# Advances in Structural Analysis of Fluoroaluminates Using DFT Calculations of ${ }^{27} \mathrm{Al}$ Electric Field Gradients 

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#### Abstract

Based on the analysis of 23 aluminum sites from 16 fluoroaluminates, the present work demonstrates the strong potential of combining accurate NMR quadrupolar parameter measurements, density functional theory (DFT)-based calculations of electric field gradients (EFG), and structure optimizations as implemented in the WIEN2k package for the structural and electronic characterizations of crystalline inorganic materials. Structure optimizations are essential for compounds whose structure was refined from usually less accurate powder diffraction data and provide a reliable assignment of the ${ }^{27} \mathrm{Al}$ quadrupolar parameters to the aluminum sites in the studied compounds. The correlation between experimental and calculated EFG tensor elements leads to the proposition of a new value of the ${ }^{27} \mathrm{Al}$ nuclear quadrupole moment $Q\left({ }^{27} \mathrm{Al}\right)=1.616( \pm 0.024) \times 10^{-29}$ $\mathrm{m}^{2}$. The DFT calculations provide the orientation of the ${ }^{27} \mathrm{Al}$ EFG tensors in the crystal frame. Electron density maps support that the magnitude and orientation of the ${ }^{27} \mathrm{Al}$ EFG tensors in fluoroaluminates mainly result from the asymmetric distribution of the Al 3p orbital valence electrons. In most cases, the definition of relevant radial and angular distortion indices, relying on EFG orientation, allows correlations between these distortions and magnitude and sign of the $V_{i i}$.


## Introduction

The electric field gradient (EFG) is a ground state property of solids which sensitively depends on the asymmetry of the electronic charge density near the probe nucleus. The EFG is defined as the second derivative of the electrostatic potential at the nucleus position written as a traceless tensor. A nucleus with a nuclear spin number $I \geq 1$ has a nuclear quadrupole moment $(Q)$ that interacts with the EFG which originates from the nonspherical charge distribution surrounding this nucleus. This interaction determines the nuclear quadrupolar frequency

$$
\begin{equation*}
v_{\mathrm{Q}}=\frac{3 e Q V_{z z}}{2 I(2 I-1) h} \tag{1}
\end{equation*}
$$

and the asymmetry parameter

$$
\begin{equation*}
\eta_{\mathrm{Q}}=\frac{V_{y y}-V_{x x}}{V_{z z}} \tag{2}
\end{equation*}
$$

The $V_{i i}$ are the eigenvalues of the EFG tensor with the convention $\left|V_{z z}\right| \geq\left|V_{x x}\right| \geq\left|V_{y y}\right|$, $e$ is the electron charge, $I$ is the nuclear spin quantum number, and $h$ is the Planck constant.

[^0]The quadrupolar parameters can be measured with nuclear magnetic resonance (NMR), nuclear quadrupolar resonance (NQR), or Mössbauer spectroscopy. Thanks to recent development of high-resolution techniques such as satellite transition spectroscopy (SATRAS) ${ }^{1,2}$ and multiple quantum magic angle spinning (MQ-MAS), ${ }^{3}$ NMR is by far the most widely used technique to accurately determine quadrupolar parameters even for compounds with several crystallographic sites. Most of the time, the assignment of the NMR resonances to the crystallographic sites is achieved using the line intensity or chemical shielding. However, when it is not possible, another strategy is to compare the experimental values of the quadrupolar parameters with those obtained by calculations from structural data.
Blaha et al. ${ }^{4}$ showed that EFG could be calculated for large infinite solids described within periodic boundary conditions, using the Full-Potential Linearized Augmented Plane-Wave (FPLAPW) method. It was successfully applied to calculate the EFG tensors and the corresponding quadrupolar parameters measured by solid-state NMR for a large number of quadrupolar spin nuclei: ${ }^{7} \mathrm{Li},{ }^{11} \mathrm{~B},{ }^{14} \mathrm{~N},{ }^{17} \mathrm{O},{ }^{23} \mathrm{Na},{ }^{25} \mathrm{Mg},{ }^{27} \mathrm{Al},{ }^{33} \mathrm{~S},{ }^{35} \mathrm{Cl},{ }^{39} \mathrm{~K}$, ${ }^{43} \mathrm{Ca},{ }^{45} \mathrm{Sc},{ }^{47,49} \mathrm{Ti},{ }^{51} \mathrm{~V},{ }^{59} \mathrm{Co},{ }^{61} \mathrm{Ni},{ }^{63} \mathrm{Cu},{ }^{67} \mathrm{Zn},{ }^{69,71} \mathrm{Ga},{ }^{87} \mathrm{Sr},{ }^{91} \mathrm{Zr}$, ${ }^{95} \mathrm{Mo},{ }^{115} \mathrm{In},{ }^{137} \mathrm{Ba},{ }^{139} \mathrm{La},{ }^{187} \mathrm{Re} .{ }^{4-27}$
These density functional theory (DFT)-based calculations assisted the interpretation of complex solid-state NMR spectra. ${ }^{6,23,25}$ DFT was also used for structure optimization by minimization of the forces acting on the nuclei. It was shown that this approach combined with experimental NMR parameters of high precision leads to improved atomic coordinates of the
crystal structure. ${ }^{11,16,17,19,23-25}$ This approach may be particularly useful for crystal structures obtained from powder X-ray diffraction (XRD), containing both light and heavier elements, which are generally less accurate. ${ }^{19,23,25}$ Furthermore, the DFT calculations provide the sign of the EFG tensor eigenvalues and the orientation of the EFG tensor eigenaxes in the crystal frame ${ }^{7,17,24}$ which cannot be determined from NMR experiments on powdered samples. Then new information about the structural environment of the quadrupolar nucleus under consideration is obtained and is discussed in term of charge distribution and visualized on electron density maps. ${ }^{7,17}$

In this work we report on DFT-based calculations and interpretation of ${ }^{27} \mathrm{Al}$ quadrupolar parameters $v_{\mathrm{Q}}$ and $\eta_{\mathrm{Q}}$ of 23 aluminum sites in 16 fluoroaluminates. These parameters, used as a basis for structure refinement, were accurately determined in previous studies for $\alpha-\mathrm{AlF}_{3},{ }^{15}$ and compounds from the $\mathrm{NaF}-$ $\mathrm{AlF}_{3},{ }^{15} \mathrm{CaF}_{2}-\mathrm{AlF}_{3},{ }^{19,28}$ and $\mathrm{BaF}_{2}-\mathrm{AlF}_{3}{ }^{23,28}$ binary systems, and $\mathrm{NaF}-\mathrm{CaF}_{2}-\mathrm{AlF}_{3}{ }^{25}$ and $\mathrm{BaF}_{2}-\mathrm{CaF}_{2}-\mathrm{AlF}_{3}{ }^{28}$ ternary systems from ${ }^{27} \mathrm{Al}$ SATRAS and MQ-MAS NMR experiments. For $\alpha-\mathrm{AlF}_{3}, \alpha-\mathrm{Na}_{3} \mathrm{AlF}_{6}$ (cryolite), and $\mathrm{Na}_{5} \mathrm{Al}_{3} \mathrm{~F}_{14}$ (chiolite), ${ }^{27} \mathrm{Al}$ EFG calculations were previously performed by some of us without optimizing the structures. ${ }^{15}$ Very recently, in a study on aluminofluoride minerals, Zhou and co-workers have determined and calculated ${ }^{27} \mathrm{Al}$ quadrupolar parameters in the latter two compounds. ${ }^{26}$ Their experimental parameters are very close to those previously published. ${ }^{15}$ For $\beta-\mathrm{CaAlF}_{5}, \beta-\mathrm{Ba}_{3} \mathrm{AlF}_{9}$, and compounds from the $\mathrm{NaF}-\mathrm{CaF}_{2}-\mathrm{AlF}_{3}$ ternary system, the structure optimizations and ${ }^{27} \mathrm{Al} \mathrm{EFG}$ calculations have been recently published. ${ }^{19,23,25}$ On the other hand, the ${ }^{27} \mathrm{Al}$ quadrupolar parameters have been recently determined for eight compounds from the $\mathrm{CaF}_{2}-\mathrm{AlF}_{3}$ and $\mathrm{BaF}_{2}-\mathrm{AlF}_{3}$ binary systems and $\mathrm{BaF}_{2}-$ $\mathrm{CaF}_{2}-\mathrm{AlF}_{3}$ ternary system. ${ }^{28}$ However, the corresponding EFGs have never been calculated, preventing a complete assignment of the NMR lines to the ${ }^{27} \mathrm{Al}$ sites. After presenting the results of the calculations, we analyze the origin of the ${ }^{27} \mathrm{Al} \mathrm{EFG}$ tensors in the 16 fluoroaluminates under investigation in which $\mathrm{AlF}_{6}{ }^{3-}$ octahedra present a wide diversity of connectivities and environments: three-dimensionally connected octahedra ( $\alpha-\mathrm{AlF}_{3}{ }^{29}$ ), isolated chains of trans- $\left(\alpha-{ }^{30}\right.$ and $\beta$ - $\left.\mathrm{CaAlF}_{5}{ }^{19}\right)$ and cis-connected octahedra ( $\alpha-,{ }^{31} \beta$ - and $\gamma-\mathrm{BaAlF}_{5}{ }^{32}$ ), rings of four cis-connected octahedra $\left(\mathrm{Ba}_{3} \mathrm{Al}_{2} \mathrm{~F}_{12}{ }^{33}\right)$, layers of corner-sharing octahedra with four or two bridging fluorine atoms $\left(\mathrm{Na}_{5} \mathrm{Al}_{3} \mathrm{~F}_{14}{ }^{34}\right)$ and isolated $\mathrm{AlF}_{6}{ }^{3-}$ octahedra $\left(\alpha-\mathrm{Na}_{3} \mathrm{AlF}_{6}{ }^{35} \mathrm{Ca}_{2} \mathrm{AlF}_{7},{ }^{36} \mathrm{Ba}_{3} \mathrm{AlF}_{9}-\mathrm{Ib},{ }^{37} \beta-\mathrm{Ba}_{3}-\right.$ $\mathrm{AlF}_{9},{ }^{38} \alpha-\mathrm{BaCaAlF}_{7},{ }^{39} \alpha-{ }^{40}$ and $\beta-\mathrm{NaCaAlF}_{6},{ }^{41}$ and $\mathrm{Na}_{2} \mathrm{Ca}_{3}-$ $\mathrm{Al}_{2} \mathrm{~F}_{14}{ }^{42}$ ). By isolated we mean $\mathrm{AlF}_{6}{ }^{3-}$ octahedra not connected to other $\mathrm{AlF}_{6}{ }^{3-}$ octahedra. Except in $\alpha-\mathrm{AlF}_{3}$, all $\mathrm{AlF}_{6}{ }^{3-}$ octahedra are connected to various alkaline or alkaline-earth polyhedra. Moreover, the 23 aluminum studied sites present various symmetries, various $\mathrm{AlF}_{6}{ }^{3-}$ octahedron distortions, and a wide range of $\nu_{\mathrm{Q}}$ and $\eta_{\mathrm{Q}}$ parameters.

