2 has stacking faults in four directions of {111} layers, fcc has no stacking fault, and cp1, cp2, cp3, and cp4 have stacking faults in only one direction of {111} layers. At  $\rho_0 = 8.0$ , D1 becomes the global minimum structure, but I+ still is the most dominant conformation despite its high energy. At  $\rho_0 = 14.0$ , T2 is the global minimum, icosahedral

TABLE 1: New Putative Global Minimum Structures at  $N \leq 80^a$ 

| Ν          | PG          | motifs      | NN         | $E_{\rm strain}$ | $\rho_0 = 3.0$             | $\rho_0 = 6.0$             | $ \rho_0 = 10.0 $                       | $\rho_0 = 14.0$ | $ ho_{ m min}$ | $ ho_{ m max}$ |
|------------|-------------|-------------|------------|------------------|----------------------------|----------------------------|-----------------------------------------|-----------------|----------------|----------------|
| 10a        | $D_{2d}$    | 2D+         | 26         | 0.0859           | -30.825058                 | -26.588332                 | -25.960250                              | -25.840646      | 17.023         |                |
| 11a        | $C_{3v}$    | dis         | 31         |                  | -37.886544                 |                            |                                         |                 | 3.491          | 3.676          |
| 24a        | $C_s$       | 2PI         | 88         | 6.7627           | -124.498381                | -89.415307                 | -81.585755                              |                 | 8.225          | 8.933          |
| 26a        | $C_s$       | 2PI,I+      | 98         | 9.6577           | -142.361067                | -99.545037                 | -88.715128                              |                 | 7.678          | 7.995          |
| 29a        | $C_s$       | 2I+         | 113        |                  | -167.108031                | -113.549356                |                                         |                 | 6.802          | 6.975          |
| 29b        | $C_s$       | 4D          | 103        | 0.2458           |                            | -111.513973                | -103.947386                             | -102.744078     | 11.061         | 11.207         |
| 32a        | $D_{3h}$    | 3T,D+       | 117        | 0.9434           | -188.968428                | -125.863078                | -117.237209                             | -115.432942     | 10.083         | 11.537         |
| 33a        | $C_s$       | 2PI,I+      | 134        | 2 (247           | -202.101265                | -131.893912                | 105 550026                              |                 | 5.725          | 5.745          |
| 34a        | $C_s$       | 31          | 128        | 3.6247           | -209.830547                | -136.524962                | -125.552036                             |                 | 8.278          | 8.579          |
| 30a<br>38a | $C_s$       | 21+<br>dia  | 152        |                  | -230.198400<br>-247.324804 | -143.144100<br>-155.568027 |                                         |                 | 4.499          | 4.834          |
| 30a<br>30a | $C_3$       | 31"         | 150        |                  | -247.324804<br>-255.643276 | -155.508927                |                                         |                 | 4.344          | 4.937          |
| 40a        | $D_{cl}$    | dis 2PI     | 171        |                  | $-265\ 425496$             | -166008623                 |                                         |                 | 4 108          | 5 025          |
| 41a        | $D_{3}$     | dis.        | 171        |                  | -273.570563                | -170.187434                |                                         |                 | 4.547          | 4.553          |
| 41b        | $C_s$       | dis,2PI     | 175        |                  | -273.338612                | -170.327675                |                                         |                 | 4.553          | 4.708          |
| 42a        | $C_s$       | dis,2I+     | 178        |                  | -285.119843                | -174.062576                |                                         |                 | 4.469          | 4.597          |
| 43a        | $C_s$       | dis,2I+     | 186        |                  | -295.132688                | -179.076290                |                                         |                 | 4.333          | 4.448          |
| 44a        | $C_{2v}$    | dis,2PI     | 191        |                  | -302.767757                | -184.833284                |                                         |                 | 4.174          | 4.831          |
| 44b        | $C_s$       | 3I          | 175        | 5.6110           | -303.600375                | -187.344244                | -171.111366                             |                 | 9.000          | 9.399          |
| 45a        | $C_s$       | dis,2PI     | 198        |                  | -314.407214                | -188.034882                |                                         |                 | 4.129          | 4.212          |
