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Magnetic Resonance Energies of Heterocyclic Conjugated Molecules

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Magnetic resonance energy (MRE), derived from ring-current diamagnetic susceptibility, can be interpreted as a kind of aromatic stabilization energy. For polycyclic conjugated hydrocarbons, this quantity correlates well with topological resonance energy (TRE). MREs for typical heterocyclic conjugated molecules were then calculated and analyzed. It was found that even for heterocycles MRE highly correlates with TRE. Thus, the MRE concept has been firmly established as a reliable indicator of aromaticity, which mediates magnetic criteria of aromaticity with energetic ones. The conformity of heterocycles to the rule of topological charge stabilization can be checked using not only TRE but also MRE.

Introduction

We have pointed out that energetic and magnetic criteria of aromaticity sometimes make different predictions on the aromaticity of a polycyclic π-system.[1−8] This aspect of aromatic molecules appears to suggest that aromaticity might be multidimensional in nature.[9−12] One, however, should note that magnetic properties associated with induced ring currents are highly dependent on molecular geometry,[1,2,13−17] whereas aromaticity must represent primarily a state of energy. Within the framework of Hückel molecular orbital theory, aromatic stabilization energy (ASE) or stabilization energy due to cyclic conjugation is dependent solely on the connectivity of conjugated atoms.[1,2,18−23] It then follows that a π-electron current induced in a polycyclic π-system[24,25] and nucleus independent chemical shift (NICS)[26] do not always reflect aromatic or antiaromatic character of the entire π-system properly.[1−8] No general relationship is conceivable between NICS and ASE for the polycyclic π-systems.

We recently succeeded in extracting a kind of ASE from ring-current diamagnetism, a magnetic response of a polycyclic π-system.[27−29] This energy quantity was called magnetic resonance energy (MRE). We evaluated MREs for many polycyclic benzenoid hydrocarbons and found that MRE correlates very well with topological resonance energy (TRE),[27−29] which is one of the well-established energetic measures of aromaticity.[2,22,23] This fact indicates explicitly that energetic and magnetic criteria of aromaticity can in principle be interpreted consistently.[27,28] Ring-current diamagnetism proved to represent a tendency of a cyclic π-system to preserve ASE at the level of individual circuits.[28] In this paper, we examine the MREs calculated for typical heterocyclic conjugated molecules and firmly establish the validity of the MRE concept as a reliable energetic indicator of aromaticity.

Theory

Our graph-theoretical variant[14−16] of the Hückel−London ring-current model[30] was used throughout this study. Ring-current diamagnetic susceptibility of a polycyclic π-system is given exactly as a sum of contributions from all possible circuits.[2,14−16] Here, circuits mean all possible cyclic paths that can be chosen from a cyclic π-system. For example, one ten-membered and two six-membered circuits can be chosen from the naphthalene π-system. Hückel parameters recommended by Van Catledge[31] were employed and are listed in Table 1.

We first evaluate the value of A_i defined for each circuit as[16]

$$A_i = 4 \prod_{m>n}^{r_i} k_{mn} \sum_{j}^{\text{occ}} \frac{P_{G-r_i}(X_j)}{P'_G(X_j)} \tag{1}$$

where r_i is a set of conjugated atoms and π-bonds that constitute the ith circuit; k_{mn} is the Hückel parameter for the resonance integral between the π-bond formed by atoms m and n; m and n run over all π-bonds that belong to r_i; $G - r_i$ is the subsystem of G, obtained by deleting r_i from G; $P_G(X)$ and $P_{G-r_i}(X)$ are the characteristic polynomials for G and $G - r_i$, respectively; X_j is the jth largest zero of $P_G(X)$; a prime added to $P_G(X)$ indicates the first derivative with respect to X; j runs over all occupied π molecular orbitals (π-MO); and all π-bonds are assumed to have equal resonance integrals. If there are degenerate π MOs, eq 1 must be replaced by others.[13−15,32] Note that A_i is determined solely by the connectivity of conjugated atoms.

