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Metal–Insulator Transitions: New Phenomena, New Questions

T. V. Ramakrishnan[1]

Department of Physics, Indian Institute of Science, Bangalore 560 012, India

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IN HONOR OF C. N. R. RAO ON HIS 60TH BIRTHDAY

The metal-to-insulator transition has reemerged as one of the major concerns of condensed matter science, due partly to the discovery of high temperature superconductivity in doped correlation induced insulators. A number of related quantum ($T = 0$) phenomena such as the superconductor–insulator and quantum Hall liquid–insulator transitions have been identified and explored. A considerable body of theoretical work also exists now. Some of this work and the questions it raises will be discussed here. © 1994 Academic Press, Inc.

1. INTRODUCTION AND BACKGROUND

1.1 Introduction

Almost a quarter of a century ago, at the Indian Institute of Technology, Kanpur, I began to learn that novel physical phenomena such as metal–insulator transitions, variable range hopping, minimum metallic conductivity, and high spin–low spin transitions are actually found in transition metal oxides which were being synthesized and explored by Professor C. N. R. Rao. That one-way traffic has continued since. In the spirit of Professor Rao's lifelong pursuit of what is new and interesting in this large area, I present here a partial view of recent happenings in the field of metal-to-insulator transitions, and try to highlight the questions that arise.

Partly due to the discovery of high temperature superconductivity in layered cuprates, there has been a great surge of interest in exploring further the passage from the insulating to the metallic state (1, 2). It is worth noting that the parent compound of high T_c superconductors, i.e., La_2CuO_4, was first identified by Ganguly and Rao (3) to be a correlation driven insulator with an antiferromagnetic ground state and a nearly two dimensional structure. Doping it with Sr produces a high temperature superconductor. The proximity of these two phases is very suggestive. Historically, the Ganguly–Rao work belongs to the previous active phase of this subject (1968–1975). In spite of considerable experimental exploration of new systems and new phenomena as well as development of concepts and methods in the present phase, our understanding is far from complete. Nearly a decade ago, Anderson wrote (4), "Metal–insulator transitions and the whole subject of the magnetic state and strongly interacting fermions constitute a set of material which I found I could never finish without its being continually superseded by some major development. I suppose this area should simply be postponed for a totally separate book. It is an important, surprisingly recalcitrant and exciting subject which has not yet gotten itself into an order that can be explained in any short space; I leave it out with regret."

This remains equally true today, as will be apparent.

In what follows, we will first mention briefly a few new classes of systems where the metal-to-insulator transition occurs. We then summarize recent extensive low temperature measurements of the continuous metal-insulator transition region in a number of oxides carried out by A. K. Raychaudhuri and co-workers at the Indian Institute of Science (5). With increasing disorder, a superconductor turns insulator (6, 7). The nature of this transition without and with a magnetic field is a subject of great current interest. The quantum Hall liquid, a novel two dimensional system, becomes an electrical insulator under certain conditions (8, 9). The nature of the insulator and of the transition are not well understood. Stable quasicrystals seem closer to being insulators, the structurally more nearly perfect they are (10). In this class of systems, none of the usual culprits, i.e., correlation, disorder, or low dimensionality, can be identified as causing near-insulator behaviour.

A detailed picture exists now of the physical phenomena attending localization of electrons on increasing disorder (11, 12). A theoretical approach to the metal–insulator transition in a disordered interacting Fermi system has been developed (13). These results agree with some exper-

[1] Also at Jawaharlal Nehru Centre for Advanced Scientific Research, Bangalore 560 012, India.

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imental features but not with all. There has recently been considerable progress in understanding the Hubbard model for correlated fermions in the limit of high spatial dimensions (14). Phenomena analogous to metal–insulator transitions can occur in a system of interacting bosons with or without disorder. These are being actively explored (7). Some of these results will be mentioned here, and open questions pointed out.

1.2. Background

The broad cause of a metal-to-insulator transition is the increasing importance of potential energy relative to kinetic energy, and consequent localization of electrons. Many successful criteria for the metal–insulator transition have been based on a comparison between potential and kinetic energies. The difficulty in understanding the details of the transition arises for the same reason; a local description in which electron motion is strongly correlated is natural for the potential-energy-dominated insulator, while the metal is more akin to a gas of free electrons. In addition, the Pauli exclusion principle places severe constraints on possible configurations. The low-energy excited states of such an assembly of localizing fermions determine many physical properties and are therefore hard to describe.