| 45b        | $C_s$       | dis,2PI     | 195        |                  | -312.722886                | -189.721433                |                                         |                 | 4.212          | 4.502          |
| 46a        | $C_s$       | dis,2PI     | 204        |                  | -325.144128                | -191.683099                |                                         |                 | 3.750          | 4.073          |
| 47a        | $C_1$       | dis,2PI     | 204        |                  | -335.821344                | -196.410327                |                                         |                 | 3.719          | 3.838          |
| 470        | $C_2$       | dis,2PI     | 209        |                  | -334.272064                | -198.601/93                |                                         |                 | 3.838          | 4.185          |
| 47C        | $C_s$       | dis,2PI     | 208        |                  | -335.023133                | -198.813474<br>-200.020214 |                                         |                 | 4.185          | 4.202          |
| 40a<br>48h | $C_s$       | dis         | 214        |                  | -343.696158                | -200.030314<br>-204.298868 |                                         |                 | 3.242          | 4 216          |
| 400<br>49a | $C_s$       | dis         | 212        |                  | -355 522137                | -207.617845                |                                         |                 | 3 840          | 3 929          |
| 50a        | $C_s$       | 31          | 204        | 9.1289           | -363.380857                | -219.163495                | -197.123775                             |                 | 3.890          | 4.937          |
| 51a        | $C_s$       | dis         | 231        | ). <u>.</u> ))   | -376.631334                | -218.675700                | 1,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,, |                 | 3.385          | 3.758          |
| 51b        | $C_1$       | 3I          | 210        | 8.5744           |                            | -225.210338                | -203.624838                             | -194.956337     | 7.144          | 7.425          |
| 53a        | $C_s$       | 4T          | 212        | 0.8191           |                            | -232.060537                | -213.889957                             | -210.925799     | 10.297         | 10.308         |
| 56a        | $C_s$       | dis         | 258        |                  | -428.496002                | -243.948769                |                                         |                 | 3.363          | 3.615          |
| 57a        | $C_{2v}$    | 3I+"        | 247        |                  | -436.630741                |                            |                                         |                 | 3.908          | 4.533          |
| 57b        | $C_{2v}$    | 3I+         | 240        | 10.5469          |                            | -257.604126                | -231.988382                             | -219.551585     | 4.533          | 4.752          |
| 57c        | $C_s$       | 3I+         | 241        | 10.7118          |                            | -257.950515                | -232.709814                             | -220.265507     | 4.752          | 5.028          |
| 58a        | $C_{2v}$    | dis         | 254        | 10 7 6 60        | -447.812676                | -257.048479                |                                         |                 | 3.978          | 4.074          |
| 58b        | $C_s$       | 31+"        | 246        | 10.5660          | 452 021 (5)                | -261.950240                | -235.992154                             | -223.520289     | 4.074          | 4.102          |
| 59a        | $C_s$       | 31+"<br>dia | 251        | 10.5884          | -453.831656<br>-400.742002 | -267.348139                | -241.012509                             | -228.486823     | 4.260          | 4.858          |
| 62h        | $C_{2v}$    | 31+''       | 200        | 11.6706          | -490.743003                | -270.030334<br>-283.133335 | -255 155600                             | -2/1 815901     | 5.107<br>4.514 | 4.514          |
| 63a        | $C_1$       | dis         | 203        | 11.5715          | -501.276459                | 205.1555555                | 255.155000                              | 241.015701      | 3.213          | 3.446          |
| 63b        | $C_{\rm s}$ | dis         | 291        |                  | -500.208115                | -281.303611                |                                         |                 | 3.446          | 4.334          |
| 64a        | $C_{3v}$    | dis         | 299        |                  | -510.627671                | -285.009129                |                                         |                 | 3.376          | 4.343          |