By combining the evaluation of the orientations of the EFG tensor elements in the crystal frame and the description of the electronic density around the aluminum atoms, a relationship is demonstrated between the ${ }^{27} \mathrm{Al}$ EFGs and chemical bonding in fluoroaluminates. It is commonly assumed that the magnitude of the quadrupolar coupling constant is related to the distortion of the polyhedron. A variety of distortion indices has been suggested ${ }^{43-45}$ and applied to tetrahedra and octahedra. They are defined over the whole polyhedron without any consideration of the EFG orientation. If the correlations are satisfactory for tetrahedra,,${ }^{9,44,46-53}$ they are less successful for octahedra. ${ }^{9,15,26,44,52,54-56}$ In the fluoroaluminates under study, we found that these distortion indices did not correlate at all with the ${ }^{27} \mathrm{Al}$ NMR quadrupolar parameters. We show that the knowledge of the

EFG orientations offers a new way to correlate sign and magnitude of the EFG with local $\mathrm{AlF}_{6}{ }^{3-}$ octahedron radial and angular distortions.

## Experimental Section

Computational Details. The DFT calculations used the fullpotential all-electron LAPW+lo approach ${ }^{57}$ which is implemented in the WIEN2k package. ${ }^{58}$ The exchange correlation potentials were calculated using the generalized gradient approximation (GGA). ${ }^{59}$ In the calculations, atomic sphere radii ( $\mathrm{R}_{\mathrm{MT}}$ ) of 1.60 a.u., 1.65 a.u., 1.65 a.u., 1.80 a.u., and 2.00 a.u. were used for $\mathrm{F}, \mathrm{Al}, \mathrm{Na}, \mathrm{Ca}$, and Ba , respectively. The core electron states were separated from the valence states by -6.0 Ry ( -7.0 Ry for barium containing compounds). By varying the total number of $k$-points in the Brillouin Zone, it was observed that $100 k$-points were sufficient to achieve a good convergence. Initial computations were performed using the crystallographic structures issued from X-ray or neutron diffraction, taking a plane wave cutoff defined by $R_{\mathrm{MT}} K_{\mathrm{MAX}}=5$ and $100 k$-points ( $K_{\mathrm{MAX}}$ is the magnitude of the largest Brillouin zone vector). In a second step, the structures were optimized with the WIEN2k code, by adjusting the atomic positions, keeping the experimental cell parameters unchanged, until the forces acting on all atoms are reduced to values lower than 2 mRy /a.u. A DFT structure optimization typically required five steps. The final calculations were performed at $R_{\mathrm{MT}} K_{\mathrm{MAX}}=8$ (1000 to 14000 plane waves, depending on the structures). For $\alpha-\mathrm{AlF}_{3}$, because of the small cell parameters, the final calculation was performed using $1000 k$-points.

## Results

Optimization Effect on Structures. Eleven WIEN2k optimized atomic coordinate sets unpublished till now are gathered as Supporting Information along with the initial structural data and the shifts of the atomic positions. As usually observed, ${ }^{19,23,25}$ the positions of the heavy atoms (i.e., $\mathrm{Ba}, \mathrm{Ca}$ ) do not vary much after optimization, while the lighter fluorine atoms show larger shifts. In most cases, the atomic position variations remain within the uncertainties inferred from the X-ray or neutron refinements. When the experimental data are obtained from single-crystal XRD, the resulting structures are very similar to the initial ones. On the contrary, the atomic positions of $\beta$ - and $\gamma$ - $\mathrm{BaAlF}_{5}$ are more significantly affected by the optimization: shifts of the atomic positions up to $0.184 \AA$ are observed for one fluorine atom (see Supporting Information). These compounds have 14 independent atoms, all of them in general positions. Thus, 42 atomic coordinates were refined from powder neutron diffraction data, ${ }^{32}$ leading to less accurate structures.

Since this paper deals with ${ }^{27} \mathrm{Al} \mathrm{EFG}$, the optimization effects are followed through the evolution of the $\mathrm{Al}-\mathrm{F}$ bond lengths and $\mathrm{F}-\mathrm{Al}-\mathrm{F}$ bond angles. Figures $1 \mathrm{a}, 1 \mathrm{~b}$, and 1 c show the minimum, mean, and maximum $\mathrm{Al}-\mathrm{F}$ bond lengths and $\mathrm{F}-\mathrm{Al}-\mathrm{F}$ angles between two adjacent $\mathrm{Al}-\mathrm{F}$ bonds, and between two opposite $\mathrm{Al}-\mathrm{F}$ bonds, respectively, for the initial and optimized structures for all studied compounds (values are reported as Supporting Information or in previous studies ${ }^{19,23}$ ). On the average, the $\mathrm{Al}-\mathrm{F}$ distances increase from 1.803 to 1.821 $\AA$. From Figure 1, it is obvious that the largest octahedron distortions, which are observed in $\beta$ - $\mathrm{CaAlF}_{5}, \beta$ - and $\gamma-\mathrm{BaAlF}_{5}$, $\beta-\mathrm{Ba}_{3} \mathrm{AlF}_{9}$, and $\alpha-\mathrm{NaCaAlF}_{6}$, are significantly reduced after optimization. Noticing that the structures of these five compounds have been determined on powdered samples from X-ray or neutron diffraction, it may be inferred that the large octahedron distortions are related to the lack of accuracy of the


Figure 1. Minimum, mean and maximum (a) $\mathrm{F}-\mathrm{Al}$ bond lengths, (b) $\mathrm{F}-\mathrm{Al}-\mathrm{F}$ angles between two adjacent $\mathrm{F}-\mathrm{Al}$ bonds and (c) $\mathrm{F}-\mathrm{Al}-\mathrm{F}$ angles between two opposite $\mathrm{F}-\mathrm{Al}$ bonds for the $(\square)$ initial, and ( $\boldsymbol{(})$ optimized structures of the studied compounds.
crystallographic structures. On the other hand, the small variations of the atomic positions in $\alpha-\mathrm{AlF}_{3}$ and $\beta-\mathrm{NaCaAlF}_{6}$ (see Supporting Information), whose structures were also refined from powder XRD data, are related to the few refined atomic coordinates.

From the optimized structures, the $\mathrm{AlF}_{6}{ }^{3-}$ octahedra appear quite rigid: dispersion of the $\mathrm{Al}-\mathrm{F}$ bond lengths is lower than $0.15 \AA$ and all $\mathrm{F}-\mathrm{Al}-\mathrm{F}$ angles between two adjacent $\mathrm{Al}-\mathrm{F}$ bonds range from 83.1 to $98.0^{\circ}$.
${ }^{27}$ Al EFGs and NMR Quadrupolar Parameters. The quality of the structural data for the fluoroaluminates under investigation may be assessed by comparison of the experimental ${ }^{27} \mathrm{Al}$ quadrupolar parameters with the EFG tensors obtained by DFT calculations. The ${ }^{27} \mathrm{Al}$ EFG tensors calculated before and
after structure optimization are given in Table 1, with the experimental quadrupolar parameters previously determined. ${ }^{15,19,23,25,28}$ Since the DFT calculations provide the sign of the EFG tensor elements, it is assumed that the experimental $v_{\mathrm{Q}}$ values exhibit the same sign as the corresponding calculated $V_{z z}$. For the multisite compounds, the assignments were performed with respect to the proportionality between $V_{z z}$ and $v_{\mathrm{Q}}$.