If the potential energy is due to a fixed random distribution of atoms or ions, electrons localize in potential fluctuations if the randomness exceeds a certain limit. This is the Anderson transition.

At the other extreme, neglecting disorder but concentrating on the interaction between electrons, the well known model due to Mott and to Hubbard imagines a lattice on which electrons hop from site to site, the kinetic energy being zt. The essence of interaction is believed to be captured by a repulsion of strength U acting between two electrons on the same site (and of opposite spin). Any longer range interaction is neglected as inessential. If there is exactly one electron per site, then for $U > \alpha\, zt$ (where α is a number of order unity), it is energetically unfavorable for electrons to move and thus create doubly occupied sites. This causes transition to a Mott insulator with a Mott–Hubbard single particle excitation gap of $(U - \alpha\, zt)$. The gap is assumed to vanish at the transition. The transition occurs *only* for commensurate filling. For *any* other electron density, one has a metal, since electrons can then always hop without creating additional doubly occupied sites. In reality, correlations and disorder are often present together, and enhance each other's effect. For example, as disorder increases, electrons diffuse slowly and in effect interact more strongly. Also, the simplified Mott model has to be enlarged, especially in transition metal oxides and chalcogenides, where oxygen or chalcogen orbitals may have energies close to d configurations of the transition metal ions, with which they overlap strongly. Sawatzky and Allen (15) pointed out that often (as in NiO and in the cuprates) the oxygen hole orbitals lie in the d hole Mott–Hubbard gap, an energy Δ_{ct} away from the top of the lower Hubbard band so that on doping, the extra holes go into the oxygen orbital rather than the d orbital. The electrical gap is Δ_{ct}, the charge transfer energy, rather than U. These are charge transfer insulators which, however, require for their existence well defined, stable atomic d configurations separated by a large energy U.

As the Anderson transition is approached, charge diffusion slows down. Many attendant transport anomalies have been predicted and observed (11, 12). The effective interaction between electrons becomes stronger, time dependent, and finally is of long range in the insulator. In the metallic limit, this interaction effect leads to a square root cusp in the density of states and and to transport anomalies, which have also been observed (11). The behavior very close to the transition (critical behavior) is influenced both by disorder and interaction (13) in ways which are not fully settled yet, as we shall see. The approach to the Mott transition is characterized by a suppression of local fluctuations in the electron number and a slowing down of electron propagation. The inverse compressibility, the effective electron mass, and the spin susceptibility are all expected to increase. (Ref. 2, p. 28). We discuss possibly related effects below. In real systems, very often disorder, Coulomb effects, and short range Mott–Hubbard correlation are present together, and influence the details of the transition.

2. THE MOTT–ANDERSON TRANSITION

2.1. Introduction

One system which was investigated extensively in the late sixties as a candidate for the Mott transition (a correlation driven transition) is V_2O_3, doped with ions isoelectronic to V^{3+} e.g. Cr^{3+} and Ti^{3+}, as well as under pressure. (See Refs. (1) and (2)). The metallic phase near the metal–insulator boundary shows an enhancement in electronic specific heat, in paramagnetic susceptibility, and in the T^2 term for electrical resistivity. These increases, of order 10–20, arise from the same cause and become larger as local charge fluctuations are suppressed and the insulator is approached (either with decreasing pressure or with larger transition metal ion substitution). There is considerable recent work by Rosenbaum and collaborators (16) on this, as well as on low temperature magnetic order in the metallic regime. A spiral magnetic phase for the metal has been predicted by Jayaprakash *et al.* (17). The observed phase is similar but there are differences which need to be explained.