| 65a        | $C_1$       | dis         | 304        |                  | -520.878839                | -289.921751                |                                         |                 | 3.510          | 4.225          |
| 66a        | $C_1$       | dis         | 310        |                  | -531.877652                | -294.567339                |                                         |                 | 3.457          | 4.228          |
| 66b        | $C_s$       | 3I+         | 286        | 13.5437          |                            | -303.262241                | -272.373110                             | -257.984208     | 4.228          | 4.255          |
| 67a        | $C_1$       | dis         | 313        |                  | -542.476797                | -299.701936                |                                         |                 | 3.505          | 4.068          |
| 67b        | $C_s$       | 3I+         | 289        | 15.7649          |                            | -308.913158                | -276.158986                             |                 | 4.114          | 5.196          |
| 68a        | $C_s$       | dis         | 318        |                  | -554.981243                | -302.707317                |                                         |                 | 3.335          | 3.438          |
| 68b        | $C_1$       | dis<br>dia  | 319        |                  | -554.520713                | -303.557452                |                                         |                 | 3.438<br>2.451 | 3.451          |
| 080<br>68d | $C_s$       | uis<br>21⊥″ | 321<br>207 |                  | -333.303223                | -303.377887<br>-312.642581 |                                         |                 | 3.431          | 5.951          |
| 68e        | $C_1$       | 5D          | 297        | 0 5643           |                            | -311,042381<br>-311,231888 | -286 246653                             | -282 604561     | 11 514         | 12 484         |
| 69a        | $C_s$       | dis         | 327        | 0.50+5           | -565,907867                | -308.764626                | 200.240055                              | 202.00+501      | 3.301          | 3.426          |
| 69b        | $C_{s}$     | dis         | 326        |                  | -565.630050                | -308.817610                |                                         |                 | 3.426          | 3.568          |
| 69c        | $C_{2v}$    | dis         | 316        |                  | -563.961350                | -311.375984                |                                         |                 | 3.568          | 3.874          |
| 69d        | $C_1$       | 3I+"        | 303        |                  | -562.944706                | -318.459290                |                                         |                 | 3.874          | 4.35           |
| 70a        | $C_1$       | dis         | 335        |                  | -577.286914                | -313.273673                |                                         |                 | 3.216          | 3.461          |
| 70b        | $C_1$       | dis         | 331        |                  | -576.341992                | -314.187108                |                                         |                 | 3.461          | 3.736          |
| 70c        | $C_5$       | 3I+"        | 309        |                  | -572.226257                | -324.143464                |                                         |                 | 3.736          | 3.954          |
| 70d        | $C_1$       | 3I+"        | 309        | 11.3497          | -571.752458                | -324.370120                |                                         |                 | 4.136          | 4.16           |
| 71a        | $C_s$       | dis         | 349        |                  | -588.345014                | -319.253314                |                                         |                 | 3.020          | 3.025          |
| /1b        | $C_s$       | dis         | 345        |                  | -588.332142                | -316.248223                |                                         |                 | 3.025          | 3.189          |
| /1C        | $C_s$       | ais<br>dia  | 332        |                  | -387.933339                | -318.4/386/                |                                         |                 | 3.189          | 5.298          |
| 720        | $C_s$       | dis         | 329<br>250 |                  | -500 106517                | -318.313013                |                                         |                 | 3.298<br>2.047 | 3.09<br>2.070  |
| 72h        | $C_1$       | dis         | 332<br>344 |                  | -599.400347                | -322.142109                |                                         |                 | 3.007          | 3.078<br>3.45  |
| 72c        | $C_s$       | dis         | 345        |                  | -599.049445                | -323.356172                |                                         |                 | 3.450          | 3.704          |