Before optimization, the agreement between calculated and experimental values for both quadrupolar frequencies $v_{\mathrm{Q}}$ and asymmetry parameters $\eta_{\mathrm{Q}}$ is not satisfactory, as shown in Figure 2, the discrepancy being larger for the structures determined by X-ray or neutron powder diffraction. A really improved agreement is obtained using the optimized structures. Two exceptions have to be mentioned: for $\alpha-\mathrm{AlF}_{3}$ and $\alpha-\mathrm{Na}_{3} \mathrm{AlF}_{6}$, a better agreement with the experimental values is obtained with the initial atomic positions. At room temperature, $\alpha-\mathrm{AlF}_{3}$ adopts a rhombohedral structure ( $\mathrm{R} \overline{3} \mathrm{c}$ ) and undergoes a first-order phase transition at approximately $450^{\circ} \mathrm{C}$ to the cubic $\alpha-\mathrm{ReO}_{3}$ structure $(\operatorname{Pm} \overline{3} \mathrm{~m}) .{ }^{29}$ In the high-temperature cubic phase, all the $\mathrm{AlF}_{6}{ }^{3-}$ octahedra are three-dimensionally connected via corner-shared fluorine atoms with $\mathrm{Al}-\mathrm{F}-\mathrm{Al}$ bond angles of $180^{\circ}$, while the octahedra are tilted along their $\overline{3}$ axis in the low-temperature rhombohedral phase, resulting in zigzag chains of octahedra and $\mathrm{Al}-\mathrm{F}-\mathrm{Al}$ bond angles of $157.1^{\circ}$ at room temperature. ${ }^{29}$ Similarly, cryolite $\mathrm{Na}_{3} \mathrm{AlF}_{6}$ is a "mixed-cation fluoride doubleperovskite", $\mathrm{Na}_{2}(\mathrm{NaAl}) \mathrm{F}_{6}$, in which the corner-sharing octahedron network is made up of alternating $\mathrm{AlF}_{6}{ }^{3-}$ and $\mathrm{NaF}_{6}{ }^{5-}$ octahedra. At ambient temperature, the monoclinic structure is distorted from the ideal cubic structure by a tilting of the octahedra, resulting in an 8 -fold coordination of the interstitial $\mathrm{Na}^{+}$ion, with the $\mathrm{Al}-\mathrm{F}-\mathrm{Na} 1$ bond angles dropping from an ideal value of $180^{\circ}$ (in a cubic perovskite) to $143.4-149.7^{\circ}$ in the room-temperature cryolite phase. ${ }^{35}$ Moreover, single crystal and powder $\mathrm{XRD}^{35,60}$ have shown that a fluctuation-induced first-order phase transition takes place in cryolite at $612{ }^{\circ} \mathrm{C}^{35}$ or $560{ }^{\circ} \mathrm{C}^{60}$ to form a high-temperature orthorhombic ${ }^{35}$ or cubic ${ }^{60}$ phase, involving a rotation of the nearly rigid $\mathrm{AlF}_{6}$ group. So in both low-temperature phases, the $\mathrm{Al}-\mathrm{F}-\mathrm{Al}(\mathrm{Na})$ bond angle can be used as one measure of the extent of tilting (or distortion). These angles decreased after optimization, corresponding to an increase of the distortions. The angles are equal to $153.1^{\circ}$ in $\alpha-\mathrm{AlF}_{3}$ and ranged from $143.0^{\circ}$ to $149.1^{\circ}$ in $\alpha-\mathrm{Na}_{3}{ }^{-}$ $\mathrm{AlF}_{6}$ (see Supporting Information). This seems to indicate that the optimized structures correspond to more distorted phases, which is consistent with the fact that WIEN2k tries to reach the lowest energy structure corresponding to 0 K . The optimized $T=0 \mathrm{~K}$ structure is different from the room temperature structure where the NMR measurements have been done, and together with the very small magnitude of the EFG, this may explain the discrepancies. For these reasons, as indicated in the caption of Figure 2, the EFG values used for these compounds are those calculated without optimization, at $R_{\mathrm{MT}} K_{\mathrm{MAX}}=8$. Mobility and/or dynamics effects could be also invoked to explain discrepancies between calculated and experimental EFGs. Obviously, such effects are not considered by the static DFT calculation. Nevertheless, the agreement between calculated and experimental quadrupolar parameters (Table 1) seems to indicate that these effects are quite negligible in the studied compounds.

The plot of the experimental $v_{\mathrm{Q}}$ as a function of the calculated $V_{z z}$ (Figure 2a) shows a convincing linear relationship. A fine agreement is also observed for the asymmetry parameter $\eta_{\mathrm{Q}}$ (Figure 2b) which is much more difficult to reproduce, ${ }^{6}$ because of its high sensitivity to variations of the atomic positions around

TABLE 1: Compound, Al Site, Site Symmetry, Experimental Quadrupolar Frequency $\boldsymbol{v}_{\mathrm{Q}, \exp }(\mathrm{kHz})$, Asymmetry Parameter $\eta_{\mathrm{Q}, \text { exp }}$, Calculated $V_{z z}\left(10^{21} \mathrm{~V} / \mathrm{m}^{2}\right)$ and Asymmetry Parameter $\eta_{\mathrm{Q}, \text { cal }}$, without (in Italic) and after Optimization, and Calculated Quadrupolar Frequency $\boldsymbol{v}_{\mathrm{Q}, \text { cal }}(\mathbf{k H z})$ after Optimization

| compound | Al site | sym | $\nu_{\text {Q,exp }}{ }^{a}$ | $\eta_{\mathrm{Q}, \mathrm{exp}}$ | $V_{z z}$ | $\eta_{\mathrm{Q}, \mathrm{cal}}$ | $V_{z z}$ | $\nu_{\text {Q,cal }}{ }^{i}$ | $\eta_{\mathrm{Q}, \mathrm{cal}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\alpha-\mathrm{AlF}_{3}$ | Al1 | $\overline{3}$ | $-32^{b}$ | $0^{b}$ | $-0.0701^{c}$ | $0.00^{c}$ | 0.0102 | -41 | 0.00 |
| $\alpha-\mathrm{Na}_{3} \mathrm{AlF}_{6}$ | Al1 | 1 | $-90^{b}$ | $0.9{ }^{\text {b }}$ | $-0.141^{c}$ | $0.86^{c}$ | 0.226 | -83 | 0.99 |
| $\mathrm{Na}_{5} \mathrm{Al}_{3} \mathrm{~F}_{14}$ | Al1 | 4/m | $-880^{b}$ | $0^{b}$ | $-1.75^{d}$ | $0.00^{d}$ | -1.52 | -890 | 0.00 |
|  | A12 | 2/m | $1200^{\text {b }}$ | $0.15{ }^{\text {b }}$ | $2.16{ }^{\text {d }}$ | $0.17^{\text {d }}$ | 2.02 | 1186 | 0.20 |
| $\alpha-\mathrm{CaAlF}_{5}$ | Al1 | 1 | $1580^{e}$ | $0.95{ }^{\text {e }}$ | 2.98 | 1.00 | 2.73 | 1598 | 0.93 |
| $\beta-\mathrm{CaAlF}_{5}{ }^{f}$ | Al1 | 1 | 1530 | 0.10 | 3.49 | 0.18 | 2.60 | 1524 | 0.09 |
| $\mathrm{Ca}_{2} \mathrm{AlF}_{7}$ | Al1 | m | $340{ }^{e}$ | $0.95{ }^{\text {e }}$ | -0.716 | 0.78 | 0.590 | 346 | 0.96 |
| $\alpha-\mathrm{BaAlF}_{5}$ | Al1 | 1 | $-1000^{e}$ | $0.09{ }^{e}$ | -2.29 | 0.12 | -1.61 | -944 | 0.12 |
| $\beta-\mathrm{BaAlF}_{5}$ | Al1 | 1 | $-550{ }^{e}$ | $0.45{ }^{\text {e }}$ | -0.903 | 0.95 | -0.929 | -544 | 0.47 |
|  | A12 | 1 | $-750{ }^{e}$ | $0.10^{e}$ | -1.03 | 0.38 | -1.24 | -725 | 0.05 |
| $\gamma-\mathrm{BaAlF}_{5}$ | Al1 | 1 | $-900^{e}$ | $0.15^{e}$ | -1.10 | 0.63 | -1.55 | -908 | 0.15 |
|  | A12 | 1 | $-1250{ }^{e}$ | $0.15{ }^{e}$ | -2.44 | 0.04 | -2.13 | -1247 | 0.17 |
| $\mathrm{Ba}_{3} \mathrm{Al}_{2} \mathrm{~F}_{12}$ | Al1 | 1 | $-560^{e}$ | $0.30^{e}$ | 1.10 | 0.29 | -0.938 | -550 | 0.33 |
| $\mathrm{Ba}_{3} \mathrm{AlF}_{9}-\mathrm{Ib}$ | Al1 | m | $75^{\circ}$ | $0.50{ }^{e}$ | 0.105 | 0.72 | 0.106 | 62 | 0.51 |
| $\beta-\mathrm{Ba}_{3} \mathrm{AlF}_{9}{ }^{\text {g }}$ | Al1 | 1 | -140 | 0.50 | 1.53 | 0.40 | $-0.376$ | -221 | 0.59 |
|  | Al2 | 2 | -210 | 0.85 | -0.555 | 0.16 | -0.416 | -244 | 0.84 |
|  | Al3 | 2 | 510 | 0.07 | 1.74 | 0.23 | 0.937 | 549 | 0.07 |
| $\alpha-\mathrm{BaCaAlF}{ }_{7}$ | Al1 | 1 | $-190^{e}$ | $0.80^{e}$ | -0.319 | 0.90 | -0.360 | -211 | 0.82 |
| $\alpha-\mathrm{NaCaAlF}_{6}$ | Al1 | 1 | $570^{h}$ | $0.25{ }^{h}$ | $-0.796^{h}$ | $0.95{ }^{h}$ | $0.980^{h}$ | 575 | $0.24{ }^{h}$ |
|  | A12 | 1 | $-440^{h}$ | $0.10^{h}$ | -0.915 | 0.85 | $-0.761^{h}$ | -446 | $0.11{ }^{h}$ |
| $\beta-\mathrm{NaCaAlF}_{6}$ | Al1 | 32 | $195^{h}$ | $0^{h}$ | 0.0469 | 0.00 | $0.375^{\text {c }}$ | 220 | $0.00^{c}$ |
|  | Al2 | 3 | $-60^{h}$ | $0^{h}$ | 0.0605 | 0.00 | $-0.102^{c}$ | -60 | $0.00^{c}$ |
| $\mathrm{Na}_{2} \mathrm{Ca}_{3} \mathrm{Al}_{2} \mathrm{~F}_{14}$ | Al1 | 3 | $65^{h}$ | $0^{h}$ | 0.126 | 0.00 | $0.132^{c}$ | 77 | $0.00^{c}$ |