2.2 Cuprates

As mentioned earlier, the "parent" cuprate is a charge transfer insulator with a charge transfer gap of 1–2 eV. On replacement of La by Sr, holes are introduced in the oxygen p band, and at a few percent Sr, the system goes metallic. The electrical properties (18) as well as the magnetic excitations (19) in this vicinity have been extensively studied, though very little is known yet about the critical regime. It seems that the ground state is either insulating or superconducting, though a small intervening metallic regime cannot yet be ruled out. If the resistance per Cu–O plane exceeds about 10 kΩ at low temperatures, superconductivity does not occur, and the system goes insulating at sufficiently low temperatures. The superconductor–insulator transition in cuprates is very similar over a large family of compounds (18); e.g., $La_{2-x}Sr_xCuO_4$ with varying Sr; $YBa_2Cu_3O_{7-y}$ with varying oxygen content; $YBa_2Cu_{3-x}M_xO_{7-y}$ where M is a metal ion such as Zn or Ni which strongly suppresses superconductivity; $Nd_{1+x}Ba_{2-x}Cu_3O_{7-y}$ the "electron" superconductor; and $Bi_2Sr_2Ca_{1-x}Y_xCu_2O_{8+y}$, the layered 2212 compound. This commonalty of continuous transition behavior calls for an explanation. The magnetic excitations in doped $La_{2-x}Sr_xCuO_4$ as it evolves from an antiferromagnetic insulator to a spin glass to a superconductor have been extensively studied (19). For $0.02 < x < 0.05$, there is no Néel order and the system shows a metallic resistivity ($\rho \propto T$) for $T > 100$ K, but crosses over to Coulomb glass localized state electrical behavior at lower temperatures. The magnetic excitation intensity in the spin glass phase is peaked around the commensurate wavevector, and has a general form $\chi''(\omega, T) = I(|\omega|, 0)\phi(\omega/T)$, the first term being the zero temperature spin excitation density, and the second describing thermal effects. The magnetic correlation length ξ seems to be a sum of two parts, one dependent on concentration only and the other only on temperature; $\xi^{-1}(x, T) = \xi^{-1}(x, 0) + \xi^{-1}(0, T)$. As x exceeds $x_c \simeq 0.05$ to 0.06, the spin excitations peak at an incommensurate wavevector q dependent on x, with a broad excitation spectrum. The relation of this detailed magnetic excitation spectrum picture to the insulator superconductor transition is not clear.

2.3. Perovskites

Stimulated by the discovery of high temperature superconductivity, a number of perovskites have recently been examined again (20, 21) to look for insulator–metal transitions and three dimensional cuprate analogues (See Ref. (2), p. 329, for an earlier review). For example, $LaTiO_3$ is a Mott insulator, with antiferromagnetic ordering of Ti^{3+} spins (d^1 configuration, $S = \frac{1}{2}$) at $T_N = 120$–150 K. Substituting La with Sr adds one hole per Sr atom, so that $La_{1-x}Sr_xTiO_3$ has x holes per unit cell in the lower Hubbard band. Such a system ought to be a metal for all $x > 0$ if it is homogeneous. However, because of disorder inevitable on doping, the system is insulating till $x_c \simeq 0.05$ and is a metal for $x > x_c$. On the metallic side of this transition, the electronic specific heat (the γT term) and the magnetic susceptibility χ are enhanced, the ratio remaining unchanged. There is a large AT^2 term in the resistivity whose coefficient increases with specific heat enhancement. The critical regime (close to x_c) has not yet been studied in detail.

The system is specially interesting as a possible three-dimensional doped Mott insulator. Simple considerations (22) suggest that for a small deviation x from half filling, χ and γ are enhanced as x^{-1} and A as x^{-2}. This is approximately the case here, though the dynamic range of enhancement is small, and $x_c \simeq 0.05$ (not zero) due to disorder effects. Also, the system is not very strongly correlated, the Mott–Hubbard gap being small, of order 0.1 eV. Since screening improves with doping, U is expected to decrease and one may be going from medium (or strong) to weak correlation as x increases. The properties of a disordered, doped oxide with $U \leq U_c$ are not known well theoretically. In contrast to doped cuprates, the normal state here seems very Fermi-liquid-like. The reasons could be that non-Fermi-liquid behavior is generically low (two and one) dimensional, or that the $LaTiO_3$ based systems are not sufficiently strongly correlated. The latter seems suggested from the small gap, and the non-Neel magnetic order quit susceptible to weak ferromagnetism on applying an external field. Another indication is the Hall effect, which is *electron*-like (and *not* hole-like as in the doped cuprates) and is approximately proportional to $(1 - x)$.

It would be of great interest to synthesize and investigate a strongly correlated, doped Mott or charge transfer insulator that is three dimensional and clean (i.e., has a small residual resistivity). It is not known if these two types of correlated insulators, Mott and charge transfer, have different kinds of insulator–metal transitions for commensurate carrier density, as a function of pressure for example. In the former case, the metallic phase may not be strongly enhanced since the ligand p band is quite broad. One likely candidate system is $NiS_{2-x}Se_x$, which is insulating for $x \leq 0.50$ and metallic for larger x. However, material composition problems and the peculiar low T resistivity saturation behavior of the insulator (see, e.g., Ref. 2, p. 246) have limited the exploration of this system.