The contribution of the ith circuit to ring-current diamagnetic susceptibility, χ_i, is formally expressed as[13−16,27,28]

$$\chi_i = 4.5\chi_0 A_i \left(\frac{S_i}{S_0}\right)^2 \tag{2}$$

where χ_0 is the benzene value and S_i and S_0 are the areas of r_i and the benzene ring, respectively. Positive and negative A_i values denote ring-current diamagnetism and paramagnetism, respectively. Circuits with positive A_i values would tend to escape from the external magnetic field because it is destabilized in the field. This tendency is nothing other than the origin of ring-current diamagnetism. Ring-current susceptibility for an entire π-system is obtained by summing χ_i over all circuits.

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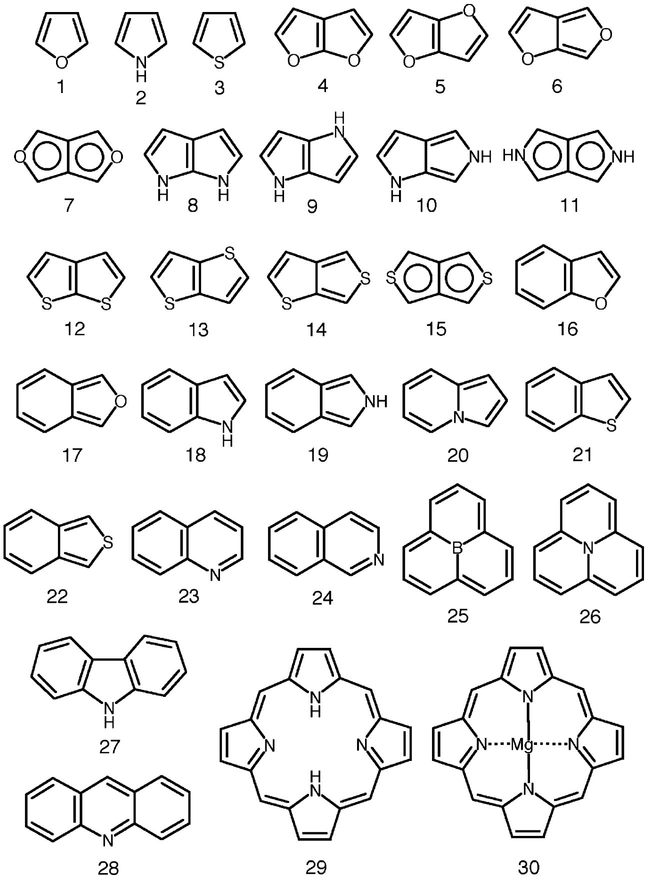

Figure 1. Heterocycles studied.

TABLE 1: Hückel Parameters Employed in This Study[31]

atom type (X)	no. of π-electrons	h_X for $\alpha_X = \alpha_C + h_X\beta_{CC}$	k_{CX} for $\beta_{CX} = k_{CX}\beta_{CC}$
C	1	0.00	1.00
B	0	−0.45	0.73
N1 (imine)	1	0.51	1.02
N2 (amine)	2	1.37	0.89
O2 (ether)	2	2.09	0.66
S2 (thioether)	2	1.11	0.69

The A_i value was called circuit resonance energy (CRE) because it can be interpreted as a contribution of a given circuit to the ASE of an entire π-system.[27−29] Therefore, the sum of A_i values for all circuits must represent a kind of ASE for an entire π-system. We refer to it as magnetic resonance energy (MRE),[27−29] which means a TRE-like quantity associated with the magnetic response of the π-system:

$$\mathrm{MRE} = \sum_{i}^{G} A_i \tag{3}$$

where i runs over all circuits in G. As in the case of TRE, MRE is a function of molecular topology only and vanishes for acyclic π-systems. A numerical measure of aromaticity usually refers to the difference between some property of the molecule and that of a nonaromatic reference system.[18−23,33−37] The definition of MRE is free from such an artificial reference.