${ }^{a}$ The sign of $\nu_{\mathrm{Q}, \exp }$ is the sign of $V_{z z}$ after optimization (except for $\alpha-\mathrm{AlF}_{3}$ and $\alpha-\mathrm{Na}_{3} \mathrm{AlF}_{6}$ ). ${ }^{b}$ From ref 15 . ${ }^{c}$ Recalculated in this work with WIEN2k06. ${ }^{d}$ Recalculated in this work with WIEN2k05. ${ }^{e}$ From ref $28 .{ }^{f}$ From ref $19 .{ }^{g}$ From ref 23. ${ }^{h}$ From ref $25 .{ }^{i} v_{\mathrm{Q}, \text { cal }}$ values are calculated from the ${ }^{27} \mathrm{Al}$ nuclear quadrupole moment obtained in this study: $Q\left({ }^{27} \mathrm{Al}\right)=1.616 \times 10^{-29} \mathrm{~m}^{2}$ and from the calculated main EFG elements, $V_{z z}$, after optimization (except for $\alpha-\mathrm{AlF}_{3}$ and $\alpha-\mathrm{Na}_{3} \mathrm{AlF}_{6}$ ).
the studied nucleus. From these linear correlations, the two ${ }^{27}$ Al NMR resonances ${ }^{28}$ of $\beta$ - and $\gamma-\mathrm{BaAlF}_{5}$ can now be assigned to their respective crystallographic sites (Table 1) as it was previously done for $\mathrm{Na}_{5} \mathrm{Al}_{3} \mathrm{~F}_{14},{ }^{15} \beta$ - $\mathrm{Ba}_{3} \mathrm{AlF}_{9},{ }^{19}$ and the two phases of $\mathrm{NaCaAlF}_{6}{ }^{25}$
${ }^{27} \mathrm{Al}$ Nuclear Quadrupole Moment. The ${ }^{27} \mathrm{Al}$ quadrupolar moment can be calculated from the slope of the linear regression (Figure 2a)

$$
\begin{equation*}
v_{\mathrm{Q}}=5.86( \pm 0.09) \times 10^{-16} V_{z z} \tag{3}
\end{equation*}
$$

using eq 1 . We obtain $Q\left({ }^{27} \mathrm{Al}\right)=1.616( \pm 0.024) \times 10^{-29} \mathrm{~m}^{2}$, which is higher than the reference value $Q\left({ }^{27} \mathrm{Al}\right)=1.466$ $( \pm 0.010) \times 10^{-29} \mathrm{~m}^{2}$ calculated by Pyykkö̈ ${ }^{61}$ as the average of the $Q$ values obtained for $\mathrm{AlF}, \mathrm{AlCl}$, and Al from microwave rotational spectra combined with EFG calculations. Nevertheless, the value we obtain is in agreement with the results of Iglesias et al. and Hansen et al. who compared experimental and calculated (WIEN code) quadrupolar parameters in polymorphics of $\mathrm{Al}_{2} \mathrm{SiO}_{5}\left(Q\left({ }^{27} \mathrm{Al}\right)=1.55 \times 10^{-29} \mathrm{~m}^{2}\right)^{7}$ and on $\mathrm{AlVO}_{4}$ $\left.\left(Q{ }^{27} \mathrm{Al}\right)=1.61 \times 10^{-29} \mathrm{~m}^{2}\right),{ }^{24}$ respectively. As previously mentioned by Iglesias et al., ${ }^{7}$ this suggests a larger value for the ${ }^{27} \mathrm{Al}$ quadrupolar moment. However, the other suggestion from Iglesias et al. incriminating a systematic Generalized Gradient Approximation (GGA) error in the description of the $\mathrm{Al}-\mathrm{X}$ bonding cannot be ruled out. As a matter of fact, the systematic increase of the $\mathrm{Al}-\mathrm{F}$ distances (from 1.803 to 1.821 A) after optimization (Figure 1a) may result in smaller EFG values and would consequently lead to a larger $Q$ value, but the perfect agreement for $\eta_{\mathrm{Q}}$, which is independent of $Q$, is a convincing argument for the accuracy of the present calculations.

Contribution from the Lattice and Valence Electrons to the ${ }^{27}$ Al EFG. We have applied an approach described in detail by Hansen et al., ${ }^{24}$ for $\mathrm{AlVO}_{4}$, where the total EFG is decomposed into contributions from the "lattice" (contributions
from outside the corresponding atomic sphere) and the "valence" electrons (from within the atomic sphere). The valence EFG can be further decomposed into p-p, s-d, p-f, and f-f partial wave contributions, and these contributions correspond to the different occupations of the corresponding orbitals (e.g. a larger/smaller occupation of $\mathrm{p}_{x}$ vs $\mathrm{p}_{z}$ orbitals). The results of this analysis are gathered as Supporting Information for all compounds under investigation. As expected and already observed by Hansen et al., ${ }^{24}$ the magnitude of the EFG tensors of ${ }^{27} \mathrm{Al}$ is dominated by the contributions of the valence electrons, and within this valence contribution, the main part is due to the p-p contributions, for all the Al sites.

## Discussion

Orientations of the ${ }^{27}$ Al EFG, Charge Densities, and $\mathrm{AlF}_{6}{ }^{3-}$ Octahedra Distortions. Ab initio calculations also provide the orientation of the EFG tensor in the crystal frame which cannot be obtained from NMR experiments on powdered samples. Thus, the WIEN2k calculations may be useful for structural interpretations of quadrupolar parameters determined from NMR studies of such samples. The three components of the directions of each calculated ${ }^{27} \mathrm{Al} \mathrm{EFG}$ tensor elements are provided as Supporting Information. The orientation of the ${ }^{27} \mathrm{Al}$ EFG tensor elements is illustrated in Figures 3-9, for typical situations among the 23 Al sites investigated. The illustrations for the remaining situations are gathered as Supporting Information. The first striking feature is that when the magnitude of $V_{z z}$ is larger than $0.75 \times 10^{21} \mathrm{~V} / \mathrm{m}^{2}$, the $V_{z z}$ direction is oriented along or nearly along two opposite $\mathrm{Al}-\mathrm{F}$ bonds. On the contrary, when the magnitude of $V_{z z}$ is lower than $0.60 \times 10^{21} \mathrm{~V} / \mathrm{m}^{2}$, the $V_{z z}$ direction is out of the $\mathrm{Al}-\mathrm{F}$ bonds.

Large $V_{z z}$ and Radial Distortions. For the aluminum sites with $V_{z z}$ larger than $0.75 \times 10^{21} \mathrm{~V} / \mathrm{m}^{2}$, Table 2 gathers the radial distortions, sorted in decreasing magnitude order, calculated as $d_{\mathrm{F}-\mathrm{Al}-\mathrm{F}}-<d>$ where $d_{\mathrm{F}-\mathrm{Al}-\mathrm{F}}$ is the sum of two opposite $\mathrm{Al}-\mathrm{F}$


Figure 2. ${ }^{27} \mathrm{Al}$ quadrupolar frequency versus calculated $V_{z z}$ and (b) experimental asymmetry parameters versus calculated ones, ( $\square$ ) before and $(\bullet)$ after optimization. (a) The solid line corresponds to a linear regression $v_{\mathrm{Q}}=5.86 \times 10^{-16} V_{z z}\left(R^{2}=0.999\right)$ after optimization (except for $\alpha-\mathrm{AlF}_{3}$ and $\alpha-\mathrm{Na}_{3} \mathrm{AlF}_{6}$ ). Error bars indicate uncertainties in quadrupolar frequency. (b) The solid line corresponds to a linear regression $\eta_{\mathrm{Q}, \text { exp. }}=0.991 \times \eta_{\mathrm{Q}, \text { cal }}\left(R^{2}=0.993\right)$ after optimization (except for $\alpha-\mathrm{AlF}_{3}$ and $\alpha-\mathrm{Na}_{3} \mathrm{AlF}_{6}$ ) and the dashed one to $\eta_{\mathrm{Q} \text {,exp. }}=$ $\eta_{\text {Q,cal }}$. Error bars indicate uncertainties in experimental asymmetry parameters.
bond lengths and $\langle d\rangle$ is the average value of the three $d_{\mathrm{F}-\mathrm{Al}-\mathrm{F}}$ in the $\mathrm{AlF}_{6}{ }^{3-}$ octahedron under consideration. $V_{z z}$ is associated with the $d_{\mathrm{F}-\mathrm{Al}-\mathrm{F}}$ value corresponding to the two opposite $\mathrm{Al}-\mathrm{F}$ bonds parallel to its direction (Figures 3-8). Except for $\mathrm{Ba}_{3}{ }^{-}$ $\mathrm{Al}_{2} \mathrm{~F}_{12}, V_{z z}$ is systematically related to the maximum value of $\left|d_{\mathrm{F}-\mathrm{Al}-\mathrm{F}}-<d>\right|$. Obviously, a $d_{\mathrm{F}-\mathrm{Al}-\mathrm{F}}-<d>$ positive (negative) value is related to a charge depletion (accumulation) in the $\mathrm{F}-\mathrm{Al}-\mathrm{F}$ direction. In all cases, $V_{z z}$ and $d_{\mathrm{F}-\mathrm{Al}-\mathrm{F}}-<d>$ have the same sign. So, $V_{z z}$ positive (negative) value is related to a charge depletion (accumulation). $V_{x x}$ and $V_{y y}$ are associated with the intermediate and the lowest $\left|d_{\mathrm{F}-\mathrm{Al}-\mathrm{F}}-\langle d\rangle\right|$ values, respectively. In Figure, $10 V_{z z}$ and $V_{x x}$ are plotted versus the