2.4. Critical Behavior

The behavior of physical quantities such as resistivity, magnetoresistance, Hall effect and density of states very close to the metal–insulator transition is a problem of

basic interest, and has been studied in several systems. The transition is discontinuous in all stoichiometric oxides, so the critical regime is not accessible. It is continuous only in disordered systems or off-stoichiometric oxides (which are necessarily disordered). In the former class several examples such as Nb_xSi_{1-x}, Si : P have been studied in depth since the late seventies (11). I review here results on off stoichiometric oxides due largely to Raychaudhuri and co-workers (5). They have studied electrical transport and tunneling in several systems such as $LaNi_{1-x}Co_xO_3$, $La_{1-x}Sr_xCoO_3$, and Na_xWO_3 down to subkelvin temperatures. The low temperature conductivity values very close to the critical x can be even lower than ($\sigma_{min}/100$).

The conductivity at low temperatures has a small $\sqrt{T}$ quantum correction which changes to a large $T^{1/3}$ term as the transition is approached. In case of $La_{1-x}Sr_xCoO_3$ the exponent increases to unity ($\sigma \propto T$) at very low conductivity values, whereas in Na_xWO_3 and related ternary oxides, the exponent stays at (1/3). The compounds are monophasic polycrystals. In some cases, single crystals have been studied. The difference in exponents could be due to the fact that the former family of compounds has magnetic ions and (at least for $La_{1-x}Sr_xCoO_3$) ferromagnetic order in the insulating phase. By contrast, Na_xWO_3 has no magnetic correlations. The tunneling conductance shows a $\sqrt{V}$ cusp as a function of bias far into the metallic phase. This deepens, and in the critical region, for small V, becomes linear. This happens for all the systems studied. There are indications of a quadratic Efros–Shklovskii–Coulomb gap in the insulating state close to the transition.

These results are suggestive of at least two classes of continuous metal–insulator transitions in disordered systems. In both, increasing disorder and slowing down of diffusion are crucial. The Coulomb interaction becomes increasingly important with disorder, becoming long ranged in the insulating phase and determining the low energy behavior. In one family of systems, magnetic or Mott correlation effects are presumably important while in the Na_xWO_3 family, they are not. In order to confirm whether these are generic features of metal–insulator transitions in disordered electronic systems with and without strong local correlation, clearly more experiments on well characterized systems are needed, as well as low temperature measurements of many more physical properties. Theoretically, the metal insulator transition in disordered electronic systems has been extensively investigated (13), and several university classes established. The microscopic scaling analyses are, however, perturbative, and cannot deal with either the insulating coulomb glass, or with strong local correlation. None of the existing theories fit the pattern of experimental observations.

3. NOVEL TYPES OF METAL–INSULATOR TRANSITIONS

3.1 Superconductor–Insulator Transitions

A superconductor, when sufficiently disordered, becomes an insulator. This phenomenon has been actively studied in thin films for nearly 20 years (6). Multilayer systems based on cuprates, as well as cuprates themselves, show, at an appropriate composition or with doping (e.g., by Ag), such a superconductor–insulator transition (18). Broadly two kinds of systems are possible. In the first, disorder is on an atomic scale, and in the second, the disorder is on a length scale larger than the Cooper pair size ξ. The latter system consists in practise of weakly coupled metallic grains, each of size greater than ξ. In the former case, effects of electron localization due to disorder are important, as also the enhanced interaction effects promoted by disorder. It is reasonable that if the Anderson localization length is less than ξ, phase coherent pairs cannot form. This happens for a resistance per square $R_\square$ of order $\simeq$8–10 $k\Omega$. Indeed, this is approximately where the superconductor–insulator transition occurs. In such an atomically disordered system, we expect that the BCS gap Δ goes to zero smoothly at the transition, as Cooper pairs do not form, and there is a Coulomb gap in the insulating phase. The former ($\Delta \to 0$) has indeed been observed in low temperature experiments (23) on thin amorphous Bi films; the superconducting transition, however, becomes broad with increasing disorder, and there is no sharp critical $R_\square$.