TABLE 1: (Continued)

| Ν   | PG    | motifs | NN  | $E_{\rm strain}$ | $\rho_0 = 3.0$ | $ \rho_0 = 6.0 $ | $\rho_0 = 10.0$ | $\rho_0 = 14.0$ | $ ho_{ m min}$ | $ ho_{ m max}$ |
|-----|-------|--------|-----|------------------|----------------|------------------|-----------------|-----------------|----------------|----------------|
| 72d | $C_2$ | dis,I+ | 325 |                  | -595.063785    | -331.393629      |                 |                 | 3.704          | 3.816          |
| 72e | $C_1$ | 3I+"   | 321 |                  | -595.064909    |                  |                 |                 | 3.816          | 4.052          |
| 73a | $C_s$ | dis    | 349 |                  | -610.843103    |                  |                 |                 | 3.015          | 3.823          |
| 73b | $C_1$ | 3I+"   | 325 |                  | -606.917536    |                  |                 |                 | 3.824          | 3.858          |
| 73c | $C_1$ | 3I+"   | 325 |                  | -605.468812    |                  |                 |                 | 3.858          | 4.108          |
| 74a | $C_1$ | 3I+"   | 330 |                  | -617.864423    |                  |                 |                 | 3.881          | 3.972          |
| 74b | $C_1$ | 3I+"   | 329 |                  | -618.802036    |                  |                 |                 | 3.972          | 4.033          |
| 75a | $C_s$ | dis    | 357 |                  | -632.435998    |                  |                 |                 | 3.225          | 3.855          |
| 75b | $C_1$ | 3I+"   | 335 |                  | -629.796590    |                  |                 |                 | 3.856          | 4.011          |
| 75c | $C_1$ | 3I+    | 328 | 18.9564          | -627.536328    | -350.984741      | -312.486515     | -295.888665     | 4.011          | 5.003          |
| 76a | $C_s$ | dis    | 368 |                  | -643.984157    |                  |                 |                 | 3.225          | 3.403          |
| 76b | $C_1$ | dis    | 364 |                  | -643.111552    |                  |                 |                 | 3.403          | 3.868          |
| 76c | $C_1$ | 3I+"   | 333 | 19.9633          | -641.116800    | -355.850733      | -316.498615     |                 | 3.868          | 4.276          |
| 77a | $C_s$ | dis    | 368 |                  | -653.480087    |                  |                 |                 | 3.438          | 3.732          |
| 77b | $C_1$ | 3I+"   | 345 |                  | -652.186718    |                  |                 |                 | 3.732          | 4.139          |
| 78a | $C_s$ | dis    | 392 |                  | -667.519832    |                  |                 |                 | 3.154          | 3.435          |
| 78b | $C_s$ | dis    | 376 |                  | -664.936478    |                  |                 |                 | 3.435          | 3.637          |
| 78c | $C_1$ | 3I+"   | 351 |                  | -663.138941    |                  |                 |                 | 3.637          | 3.932          |
| 79a | $C_1$ | dis    | 400 |                  | -678.920223    |                  |                 |                 | 3.185          | 3.399          |
| 79b | $C_1$ | dis    | 378 |                  | -678.786959    |                  |                 |                 | 3.399          | 3.427          |
| 79c | $C_1$ | dis    | 382 |                  | -677.838383    |                  |                 |                 | 3.427          | 3.493          |
| 79d | $C_2$ | 3I+"   | 357 |                  | -674.451312    |                  |                 |                 | 3.493          | 4.005          |
| 80a | $C_1$ | dis    | 403 |                  | -690.518910    |                  |                 |                 | 3.041          | 3.15           |
| 80b | $C_1$ | dis    | 407 |                  | -690.458744    |                  |                 |                 | 3.151          | 3.332          |
| 80c | $C_1$ | dis    | 384 |                  | -690.053940    |                  |                 |                 | 3.332          | 3.418          |
| 80d | $C_1$ | dis    | 387 |                  | -689.067555    |                  |                 |                 | 3.418          | 3.488          |
| 80e | $C_1$ | 3I+"   | 363 |                  | -685.686196    |                  |                 |                 | 3.488          | 4.024          |

<sup>*a*</sup> PG gives the point group. NN gives the nearest-neighbor contacts.  $\rho_{min}$  and  $\rho_{max}$  give the range of  $\rho_0$  for which the structure is the global minimum.  $E_{strain}$  gives the strain energy at  $\rho_0 = 10.0$ . All energies are given in units of the pair well depth,  $\epsilon$ . If the structure is unstable, no value of energy is given. The motifs are classified by the packing styles (see ref 25), and the "" means the structure is distorted.



**Figure 3.** Some typical structures of the newly located global minimum at  $N \leq 80$ .

structures are unstable, and the energy of T1 increases much but becomes the most predominant conformation.