TABLE 2: Compound, Site, Sum of Two Opposite Al-F Bond Lengths $d_{\mathrm{F}-\mathrm{Al}-\mathrm{F}}(\mathrm{A}), \boldsymbol{d}_{\mathrm{F}-\mathrm{Al}-\mathrm{F}}-<d>$ (A) with $<d>$ the Mean $d_{\text {F-Al-F }}$ Distance, and Calculated ${ }^{27} \mathrm{Al}$ EFG Tensor $V_{i i}$ $\left(10^{21} \mathrm{~V} / \mathrm{m}^{2}\right)$ after WIEN2k Optimization

| compound | site | $d_{\text {F-Al-F }}$ | $d_{\text {F-Al-F }}-<d>$ | $V_{i i}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{Na}_{5} \mathrm{Al}_{3} \mathrm{~F}_{14}$ | Al1 | 3.599 | -0.008 | $V_{z z}=-1.52$ |
|  |  | 3.611 | 0.004 | $V_{x x}=0.76$ |
|  |  | 3.611 | 0.004 | $V_{y y}=0.76$ |
|  | Al2 | 3.673 | 0.041 | $V_{z z}=2.02$ |
|  |  | 3.611 | -0.021 | $V_{x x}=-1.21$ |
|  |  | 3.611 | -0.021 | $V_{y y}=-0.81$ |
| $\alpha-\mathrm{CaAlF}_{5}$ | Al1 | 3.748 | 0.110 | $V_{z z}=2.73$ |
|  |  | 3.530 | -0.107 | $V_{x x}=-2.63$ |
|  |  | 3.634 | -0.003 | $V_{y y}=-0.10$ |
| $\beta-\mathrm{CaAlF}_{5}$ | Al1 | 3.739 | 0.100 | $V_{z z}=2.60$ |
|  |  | 3.583 | -0.056 | $V_{x x}=-1.41$ |
|  |  | 3.595 | -0.044 | $V_{y y}=-1.19$ |
| $\alpha-\mathrm{BaAlF}_{5}$ | Al1 | 3.583 | -0.062 | $V_{z z}=-1.61$ |
|  |  | 3.680 | 0.034 | $V_{x x}=0.90$ |
|  |  | 3.674 | 0.028 | $V_{y y}=0.71$ |
| $\beta-\mathrm{BaAlF}_{5}$ | Al1 | 3.619 | -0.035 | $V_{z z}=-0.929$ |
|  |  | 3.683 | 0.029 | $V_{x x}=0.682$ |
|  |  | 3.661 | 0.006 | $V_{y y}=0.247$ |
|  | Al2 | 3.595 | -0.041 | $V_{z z}=-1.24$ |
|  |  | 3.657 | 0.021 | $V_{x x}=0.65$ |
|  |  | 3.656 | 0.020 | $V_{y y}=0.59$ |
| $\gamma-\mathrm{BaAlF}_{5}$ | Al1 | 3.574 | -0.062 | $V_{z z}=-1.55$ |
|  |  | 3.669 | 0.032 | $V_{x x}=0.89$ |
|  |  | 3.667 | 0.030 | $V_{y y}=0.65$ |
|  | Al2 | 3.545 | -0.097 | $V_{z z}=-2.13$ |
|  |  | 3.709 | 0.068 | $V_{x x}=1.24$ |
|  |  | 3.670 | 0.029 | $V_{y y}=0.88$ |
| $\mathrm{Ba}_{3} \mathrm{Al}_{2} \mathrm{~F}_{12}$ | Al1 | 3.650 | 0.013 | $V_{x x}=0.623$ |
|  |  | 3.627 | -0.010 | $V_{z z}=-0.938$ |
|  |  | 3.634 | -0.003 | $V_{y y}=0.315$ |
| $\beta-\mathrm{Ba}_{3} \mathrm{AlF}_{9}$ | Al3 | 3.714 | 0.053 | $V_{z z}=0.937$ |
|  |  | 3.634 | -0.027 | $V_{x x}=-0.502$ |
|  |  | 3.634 | -0.027 | $V_{y y}=-0.435$ |
| $\alpha-\mathrm{NaCaAlF}_{6}$ | Al1 | 3.709 | 0.057 | $V_{z z}=0.980$ |
|  |  | 3.631 | -0.036 | $V_{x x}=-0.610$ |
|  |  | 3.616 | -0.021 | $V_{y y}=-0.371$ |
|  | Al2 | 3.608 | -0.032 | $V_{z z}=-0.761$ |
|  |  | 3.652 | 0.020 | $V_{x x}=0.423$ |
|  |  | 3.659 | 0.012 | $V_{y y}=0.338$ |

$d_{\mathrm{F}-\mathrm{Al}-\mathrm{F}}-<d>$ associated values, demonstrating a linear relationship except for $\mathrm{Ba}_{3} \mathrm{Al}_{2} \mathrm{~F}_{12}$ and $\mathrm{Na}_{5} \mathrm{Al}_{3} \mathrm{~F}_{14}$ which will be discussed later.

To go further, we drew electronic density maps for particular $\mathrm{F}-\mathrm{Al}-\mathrm{F}$ planes. We begin the discussion with the case of $\alpha-\mathrm{CaAlF}_{5}$ for which all the $V_{i i}$ elements are oriented along the $\mathrm{Al}-\mathrm{F}$ bonds. $\alpha-\mathrm{CaAlF}_{5}$ contains isolated chains of $\mathrm{AlF}_{6}{ }^{3-}$ octahedra sharing trans-connected fluorine (F1) atoms. ${ }^{30}$ The symmetry of the Al site is $\overline{1} . V_{z z}$ is positive and nearly oriented along the longest $\mathrm{Al}-\mathrm{F} 1$ bond, $V_{x x}$ is negative and oriented along the shortest $\mathrm{Al}-\mathrm{F} 3$ bond, and $V_{y y}$, also negative, is oriented along the $\mathrm{Al}-\mathrm{F} 2$ bond (Figure 3a) whose length is intermediate between $\mathrm{Al}-\mathrm{F} 1$ and $\mathrm{Al}-\mathrm{F} 3$ ones (Table 2). Figure 3 b shows a contour map of the calculated valence electron density around the Al site in the $\mathrm{F} 1-\mathrm{Al}-\mathrm{F} 3$ plane, taking into account the following valence states $\mathrm{F}-2 \mathrm{p}, \mathrm{Al}-3 \mathrm{~s}+3 \mathrm{p}$, and $\mathrm{Ca}-4 \mathrm{~s}$. Large electron densities at F atoms are demonstrated which can be related to the nearly fully occupied 2 p orbitals. Small densities on Ca and Al atoms correspond to the 4 s and $3 \mathrm{~s}+3 \mathrm{p}$ orbitals, respectively, which are mostly unoccupied. A more precise picture of the overlap between Al and F orbitals can be seen in difference electron density $\Delta \rho$ maps. $\Delta \rho$ represents the difference between the crystalline electron density and the superposition of electron densities from the neutral atoms. In these figures, negative lines represent those places where the electron density is lower in the crystal than the superposition of neutral atoms


Figure 3. (a) Orientation of the calculated ${ }^{27} \mathrm{Al} \mathrm{EFG}$ tensor in $\alpha-\mathrm{CaAlF}_{5}$. The vector lengths are proportional to the magnitude of the contributions. $\mathrm{Al}-\mathrm{F}$ distances ( $\AA$ ) and F labels are indicated. (b) Valence electron density and difference electron density $(\Delta \rho$ ) in the (c) F1-Al-F3 and (d) $\mathrm{F} 2-\mathrm{Al}-\mathrm{F} 3$ plane in $\alpha-\mathrm{CaAlF}_{5}$. Atom labels are indicated. The contour intervals are in units of $0.05 \mathrm{e} . \AA^{-3}$. (c and d) Solid, dotted, and dashed lines correspond to positive, zero, and negative $\Delta \rho$, respectively.


Figure 4. (a) Orientation of the calculated ${ }^{27} \mathrm{Al} \mathrm{EFG}$ tensor in $\beta-\mathrm{CaAlF}_{5}$. The vector lengths are proportional to the magnitude of the contributions. $\mathrm{Al}-\mathrm{F}$ distances ( $\AA$ ) and F labels are indicated. Difference electron density $(\Delta \rho$ ) in the (b) F4-A1-F5 and (c) F3-Al-F5 plane in $\beta$-CaAlF5. Atom labels are indicated. The contour intervals are in units of $0.05 \mathrm{e}^{\AA} \AA^{-3}$. Solid, dotted, and dashed lines correspond to positive, zero, and negative $\Delta \rho$, respectively.
(e.g., the positively charged Al and Ca ions), while positive lines indicate those regions where the electron density is higher in the crystal than the superposition of neutral atoms (e.g., the negatively charged fluorine ions). ${ }^{7}$ For $\alpha-\mathrm{CaAlF}_{5}$ (Figures 3c and 3d), F atoms are highly nonspherical. Difference densities are predominantly positive at F atoms and negative at Al and Ca atoms. However, one can see a small negative difference density at F atoms in the directions toward Al due to some unoccupied antibonding F-p- $\sigma$ states. In other words, F is not
exactly a $\mathrm{F}^{-}$ion. Because of some covalent interaction, the bonding states have low energy and the antibonding F-p states, of high energy (above Fermi Energy), are partially unoccupied. Figures 3c and 3d show differences between the electronic clouds of F1 and F2 or F3 which are due to the local connections they have with their nearest neighbors: F1 bridges two aluminum atoms whereas F2 and F3 are nonbridging fluorine atoms and have a calcium atom as second neighbor. These differences come from the large redistribution of the F-charge,


Figure 5. Orientation of the calculated ${ }^{27} \mathrm{Al}$ EFG tensor in (a) Al1 and (b) Al2 in $\gamma$ - $\mathrm{BaAlF}_{5}$. For each, $\mathrm{Al}-\mathrm{F}$ distances ( $\AA$ ) and atom labels are indicated and the vector lengths are proportional to the magnitudes of the contributions. Difference electron density ( $\Delta \rho$ ) in the (c) F5-Al1-F9, (d) F6-A12-F10, (e) F3-A11-F9, and (f) F9-A12-F10 plane in $\gamma-$ BaAlF $_{5}$. Atom labels are indicated. The contour intervals are in units of 0.05 e $\AA^{-3}$. Solid, dotted, and dashed lines correspond to positive, zero, and negative $\Delta \rho$, respectively.