Results and Discussion

TREs and MREs calculated for 30 typical heterocycles (**1**−**30** in Figure 1), together with those for benzene, are listed in Table 2. These two quantities were plotted in Figure 2. With the only exception of azaphenalene (**26**), MRE is more or less smaller than TRE. Figure 1 clearly shows that even for heterocycles MRE highly correlates with TRE with a correlation coefficient of 0.992. Such a trend in MRE is essentially the same as that for polycyclic conjugated hydrocarbons.[27−29] Both types of resonance energies are almost tantamount to each other. We can then safely say that the MRE concept is very meaningful and useful not only for polycyclic conjugated hydrocarbons but also for heterocycles.

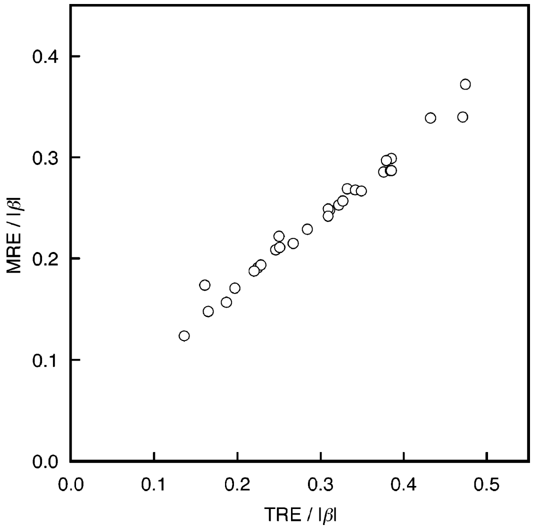

Figure 2. Correlation between TREs and MREs for heterocycles **1**−**30**.

TABLE 2: TREs and MREs for Benzene and Typical Heterocyclic Molecules

species	TRE/|β|	MRE/|β|
benzene	0.273	0.222
furan (**1**)	0.136	0.124
pyrrole (**2**)	0.246	0.209
thiophene (**3**)	0.197	0.171
furo[3,2-*b*]furan (**4**)	0.224	0.191
furo[2,3-*b*]furan (**5**)	0.228	0.194
furo[3,4-*b*]furan (**6**)	0.165	0.148
furo[3,4-*c*]furan (**7**)	0.187	0.157
pyrrolo[3,2-*b*]pyrrole (**8**)	0.385	0.299
pyrrolo[2,3-*b*]pyrrole (**9**)	0.379	0.297
pyrrolo[3,4-*b*]pyrrole (**10**)	0.332	0.269
pyrrolo[3,4-*c*]pyrrole (**11**)	0.342	0.268
thieno[3,2-*b*]thiophene (**12**)	0.311	0.248
thieno[2,3-*b*]thiophene (**13**)	0.309	0.249
thieno[3,4-*b*]thiophene (**14**)	0.251	0.211
thieno[3,4-*c*]thiophene (**15**)	0.267	0.215
benzo[*b*]furan (**16**)	0.322	0.253
benzo[*c*]furan (**17**)	0.220	0.188
indole (**18**)	0.376	0.286
isoindole (**19**)	0.327	0.257
indolizine (**20**)	0.309	0.242
benzo[*b*]thiophene (**21**)	0.349	0.267
benzo[*c*]thiophene (**22**)	0.284	0.229
quinoline (**23**)	0.384	0.287
isoquinoline (**24**)	0.385	0.287
boraphenalene (**25**)	0.250	0.222
azaphenalene (**26**)	0.161	0.174
carbazole (**27**)	0.552	0.423
acridine (**28**)	0.471	0.340
free-base porphine (**29**)	0.432	0.339
metal(II) porphine (**30**)	0.474	0.372