**3.2. New Global Minimum Structures at**  $N \leq 80$ . Morse clusters for cluster size  $N \leq 80$  have been systematically studied by various global optimization methods,<sup>23,24,29–31</sup> and the global minimum structures are available online from the Cambridge Cluster Database (CCD).<sup>16</sup> To check with the results in CCD, we also optimized Morse clusters at  $N \leq 80$ , and found out a number of new putative global minimum structures for a certain range of  $\rho_0$  as listed in Table 1.

Most of the new global minima are disordered clusters or distorted icosahedral clusters and similar to the structures given in CCD. Some typical or novel structures given in Table 1 are plotted in Figure 3: 10a is polydecahedral; 11a is disordered; 32a is tetrahedral (based on the 3-atom edged tetrahedron); 38a is disordered; 39a is distorted icosahedral, where the {100} faces are distorted to {111} faces to have more nearest-neighbors; 40a is the magic 38-atom polyicosahedron<sup>26,27</sup> plus two vertices;

41a is 38a plus three atoms; 44b is icosahedral; 53a is tetrahedral (based on the 4-atom edged tetrahedron); 57a is Ih55 plus two atoms and little distorted; 64a is the magic 61-atom  $T_d^{27}$  plus three atoms; 70c is one-vertex missed Ih55 plus a regular distorted anti-layer cap.

**3.3. Global Minimum Structures at 81**  $\leq N \leq$  **160.** To further investigate the relationship between the global minimum structures and the range of the potential at larger cluster sizes, we located the putative global minima of Morse clusters at 81  $\leq N \leq$  160 with potential range  $\rho_0 \geq 3.0.^{32}$  For  $\rho_0 < 3.0$ , the potential is too long-ranged, so we think it may be unreasonable for real systems.

At the middle-ranged potential, icosahedral clusters are predominant in potential energy. Figure 4 plots some typical icosahedral minima: 83E, 89F, 92E, 99E, 116C, 135B, and 137E are Ih55 plus various incomplete regular Mackay overlayers; 81F, 85C, 88C, 95D, 106E, 115B, and 127C are Ih55 plus regular anti-Mackay overlayers, 156F is 6-vertices missed Ih147 plus a regular anti-Mackay cap and little distorted, and 157F is Ih147 plus anti-Mackay overlayers; 81E, 88B, 106D, 115A, and 127A are distorted 81F, 88C, 106E, 115B, and 127C, respectively, but unlike 85C, 85B is 3-vertices missed Ih55 plus distorted anti-Mackay overlayers; 87E is a magic number Ih55based polyicosahedron, and 101F, 111F, and 152G are 87E plus regular anti-layers. The distorting can make the isosahedral clusters more strained but, on the other hand, more compact and have more nearest-neighbors. Therefore, at small  $\rho_0$  (about smaller than 4.0), the distorting of icosahedral clusters may decrease the potential energy.

At very small  $\rho_0$ , due to the very strong long-distance interaction, there are many packing styles to make the structures more compact and spherical. These packing styles are called disordered packing, where the coordination number of one atom



Figure 4. Global minima of Morse clusters with icosahedral packing.



Figure 5. Global minima of Morse clusters with disordered packing. The numbers enclosed by "[]" are the size of the core.



Figure 6. Global minima of Morse clusters with (a) tetrahedral packing and (b) magic number Marks decahedral packing or decahedral packing with anti-layers.

may be larger than 12. Figure 5 plots the magic numbers of the distorted clusters: 89B, 91A, and 101B are a 39-atom core plus compact outer shells; 91B is a distorted decahedron plus two distorted caps; 92D is similar to a rearrangement of the icosahedral 92E and is known as the global minimum of sodium clusters;<sup>32</sup> 96C has a very strange symmetry ( $S_6$ ); 101A and 105A are largely compressed core plus spherical outer shells; 107B is the 49-atom icosahedron (one {111} face missed Ih55) plus a compact outer shell; 110A has a 55-atom core which is a little-distorted ordered isomer of Ih55; 126B, 132C, 136A, 141A, 141B, and 146B are a 71-atom icosahedral core plus various outer shells (the distorting styles may be different); 106C, 128A, and 149A both have largely distorted core to make the structures spherical enough; 129E is similar to an Ih55-based polyicosahedron (in three directions); 154A is the 23-