Figure 6. Orientation of the calculated ${ }^{27} \mathrm{Al} \mathrm{EFG}$ tensor for (a) $\mathrm{Al3}$ in $\beta-\mathrm{Ba}_{3} \mathrm{AlF}_{9}$, (b) All , and (c) $\mathrm{Al2}$ in $\alpha-\mathrm{NaCaAlF}_{6}$. For each, $\mathrm{Al}-\mathrm{F}$ distances $(\AA)$ and atom labels are indicated and the vector lengths are proportional to the magnitudes of the contributions.
which is moved away from the Al direction into the Ca direction. The lack of nonspherical density around Ca shows that the $\mathrm{F}-\mathrm{Ca}$ bond has a more ionic character than the $\mathrm{F}-\mathrm{Al}$ bond. Figures 3 c and 3 d also show that the electron density between Al and F atoms is higher and the $\mathrm{Al}-\mathrm{F}$ bond length shorter when F is a nonbridging atom. In that case, $V_{z z}$ is positive and oriented along the direction of the longest $d_{\mathrm{F}-\mathrm{Al}-\mathrm{F}}\left(d_{\mathrm{Fl}-\mathrm{Al}-\mathrm{F} 1}=3.748\right.$ $\AA$ ) corresponding to a depletion of charge, whereas $V_{x x}$ is negative and oriented along the direction of the shortest $d_{\mathrm{F}-\mathrm{Al}-\mathrm{F}}$ $\left(d_{\mathrm{F} 3-\mathrm{Al}-\mathrm{F} 3}=3.530 \AA\right)$ corresponding to an accumulation of charge. It may be outlined that $d_{\mathrm{F} 2-\mathrm{Al}-\mathrm{F} 2}(3.634 \AA)$ is nearly equal to the average of the two above-mentioned distances. The marked contrast between these distances, which is also found for the EFG tensor elements, appears to correlate with the alignment of these elements along the $\mathrm{Al}-\mathrm{F}$ bonds and results in an $\eta_{\mathrm{Q}}$ value close to 1 .
$\beta$-CaAlF 5 also presents isolated chains of $\mathrm{AlF}_{6}{ }^{3-}$ octahedra sharing trans-connected fluorine atoms. ${ }^{19}$ Six different $\mathrm{Al}-\mathrm{F}$
bond lengths result in a lower symmetry of the Al site (1 instead of $\overline{1}$ in $\alpha-\mathrm{CaAlF}_{5}$ ). F4 is the bridging fluorine atom and the $\mathrm{AlF}_{6}{ }^{3-}$ octahedra are elongated along the $\mathrm{F} 4-\mathrm{Al}-\mathrm{F} 4$ direction. As observed for $\alpha-\mathrm{CaAlF}_{5}, V_{z z}$ is positive and oriented along these $\mathrm{Al}-\mathrm{F} 4$ bonds (Figure 4a) corresponding to a charge depletion. This is in agreement with the two similar difference electron density maps in the $\mathrm{F} 4-\mathrm{Al}-\mathrm{F} 5$ plane for $\beta-\mathrm{CaAlF}_{5}$ (Figure 4 b ) and in the $\mathrm{F} 3-\mathrm{Al}-\mathrm{F} 1$ plane for $\alpha-\mathrm{CaAlF}_{5}$ (Figure 3c). $V_{x x}$ and $V_{y y}$ lie approximately in the plane of the $\mathrm{AlF}_{6}{ }^{3-}$ octahedra containing the nonbridging fluorine atoms (Figure 4a). In contrast with $\alpha-\mathrm{CaAlF}_{5}, V_{x x}$ and $V_{y y}$ are not directed along the $\mathrm{Al}-\mathrm{F}$ bonds, $V_{x x}$ being nearer the direction of the shortest $d_{\mathrm{F}-\mathrm{Al}-\mathrm{F}}\left(d_{\mathrm{F} 1-\mathrm{Al}-\mathrm{F} 5}\right)$. Despite significantly different $\mathrm{Al}-\mathrm{F}$ distances, the sums of two opposite bond lengths are indeed nearly identical $\left(d_{\mathrm{F} 3-\mathrm{Al}-\mathrm{F} 2}=3.596 \AA\right.$ and $\left.d_{\mathrm{Fl} 1-\mathrm{Al}-\mathrm{F} 5}=3.583 \AA\right)$ resulting in an asymmetry of the electronic density distribution around the aluminum atom: a concentration of charge between Al and F 1 and Al and F 2 and a depletion of charge between Al


Figure 7. Orientation of the calculated ${ }^{27} \mathrm{Al}$ EFG tensor for $\mathrm{Ba}_{3} \mathrm{Al}_{2} \mathrm{~F}_{12}$. $\mathrm{Al}-\mathrm{F}$ distances $(\AA)$ and atom labels are indicated. The vector lengths are proportional to the magnitudes of the contributions.
and F3 and Al and F5 (Figure 4c). The small difference between the two $\mathrm{F}-\mathrm{Al}-\mathrm{F}$ distances may also correlate with the closeness of the $V_{x x}$ and $V_{y y}$ values which results in a small $\eta_{\mathrm{Q}}$ parameter.

After discussing compounds containing isolated chains of trans-connected $\mathrm{AlF}_{6}{ }^{3-}$ octahedra, we continue with $\alpha-$-, ${ }^{31} \beta$-, and $\gamma-\mathrm{BaAlF}_{5}^{32}$ which present isolated chains of cis-connected octahedra and where the two bridging fluorine atoms are no longer along two opposite $\mathrm{Al}-\mathrm{F}$ bonds. $V_{z z}$ is negative and is nearly oriented along the direction corresponding to the shortest $\mathrm{F}-\mathrm{Al}-\mathrm{F}$ distance perpendicular to the $\mathrm{F}-\mathrm{Al}-\mathrm{F}$ plane containing these two bridging atoms. $V_{x x}$ and $V_{y y}$ are positive, lie in this plane and are not directed along the $\mathrm{Al}-\mathrm{F}$ bonds except for the Al 2 site in $\gamma-\mathrm{BaAlF}_{5}$ (Table 2, Figures 5a and 5b and Supporting Information). Thus, difference density maps are presented for the All and $\mathrm{Al2}$ sites in $\gamma-\mathrm{BaAlF}_{5}$ which correspond to the two cases encountered in the three $\mathrm{BaAlF}_{5}$ phases under investigation. Figure 5c presents the difference density map in the F5-Al1-F9 plane which contains the F9 bridging fluorine atom and the F1, F5, and F7 nonbridging fluorine atoms. In this case, $V_{z z}$ is directed along the F5-Al1-

F7 direction where a charge concentration is observed. Figure 5d shows a similar behavior in the $\mathrm{F} 6-\mathrm{Al} 2-\mathrm{F} 10$ plane with the F10 bridging atom and $V_{z z}$ along the F6-Al2-F8 direction. Figures 5 e and 5 f present difference density maps in the orthogonal planes, for Al1 and Al2, respectively. Except for Al 2 in $\gamma-\mathrm{BaAlF}_{5}$, the orientation of $V_{x x}$ and $V_{y y}$ may be explained, as for $\beta-\mathrm{CaAlF}_{5}$, by nearly identical $d_{\mathrm{F}-\mathrm{Al}-\mathrm{F}}$. As shown on the difference density maps, a charge concentration is observed between aluminum and nonbridging fluorine atoms while a depletion occurs between aluminum and bridging fluorine atoms, consistent with the closeness of the $V_{x x}$ and $V_{y y}$ values and with the small $\eta_{\mathrm{Q}}$ parameters. The particular behavior for Al 2 in $\gamma$ - $\mathrm{BaAlF}_{5}$ may be due to the large $\mathrm{d}_{\mathrm{F} 4-\mathrm{Al2}-\mathrm{Fl} 1}$.

Among the compounds listed in Table 2, three of them contain isolated $\mathrm{AlF}_{6}{ }^{3-}$ octahedra for which $V_{x x}$ and $V_{y y}$ are not directed along the $\mathrm{Al}-\mathrm{F}$ bonds. Two cases are analogous to $\beta-\mathrm{CaAlF}_{5}$ : $\mathrm{Al3}$ in $\beta-\mathrm{Ba}_{3} \mathrm{AlF}_{9}$ and All in $\alpha-\mathrm{NaCaAlF}_{6}$. The $V_{z z}$ elements lie nearly in the direction corresponding to the longest $d_{\mathrm{F}-\mathrm{Al}-\mathrm{F}}$ (Figures 6 a and 6 b ) in agreement with their positive values. On the other hand, for Al 2 in $\alpha-\mathrm{NaCaAlF}_{6}, V_{z z}$ lies nearly in the direction corresponding to the shortest $d_{\mathrm{F}-\mathrm{Al}-\mathrm{F}}$ (Figure 6c) in agreement with a negative value, as observed in $\alpha-\mathrm{BaAlF}_{5}$, $\beta-\mathrm{BaAlF}_{5}$ and All in $\gamma-\mathrm{BaAlF}_{5}$. For these isolated octahedra, the $\eta_{\mathrm{Q}}$ values are rather small and in agreement with equal (A13 in $\beta-\mathrm{Ba}_{3} \mathrm{AlF}_{9}$ ) or very close ( All and $\mathrm{Al2}$ in $\alpha-\mathrm{NaCaAlF}_{6}$ ) $d_{\mathrm{F}-\mathrm{Al}-\mathrm{F}}$ in the equatorial plane perpendicular to $V_{z z}$.

For all cases described up to now, we clearly demonstrate the influence of the radial distortions $d_{\mathrm{F}-\mathrm{Al}-\mathrm{F}}-<d>$ on the charge density distribution around the aluminum atoms which induces magnitude and sign of the EFG tensor elements (Figure 10) and enforces the direction of $V_{z z}$. Furthermore the larger the difference between the $d_{\mathrm{F}-\mathrm{Al}-\mathrm{F}}$ in the equatorial plane, the closer $V_{x x}$ to one $\mathrm{F}-\mathrm{Al}-\mathrm{F}$ direction corresponding to the shortest $d_{\mathrm{F}-\mathrm{Al}-\mathrm{F}}$ for the trans-connected octahedra, the longest $d_{\mathrm{F}-\mathrm{Al}-\mathrm{F}}$ for the cis-connected octahedra.