TREs indicate that furan (**1**) is much less aromatic than pyrrole (**2**) and thiophene (**3**). These molecules have more or less smaller TREs than benzene. The same thing can be said on the basis of MREs. Because heterobicycles **4**−**15** are essentially of the same geometry with the same number of π-electrons, the relative magnitudes of TREs can be associated naively with the order of aromaticity. The orders of aromaticity of furofurans, pyrrolopyrroles, and thienothiophenes thus determined are **4** ≈ **5** > **7** > **6**, **8** ≈ **9** > **11** > **10**, and **12** ≈ **13** > **15** > **14**, respectively.[38] It is noteworthy that exactly the same orders of aromaticity are obtained using the MREs. Interestingly, **7**, **11**, and **15** have larger TREs and larger MREs than **6**, **10**, and **14**, respectively, although the former species are energetically much less stable than the latter ones.[39,40] In this sense, **7**, **11**, and **15** are among the rare examples that are energetically unstable but fully aromatic. As pointed out previously,[38] aromaticity is not the only determinant of kinetic stability.

TREs support our view that benzo[*b*]furan (**16**), indole (**18**), and benzo[*b*]thiophene (**21**) are more aromatic than benzo[*c*]-furan (**17**), isoindole (**19**), and benzo[*c*]thiophene (**22**), respectively. Such relative aromaticities can be reproduced readily in terms of MREs. Both TREs and MREs indicate that indolizine (**20**) is less aromatic than isoindole (**19**). Note that only one Kekulé structure can be written for **17**, **19**, **20**, and **22**; these heterobicycles are not anyways highly aromatic. Quinoline (**23**) must be similar in aromaticity to isoquinoline (**24**), because an imine-type nitrogen atom is iso-π-electronic with an sp^2-carbon atom, providing one electron to the π-system. In fact, they have nearly the same TREs and MREs. Boraphenalene (**25**) and azaphenalene (**26**) are iso-π-electronic with, but much less aromatic than, phenalenium (**33**) and phenalenide (**34**) ions, respectively.[41]

Porphyrins and metalloporphyrins are among the most important natural products. The parent compounds of porphyrins and metalloporphyrins are free-base porphine (**29**) and metal(II) porphine (**30**), respectively. Both are fully conjugated 26-π-electron systems with a moderate degree of aromaticity. We assume that the central metal ion in **30** does not take part in the π-system and that all nitrogen atoms coordinated to the metal(II) ion are of imine-type. Interestingly, **29** and **30** happen to have essentially the same TRE and MRE as tricyclic anthracene (**32**). Thus, MRE can be used safely to compare the ASEs of a wide variety of π-systems. Recent studies suggest that the total aromatic pathway in **29** and **30** may consist of a combination of several routes (i.e., several circuits in our terminology).[42−44] Gutman showed that aromaticity indeed arises from a set of all possible circuits in a π-system.[45]

According to Gimarc's topological charge stabilization rule,[46] the best placement of electronegative heteroatoms is at the positions with the greatest charge in the uniform reference frame (i.e., the iso-π-electronic, isostructural hydrocarbon). π-Electron densities in the uniform reference frames for **1**−**30** are presented in Figure 3, and their TREs and MREs are presented in Table 3. Many heterocycles are less aromatic with smaller TREs and smaller MREs than their respective uniform reference frames. Relative aromaticities of isomeric heterobicycles (**4**−**15**), benzofurans (**16**, **17**), indoles (**18**−**20**), and benzothiophenes (**21**, **22**), predicted using TREs and MREs, are fully consistent with this rule.[46] Those that obey the rule of topological charge stabilization have larger TREs and larger MREs than others. It is noteworthy that some heterocycles (**18**, **27**, **29**, and **30**) are more aromatic than their respective uniform reference frames. These four heterocycles fully conform to the rule of topological charge stabilization.[46] Heterophenalenes **25** and **26** disobey the rule and so are much less aromatic than their respective uniform reference frames.