atom icosahedron plus two layers of outer shells, and 156A is similar to 154A but the core is 25-atom disordered. Most magic numbers of the distorted global minima are core—shell clusters; i.e., an ordered core (icosahedral or decahedral, may be little distorted) plus a spherical outer shell, where the packing styles of the core and the outer shell are different. The icosahedra plus complete anti-Mackay overlayers (115A, 115B, 127A, and 127C) are also core—shell clusters. Core—shell clusters can be sufficient spherical and compact, and so are favored for the very long-ranged potentials.

At very large  $\rho_0$ , the potential is very short-ranged, so icosahedral clusters become too strained, and the global minima are close-packed or decahedral clusters, which have been sufficiently discussed in the literatures.<sup>29,33</sup> However, between the middle- and short-ranged potential (near  $\rho_0 = 8.0$ ), we found



Figure 7. Plots of the energies of the global minima as a function of cluster size  $10 \le N \le 160$  for various clusters (as labeled). *E* is the energy of the global minima, and  $E_{ave}$  is a four-parameter fit of the global minima. Downward peaks represent the most stable magic numbers compared to the neighbors. Data of LJ clusters are from the CCD,<sup>16</sup> and data of C60 molecular clusters are from ref 21.



**Figure 8.** Zero temperature "phase diagram" showing the variation of the lowest-energy structure with *N* and  $\rho_0$ . Labeled are the structural types: close-packed, decahedral, icosahedral (2I, 3I, and 4I), and disordered, where 2I is based on Ih13, 3I is based on Ih55, and 4I is based on Ih147 (may be incomplete or have anti-layers). The distorted icosahedra are included in the area of icosahedral motifs, while the polyicosahedra are not included.

some Leary tetrahedron-like<sup>28</sup> global minima. As shown in Figure 6a, 98I is the Leary tetrahedron, 89G is 98I minus a cap, 84J is 89G minus a patch, 159J is the next magic number of Leary tetrahedron (based on the 5-atom edged tetrahedron), 158M is 159J minus one atom, and 160H is 159J plus one atom. Moreover, there are also some global minima with decahedral packing plus anti-Mackay overlayers (D+). As shown in Figure 6b, each of the D+ global minima has a rather regular anti-Mackay cap or surface, where 89H is most interesting, which seems like both D+ and tetrahedral motifs.

**3.4.** Sequence Analysis of the Global Minima. Figure 7 plots the sequences of the energy of the global minima at various  $\rho_0$  to find out the most stable magic numbers. First, as shown in Figure 7a, for the very long-ranged potential ( $\rho_0 = 3.0, 3.2$ ), at N > 80, the most stable structures are the core-shell clusters (101A, 105A, 110A, 115A, 136A, 141A, 149A, and 154A). The magic number Ih45 is also a core-shell cluster which has an Ih13 core and a complete anti-Mackay outer shell. Second,

Figure 7b shows that, with  $\rho_0$  increasing ( $\rho_0 = 3.5, 3.7, and$ 4.0), peaks of the core-shell magic numbers gradually disappear or become weaker, and the icosahedral magic numbers (Ih55, 71, 81E, 88B, 95C, 135-147) appear, where Ih55 and 135-147 are Mackay icosahedra (I), while 71, 81E, 88B, 95C, 115A, and 115B are Ih55 plus various anti-Mackay overlayers (I+, may be little distorted). 135B is the 12-vertices missed Ih147, which is more spherical, and so is more dominant than Ih147 at smaller  $\rho_0$ . Next, Figure 7c compares the energetic sequences of Morse clusters at  $\rho_0 = 6.0$  and the LJ clusters, which shows that the sequences of the two potentials have very similar outlines except for very few cases (Dh75, 87E, and 137E). Morse potential at  $\rho_0 = 6.0$  and the LJ potential are very similar near the curve bottom, and the difference is that the interaction of Morse potential is too weak at long distances. Dh75 has fewer nearest-neighbor contacts (NN) compared to relative icosahedral clusters (I) but is more spherical, so it has stronger non-NN interactions and favors more the potentials with stronger long