Let us now discuss the three exceptions, Al 1 and Al 2 in $\mathrm{Na}_{5}-$ $\mathrm{Al}_{3} \mathrm{~F}_{14}$ and $\mathrm{Ba}_{3} \mathrm{Al}_{2} \mathrm{~F}_{12}$, for which the previously mentioned correlation does not hold. $\mathrm{Ba}_{3} \mathrm{Al}_{2} \mathrm{~F}_{12}$ presents rings of four cisconnected octahedra. ${ }^{33}$ As already observed for compounds exhibiting the same kind of octahedron connectivity, $V_{z z}$ is perpendicular to the plane containing the two bridging fluorine

TABLE 3: Compound, Site, $d_{\mathrm{F}-\mathrm{Al}-\mathrm{F}}-<d>(\AA)$, Calculated ${ }^{27} \mathrm{Al}$ EFG Tensor $V_{i i}\left(10^{21} \mathrm{~V} / \mathrm{m}^{2}\right)$ after WIEN2k Optimization (except for $\alpha-\mathrm{AlF}_{3}$ and $\alpha-\mathrm{Na}_{3} \mathrm{AlF}_{6}$ ), and Angular Distortions $\alpha$ and $\beta$ (deg)

| compound | site | $d_{\mathrm{F}-\mathrm{Al}-\mathrm{F}}-<d>$ | $V_{i i}$ orthogonal to opposite faces | $\alpha$ | $V_{i i}$ bisector of two adjacent Al-F bonds | $\beta$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\alpha-\mathrm{AlF}_{3}$ | Al1 | 0. | $V_{z z}=-0.0701$ | 0.02 | $V_{x x}=V_{y y}=0.0350$ | -0.02 |
| $\alpha-\mathrm{Na}_{3} \mathrm{AlF}_{6}$ | Al1 | 0.0084 |  |  | $V_{z z}=-0.141$ | -0.6 |
|  |  | -0.0080 |  |  | $V_{x x}=0.131$ | 0.6 |
|  |  | -0.0004 |  |  |  |  |
| $\mathrm{Ca}_{2} \mathrm{AlF}_{7}$ | Al1 | -0.0256 | $V_{z z}=0.590$ | -1.6 |  |  |
|  |  | 0.0128 | $V_{x x}=-0.577$ | 1.5 |  |  |
|  |  | 0.0128 |  |  |  |  |
| $\mathrm{Ba}_{3} \mathrm{AlF}_{9}-\mathrm{Ib}$ | Al1 | 0.0031 | $V_{x x}=-0.080$ | -0.2 | $V_{z z}=0.106$ | 0.4 |
|  |  | -0.0016 |  |  |  |  |
|  |  | -0.0016 |  |  |  |  |
| $\beta-\mathrm{Ba}_{3} \mathrm{AlF}_{9}$ | Al1 | 0.0176 |  |  | $V_{z z}=-0.376$ | 0.8 |
|  |  | -0.0155 |  |  | $V_{x x}=0.299$ | -0.9 |
|  |  | -0.0022 |  |  |  |  |
|  | Al2 | $0.0073$ | $V_{z z}=-0.416$ | -0.1 | $V_{x x}=0.383$ | 2.6 |
|  |  | -0.0037 |  |  |  |  |
|  |  | -0.0037 |  |  |  |  |
| $\alpha-\mathrm{BaCaAlF}{ }_{7}$ | Al1 | -0.0035 | $V_{z z}=-0.360$ | $-1.3$ |  |  |
|  |  | 0.0028 | $V_{x x}=0.328$ | 1.4 |  |  |
|  |  | 0.0007 |  |  |  |  |
| $\beta-\mathrm{NaCaAlF}_{6}$ | Al1 | 0. | $V_{z z}=0.375$ | 1.8 | $V_{y y}=-0.187$ | -1.6 |
|  | Al2 | 0. | $V_{z z}=-0.102$ | -0.1 | $V_{y y}=0.051$ | 0.1 |
| $\mathrm{Na}_{2} \mathrm{Ca}_{3} \mathrm{Al}_{2} \mathrm{~F}_{14}$ | Al1 | 0. | $V_{z z}=0.132$ | 0.6 | $V_{x x}=-0.066$ | -0.5 |



Figure 8. Orientation of the calculated ${ }^{27} \mathrm{Al} \mathrm{EFG}$ tensor for (a) All and (b) $\mathrm{Al2}$ in $\mathrm{Na}_{5} \mathrm{Al}_{3} \mathrm{~F}_{14}$. For each, $\mathrm{Al}-\mathrm{F}$ distances ( $\AA$ ) and atom labels are indicated and the vector lengths are proportional to the magnitudes of the contributions. Difference electron density ( $\Delta \rho$ ) in the (c) F1-A11-F2, (d) F2-Al1-F2, and (e) F2-Al2-F3 planes in $\mathrm{Na}_{5} \mathrm{Al}_{3} \mathrm{~F}_{14}$. Solid, dotted, and dashed lines correspond to positive, zero, and negative $\Delta \rho$, respectively. Atom labels are indicated. The contour intervals are in units of $0.05 \mathrm{e}^{-3} \AA^{-3}$.
( F 1 and F 2 ) and the aluminum atoms and parallel to the direction of $d_{\mathrm{F} 6-\mathrm{Al}-\mathrm{F} 8}$, which is the shortest $d_{\mathrm{F}-\mathrm{Al}-\mathrm{F}}$ in this $\mathrm{AlF}_{6}{ }^{3-}$ octahedron (Figure 7). $V_{x x}$ and $V_{y y}$ are not directed along the $\mathrm{Al}-\mathrm{F}$ bonds. In contrast to the other compounds exhibiting octahedron cis-connectivity, the shortest $d_{\mathrm{F}-\mathrm{Al}-\mathrm{F}}$ related to $V_{z z}$ does not correspond to the largest $\left|d_{\mathrm{F}-\mathrm{Al}-\mathrm{F}}-\langle d\rangle\right|$ value. It may be due to geometrical constraints in the original tetramer ring which contains rather short bonds between Al and bridging F1 atoms. At that point of the discussion, it may be outlined that for the cis-connected octahedra, the $\eta_{\mathrm{Q}}$ parameter values are smaller than 0.5 which mirror small differences between the sums of two opposite bonds in the plane containing the two bridging fluorine atoms.

The chiolite $\mathrm{Na}_{5} \mathrm{Al}_{3} \mathrm{~F}_{14}$ consists of layers of corner-sharing $\mathrm{AlF}_{6}{ }^{3-}$ octahedra which contain two types of $\mathrm{AlF}_{6}{ }^{3-}$ octahedra in the ratio $1: 2 . \mathrm{AllF}_{6}{ }^{3-}$ contains four bridging fluorine atoms
(Figure 8a), and the symmetry of the All site is $4 / \mathrm{m}$. $\mathrm{Al2F}_{6}{ }^{3-}$ contains two trans-connected fluorine atoms (Figure 8b) and the symmetry of the Al2 site is $2 / \mathrm{m} .{ }^{34}$ For Al1, $V_{z z}$ is parallel to the fourfold axis (F1-A11-F1 direction) which is directed along the c crystallographic axis. $V_{x x}$ and $V_{y y}$ are equal resulting in $\eta_{\mathrm{Q}}=0$ in agreement with the $4 / \mathrm{m}$ symmetry. They are oriented along the $a$ and the $b$ crystallographic axes, respectively. Despite small radial distortions, the magnitude of the EFG tensor elements is high. $V_{z z}$ is negative, in agreement with a charge concentration in the $\mathrm{F} 1-\mathrm{Al}-\mathrm{F} 1$ direction as demonstrated in the difference density map in the $\mathrm{F} 1-\mathrm{Al} 1-\mathrm{F} 2$ plane (Figure 8c). Figure 8d shows the four-fold symmetry in the equatorial plane where the presence of four bridging fluorine atoms results in strong charge depletion around the All atom. The present case demonstrates that the charge density distributions around aluminum atoms mostly arise from the different nature (bridging


Figure 9. Orientation of the calculated ${ }^{27} \mathrm{Al}$ EFG tensor for (a) A 12 in $\beta-\mathrm{NaCaAlF}_{6}$ and (b) in $\alpha-\mathrm{BaCaAlF}_{7}$. For each, $\mathrm{Al}-\mathrm{F}$ distances ( $\AA$ ) and atom labels are indicated and the vector lengths are proportional to the magnitudes of the contributions.
or nonbridging) of the fluorine atoms in the $\mathrm{AlF}_{6}{ }^{3-}$ octahedron and are not systematically related to radial distortions. This case questions the statement commonly assumed in the literature: "the larger the EFG, the larger the polyhedron distortions". For Al2, $V_{y y}$ is parallel to the two-fold axis directed along the $c$ crystallographic axis and bisecting the largest F3-A12-F3 angles in the equatorial plane. $V_{z z}$ is perpendicular to this plane and nearly oriented along the F2-A12-F2 direction. The sign of these EFG tensor elements (Table 2) is in agreement with the radial distortions and with charge density distributions shown in Figure 8e: a charge depletion along F2-A12-F2 and a charge concentration along F3-A12-F3. Nevertheless, as for All, the magnitude of the EFG tensor elements does not correlate with the radial distortions (Table 2 and Figure 10). Despite equal $\mathrm{Al}-\mathrm{F}$ distances in the equatorial plane, $V_{x x}$ is not equal to $V_{y y}$ and $\eta_{\mathrm{Q}}$ is different from zero, highlighting the influence of the angular distortion on the charge density distribution. $V_{y y}$ bisects the largest $\mathrm{F} 3-\mathrm{Al} 2-\mathrm{F} 3$ angles $\left(91.8^{\circ}\right)$. Then, in this direction, one may expect a smaller charge concentration than along the $V_{x x}$ direction bisecting the smallest F3-A12-F3 angles $\left(88.2^{\circ}\right)$. In the compounds under investigation previously mentioned the influence of the angular distortion is hidden by the large radial distortions.

Small $V_{z z}$ and Angular Distortions. Now we move to the discussion of the compounds where $\mathrm{AlF}_{6}{ }^{3-}$ octahedra are characterized by magnitude of $V_{z z}$ smaller than $0.60 \times 10^{21} \mathrm{~V} / \mathrm{m}^{2}$ (Table 1). The $\mathrm{AlF}_{6}{ }^{3-}$ octahedra which contain fluorine atoms either all bridging $\left(\alpha-\mathrm{AlF}_{3}\right)$ or all nonbridging present small radial distortions (Table 3) related to small charge concentrations and depletions, which could explain why the EFG directions are out of the Al-F bonds (Figure 9 and Supporting Information).