Discussions so far made on the aromaticity of heterocycles, e.g., the ones based on Dewar[34−37] and Hess−Schaad[19−21] resonance energies, generally apply to the present set of molecules. One, however, should remember that these types of resonance energies cannot be estimated properly not only for charged species but also for such heterocycles as **7**, **11**, and **15**, for which classical resonance structures cannot be written. Azaphenalene (**26**) has once been predicted to be antiaromatic with negative Dewar resonance energy although it is iso-π-electronic with the aromatic phenalenide ion (**34**).[35] As has been seen above, these problems can be solved using the TRE or MRE concept. The collinearity between TRE and MRE is deteriorated when they have negative values.[29] MRE sometimes overestimates antiaromaticity when antiaromatic species are dealt with.[29]

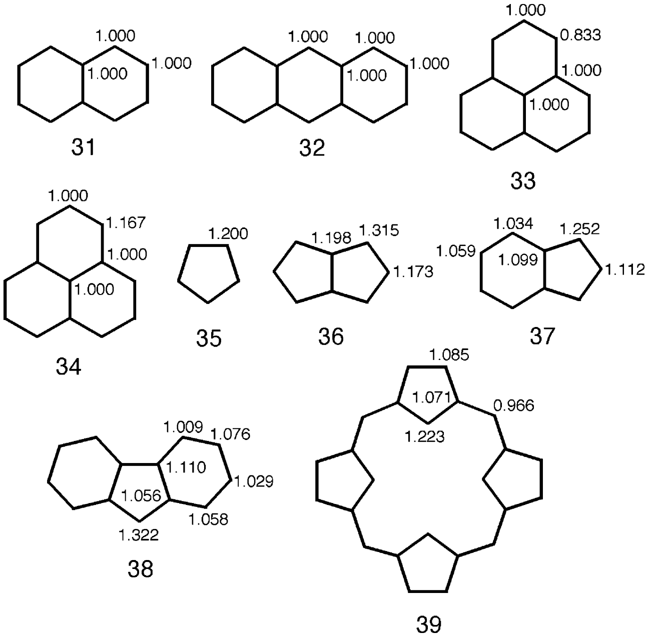

Figure 3. π-Electron densities in the uniform reference frames for heterocycles **1**−**30**.

TABLE 3: TREs and MREs of the Uniform Reference Frames for Heterocycles 1−30

species	TRE/|β|	MRE/|β|
naphthalene (**31**)	0.389	0.289
anthracene (**32**)	0.475	0.341
phenalenium ion (**33**)	0.410	0.310
phenalenide ion (**34**)	0.410	0.310
cyclopentadienide ion (**35**)	0.317	0.259
pentalene dianion (**36**)	0.464	0.346
indenide ion (**37**)	0.364	0.275
fluorenide ion (**38**)	0.465	0.340
porphyrinoid hydrocarbon dianion (**39**)	0.394	0.323

Concluding Remarks

Haddon once noted that, as far as aromatic annulenes are concerned, ASE is analytically related to the intensity of the induced ring current.[47,48] This was indeed one of the first attempts to consistently interpret the energetic and magnetic criteria of aromaticity. We have managed to extend this idea to include all kinds of polycyclic π-systems.[1,2] MRE proposed for this purpose is an energetic indicator of aromaticity in appearance but is derived theoretically from ring-current diamagnetism, which can be observed directly or indirectly by experiment.[27−29] This quantity proved to be useful as a reliable measure of aromaticity for a wide variety of π-systems, in the sense that MRE highly correlates in magnitude with TRE. This fact justifies not only the utility of the MRE concept but also that of the TRE one. MRE represents one of the very realistic ASEs, in the sense that no artificial reference structures is necessary to define it.

As stated previously,[28] MRE has an advantage in that it can be evaluated additively with respect to individual circuits. In this context, it may be worthy of mention that Gutman et al. attempted to attribute TRE explicitly to a set of circuits in a π-system.[49] In 1977 Bosanac and Gutman devised a hypothetical π-system in which cyclic motion of π-electrons around a given circuit is forbidden and then defined a kind of CRE for a polycyclic π-system.[50−52] Their CRE, called the cyclic conjugation energy (CCE), is indeed comparable in magnitude to our CRE.[28] Gutman et al. recently found that this way of reasoning is useful for attributing TRE to individual circuits and their combination.[49]

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