distance interaction. 137E is the one-face missed Ih147, which also has large NN but is less spherical and so favors more the Morse potential. 87E is the polyicosahedral (PI) cluster, which has similar NN with relative I+ motifs but is much less spherical than relative I+ or I motifs, and so it is even not the global minimum for LJ clusters. However, 87E is less strained than relative I+ motifs and has more NN than relative I motifs, so it acts as a magic number for Morse clusters at  $\rho_0 = 6.0$ . Then, as shown in Figure 7d, with  $\rho_0$  increasing ( $\rho_0 = 7.0, 8.0, 9.0$ , and 10.0), peaks of the icosahedral magic numbers (39, 71, Ih55, 87E, 116C, and Ih147) gradually disappear, and peaks of decahedral or fcc magic numbers (38, Dh75, 101H, and 146D) appear but are weaker than those of the icosahedral magic numbers at smaller  $\rho_0$ . Last, Figure 7e plots the sequences of the Morse clusters at  $\rho_0 = 14.0$  and the C60 molecular clusters with the Girifalco potential,<sup>34</sup> which are both clusters with very short-ranged potentials. For a short-ranged potential, NN is most important in determining the potential energy; i.e., isomers with the same NN and same packing style (decahedral or closepacked) will have very similar potential energy. Therefore, it can be seen that the two clusters have very similar energy sequences of the global minima, although the motifs of the global minima may be different.

Figure 7 shows that, with  $\rho_0$  increasing, the sequences of the most stable magic numbers are disordered core-shell clusters, icosahedral clusters, decahedral and close-packed clusters. Finally, to give an overall view of how the global minima depend upon N and  $\rho_0$ , following Doye et al.,<sup>23,24</sup> Figure 8 plots the zero temperature "phase diagram" for  $10 \le N \le 160$ , and  $\rho_0 \geq 3.0$ . With  $\rho_0$  increasing, structural transition may be disordered to icosahedral, icoshahedral with anti-layers to icoshahedral without anti-layers (2I to 3I, and 3I to 4I), icosahedral to decahedral, decahedral to close-packed, and icosahedral directly to close-packed (38-40, 59, and 91). The blanks between 2I and 3I are disordered structure (e.g., 38a, 40a, and 41a as shown in Figure 3), the blanks between 3I and 4I are the Ih55-based polyicosahedra (e.g., 87E as shown in Figure 4), and the blanks between icosahedral and decahedral are the Leary tetrahedron-like structures (as shown in Figure 6a).

## 4. Conclusions

In conclusion, with the dynamic lattice searching (DLS) method, we have attempted to find the putative global minimum structures of Morse clusters for cluster size  $N \leq 160$  as a function of the range of the potential (with potential range  $\rho_0$  $\geq$  3.0), which is notoriously difficult for unbiased global optimization methods. Compared to the results for  $N \leq 80$  listed in the Cambridge Cluster Database (CCD), a number of new putative global minima were given. We studied the structural and conformational distribution of  $M_{100}$  for different  $\rho_0$  to know about the funnel information of the energy landscape. The sequences of the global minima as a function of  $\rho_0$  and N were studied, and the zero temperature "phase diagram" was given to find out how the global minima depend upon N and  $\rho_0$ . The global minima of Morse clusters can act as a structural bank, which may be helpful in determining the global minimum structures of other atomic or molecular clusters.

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**Supporting Information Available:** Table of the global minima of Morse clusters at  $81 \le N \le 160$ . This material is available free of charge via the Internet at http://pubs.acs.org.

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(32) Table of the global minima of Morse clusters along with their motifs, energies, point groups, number of nearest-neighbors, strain energies, and the values of  $\rho_0$  for which they are probably the lowest energy minimum for  $81 \le N \le 160$  can be found in the Supporting Information. Alternatively, the table and the coordinates are available from the webpage: http:// staff.ustc.edu.cn/~clj/morse/table.html.

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