In this situation, angular distortions should play a significant role. Among these compounds, three of them contain $\mathrm{AlF}_{6}{ }^{3-}$ octahedra with a three-fold axis on the aluminum sites: $\alpha-\mathrm{AlF}_{3},{ }^{29}$ $\beta-\mathrm{NaCaAlF}_{6},{ }^{41}$ and $\mathrm{Na}_{2} \mathrm{Ca}_{3} \mathrm{Al}_{2} \mathrm{~F}_{14} .{ }^{42}$ The aluminum site symmetries impose the $V_{z z}$ direction along the three-fold axis, i.e., orthogonal to opposite faces, $\eta_{\mathrm{Q}}=0$ and no radial distortions. $V_{x x}$ and/or $V_{y y}$ bisect or nearly bisect the angle between two adjacent $\mathrm{Al}-\mathrm{F}$ bonds for the four sites under consideration (Figure 9a, Supporting Information, and Table 3). For the other compounds, $\alpha-\mathrm{Na}_{3} \mathrm{AlF}_{6}, \mathrm{Ca}_{2} \mathrm{AlF}_{7}, \mathrm{Ba}_{3} \mathrm{AlF}_{9}-\mathrm{Ib}, \beta-\mathrm{Ba}_{3} \mathrm{AlF}_{9}(\mathrm{All}$ and Al 2 ), and $\alpha-\mathrm{BaCaAlF}_{7}$, two of the main tensor element directions are almost orthogonal to opposite faces and/or nearly bisect the angle between two adjacent $\mathrm{Al}-\mathrm{F}$ bonds (Figure 9b, Supporting Information, and Table 3). Thus, the angular distortions are measured through two parameters depending on the $V_{i i}$ direction, defined as

$$
\alpha=\frac{1}{6} \sum_{i=1}^{6} \alpha_{i}-90^{\circ} \text { and } \beta=\frac{1}{2} \sum_{i=1}^{2} \beta_{i}-90^{\circ}
$$



Figure 10. $V_{z z}$ and $V_{x x}$ EFG tensor elements vs radial distortion $d_{\mathrm{F}-\mathrm{Al}-\mathrm{F}}$ $-<d>$ for Al sites with an absolute value of $V_{z z}$ larger than $0.75 \times$ $10^{21} \mathrm{~V} / \mathrm{m}^{2}$. The solid line corresponds to the linear regression $(a=$ $23.6 \times 10^{31} \mathrm{~V} \mathrm{~m}^{-3}, b=-0.029 \times 10^{21} \mathrm{~V} \mathrm{~m}^{-2}, R^{2}=0.984$ ) for which $\mathrm{Na}_{5} \mathrm{Al}_{3} \mathrm{~F}_{14}$ and $\mathrm{Ba}_{3} \mathrm{Al}_{2} \mathrm{~F}_{12}$ are not taken into account (the corresponding symbols are surrounded by circles).


Figure 11. $V_{i i} \mathrm{EFG}$ tensor elements vs angular distortion $\alpha$ for some Al sites with magnitude of $V_{z z}$ smaller than $0.60 \times 10^{21} \mathrm{~V} / \mathrm{m}^{2}$. The solid line corresponds to the linear regression $\left(a=0.246 \times 10^{21} \mathrm{~V}\right.$ $\left.\mathrm{m}^{-2} \mathrm{deg}^{-1}, b=-0.038 \times 10^{21} \mathrm{~V} \mathrm{~m}^{-2}, R^{2}=0.993\right)$.
where $\alpha_{i}$ are the six angles between two adjacent $\mathrm{Al}-\mathrm{F}$ bonds involving fluorine atoms belonging to octahedron faces orthogonal to the $V_{i i}$ direction, and $\beta_{i}$ the two angles between two adjacent $\mathrm{Al}-\mathrm{F}$ bonds which $V_{i i}$ bisects. When there is no radial distortion, a positive (negative) angular distortion corresponds to a charge depletion (concentration) in the $V_{i i}$ direction and then to a positive (negative) $V_{i i}$ value. Table 3 gathers the $\alpha$ and $\beta$ values for the ten sites under consideration. For the smallest radial distortions encountered in $\beta-\mathrm{NaCaAlF}_{6}, \mathrm{Na}_{2} \mathrm{Ca}_{3}{ }^{-}$ $\mathrm{Al}_{2} \mathrm{~F}_{14}, \mathrm{Ba}_{3} \mathrm{AlF}_{9}-\mathrm{Ib}$, and $\alpha-\mathrm{BaCaAlF}{ }_{7}$, a linear correlation exists between $V_{i i}$ and the angular distortion parameters as shown for $\alpha$ in Figure 11. For $\alpha-\mathrm{AlF}_{3}$, the discrepancy is probably due to the very small value of the angular distortion which makes the correlation questionable. For intermediate radial distortions ( $\alpha$ $\mathrm{Na}_{3} \mathrm{AlF}_{6}$ and Al 2 in $\beta-\mathrm{Ba}_{3} \mathrm{AlF}_{9}$ ), only the signs of the $V_{i i}$ are in agreement with the signs of the angular distortions. For the largest radial distortions $\left(\mathrm{Ca}_{2} \mathrm{AlF}_{7}\right.$ and All in $\beta$ - $\left.\mathrm{Ba}_{3} \mathrm{AlF}_{9}\right)$, no correlation at all can be found. The lack of quantitative correlation may be explained by the superposition between both angular and radial distortions.

The present analysis which relies on optimized structures shows that the EFG orientation is the essential information to define the relevant distortion index. When $V_{z z}$ is oriented along an $\mathrm{Al}-\mathrm{F}$ bond, $V_{i i}$ values correlate with radial distortions (Figure
10). Obviously, since the correlation between $V_{i i}$ and the radial distortion indices is fine, the $\eta_{\mathrm{Q}}$ parameter can be predicted from these indices. When $V_{i i}$ are out of $\mathrm{Al}-\mathrm{F}$ bonds and the radial distortions are small, $V_{z z}$ values correlate with angular distortions (Figure 11).

## Conclusions

The present work demonstrates that accurate NMR quadrupolar parameters represent valuable experimental data for evaluation and refinement of inorganic structures when combined with high level DFT calculations and structure optimizations as implemented in the WIEN2k package. This is illustrated for 16 fluoroaluminates which exhibit 23 aluminum sites. Optimizations are essential for compounds whose structure was refined from powder diffraction data usually less accurate. The shifts of the atomic positions lead to reduced octahedron distortions. The optimized structures provide a reliable assignment of the ${ }^{27} \mathrm{Al}$ quadrupolar parameters to the aluminum sites in the studied compounds. The correlation between experimental and calculated EFG tensor elements allows the determination of a new value of the ${ }^{27} \mathrm{Al}$ nuclear quadrupole moment with an improved accuracy $Q\left({ }^{27} \mathrm{Al}\right)=1.616( \pm 0.024) \times 10^{-29} \mathrm{~m}^{2}$.

Moreover the DFT calculations provide the orientation of the ${ }^{27} \mathrm{Al}$ EFG tensors in the crystal frame and allow a quantitative interpretation of the tensor element orientations and magnitudes in terms of electron densities and octahedron distortions. Electron density maps support that the magnitude and orientation of the ${ }^{27} \mathrm{Al} \mathrm{EFG}$ tensors in fluoroaluminates mainly result from the asymmetric distribution of the $\mathrm{Al} \mathrm{3p}$ orbital valence electrons.

For most of the aluminum sites with values of $V_{z z}$ higher than $0.75 \times 10^{21} \mathrm{~V} / \mathrm{m}^{2}$, the sorted $V_{i i}$ tensor elements are directly proportional to the three sorted $d_{\mathrm{F}-\mathrm{Al}-\mathrm{F}}-<d>$ radial distortions. Charge concentrations or depletions are located along the $V_{z z}$ direction corresponding to the highest $\left|d_{\mathrm{F}-\mathrm{Al}-\mathrm{F}}-\langle d\rangle\right|$. The largest EFGs are observed for isolated chains of cornersharing $\mathrm{AlF}_{6}{ }^{3-}$ octahedra involving two bridging fluorine atoms.

For sites with values of $V_{z z}$ lower than $0.60 \times 10^{21} \mathrm{~V} / \mathrm{m}^{2}$, the $\mathrm{AlF}_{6}{ }^{3-}$ octahedra are isolated (except $\alpha-\mathrm{AlF}_{3}$ ) and all the $V_{i i}$ are out the $\mathrm{Al}-\mathrm{F}$ bonds. Then, the angular distortions have to be taken into account. When the radial distortions are very small, the definition of relevant indices allows correlation between angular distortions, which are the predominant factor, and magnitude and sign of the tensor elements.

To conclude, this study shows the strong potential of combining EFG DFT calculations and accurate quadrupolar NMR parameter measurements for the structural and electronic characterizations of crystalline inorganic materials.

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Supporting Information Available: Space groups, unit cell dimensions, and structure determination method of the compounds under investigation, initial fractional atomic coordinates, WIEN2k optimized fractional atomic coordinates and corresponding atomic displacements, $\mathrm{Al}-\mathrm{F}$ bond lengths, $\mathrm{F}-\mathrm{Al}-\mathrm{F}$ and $\mathrm{Al}-\mathrm{F}-\mathrm{Al}$ bond angles as deduced from X-ray or neutron structure refinement and WIEN2k optimization, contributions from the lattice and valence-electron parts of the calculated main EFG elements, components of the directions of the calculated ${ }^{27} \mathrm{Al}$ EFG tensor elements after WIEN2k optimization (except $\alpha-\mathrm{AlF}_{3}$ et $\alpha-\mathrm{Na}_{3} \mathrm{AlF}_{6}$ ) expressed in the crystallographic axis for
the studied compounds, and orientation of the calculated ${ }^{27} \mathrm{Al}$ EFG tensor for $\alpha-\mathrm{BaAlF}_{5}, \mathrm{All}$ and $\mathrm{Al2}$ in $\beta-\mathrm{BaAlF}_{5}, \alpha-\mathrm{AlF}_{3}$, All in $\beta$ - $\mathrm{NaCaAlF}_{6}, \mathrm{Na}_{2} \mathrm{Ca}_{3} \mathrm{Al}_{2} \mathrm{~F}_{14}, \alpha-\mathrm{Na}_{3} \mathrm{AlF}_{6}, \mathrm{Ca}_{2} \mathrm{AlF}_{7}, \mathrm{All}$ and Al 2 in $\beta-\mathrm{Ba}_{3} \mathrm{AlF}_{9}$, and $\mathrm{Ba}_{3} \mathrm{AlF}_{9}-\mathrm{Ib}$. This material is available free of charge via the Internet at http://pubs.acs.org.

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