In the large grain limit, Cooper pairs always exist within grains; the question is of their phase coherence. One thus expects that Δ_{BCS} remains virtually unchanged with disorder, as is observed (24). Based on viewing the continuous, quenched disorder induced $T = 0$ (quantum) phase transition as being formally similar in scaling behavior to a thermal phase transition without disorder, a number of predictions for relations between exponents have been made. These have not yet been conclusively verified either experimentally or in computer simulations. One of the predictions is that the transition sheet resistance is akin to a critical amplitude in a conventional second order phase transition, and hence is universal (25). This is an area of considerable activity and controversy since both the experimental and numerical results suggest an $R_\square$ in the range 6–12 $k\Omega$ (26) and not a single fixed value. Whether this is due to there being several universality classes, or to conditions for universal behavior not being met, or to some basic inadequacy of theory, is not clear. The superconductor–insulator transition is also influenced strongly by an external magnetic field. A phase diagram in the field-temperature-disorder domain has been suggested, and predictions for physical properties

have been made (27). Though there are a few experiments relevant to these scaling theory predictions (28), the picture is not clear. Experiments on more and better characterized systems are needed. The theory is largely phenomenological, with some computer simulation results. Systems with nonzero Cooper pair amplitude but phase disorder are similar to a collection of interacting bosons in a random potential. The corresponding transition is one of loss of superfluidity by Bose glass formation, which is another area of considerable activity. (See Section 4).

3.2. Quantum Hall Liquid–Insulator Transition

The quantum Hall liquid is a new state of matter, found at very low temperatures and high magnetic fields in a gas of two dimensionally confined interacting, highly mobile electrons. Under such conditions, one finds that around certain integer or odd denominator rational fillings ν of the Landau levels the transverse (or Hall) conductivity is quantized, i.e., $\sigma_{xy} = \nu(e^2/h)$, and the longitudinal conductivity $\sigma_{xx} = 0$. The electron flow is dissipationless. The Hall conductivity has a plateau, i.e., is unchanged as filling is varied around ν. The Hall state is stable because there is a gap against quasiparticle excitations. In the last few years, transitions from both integer and fractional quantum Hall ground states to insulating states have been observed. In the former case, the transition is continuous, and seems to take place at a conductance of order (e^2/h), and for $\nu \simeq 2$ (9). In the latter, around $\nu = \frac{1}{5}$, there is transition to an insulating state whose electrical activation energy vanishes as one tends toward the critical filling (8). The nature of the insulating state and of the transition are not clear. Is the insulator dominated by disorder, or is it a Wigner crystal of localized electrons, or both? Considering the low electron density, a Wigner crystal seems likely; however, disorder is necessarily present. No direct experimental evidence for the Wigner crystal is available yet, though optical signatures have been suggested. For the large ($\nu \simeq 2$) filling transition, the insulator is more likely an Anderson insulator in a strong magnetic field.

These transitions are interesting because disorder plays a dual role. It helps localize excitations (which are extra electrons or quasiparticles) with respect to the commensurate filling state. These are necessarily present when the filling is different from the commensurate value. Without such a localization of excitations, the Hall plateau is not possible. However, with increasing disorder, the quantum Hall state itself seems unstable and the plateau dissappears. In the fractional quantum Hall state, the ground state Laughlin wave function has electrons perfectly correlated with each other as regards relative pair angular momentum l (e.g., for $\nu = \frac{1}{3}$, $l = 3$). A sufficiently strong random potential can destroy this correlation and thus the quantum Hall state even at commensurate filling. It is not known how this happens.

3.3 Quasicrystals

Thermodynamically stable quasicrystals such as $Al_xCu_yFe_{1-x-y}$ with $x \simeq 0.625$ and $y \simeq 0.25$ have been known since 1988. They have extremely good long range order, with positional correlation lengths exceeding 8,000 Å. Their electrical properties are very unusual (10). The zero temperature resistivities are very large, of order 10 to $30 \times 10^{-3}\ \Omega$ cm, *much* larger than the Mott maximum metallic resistivities of about $5 \times 10^{-3}\ \Omega$ cm. In a recently synthesized Al–Re–Cu system, the resistivity is about 0.2 Ω cm at the lowest temperature measured (29). The conductivity increases with temperature, linearly over a certain range. The temperature dependent conductivity increases are quite large, and seem to be intrinsic (30), while the $T = 0$ conductivity seems dependent on residual structural imperfections, etc. Slow annealing, which improves the structural perfection, *increases* the resistivity. The Hall coefficient corresponds to there being $\sim 10^{20}$ electron/cm^3, whereas from electron count, the electron density is about 100 times larger. Also, the $T = 0$ Hall constant changes sign with composition. The ac conductivity is distinctly non-Drude-like, $\sigma(\omega)$ rising approximately linearly with frequency till about $\hbar\omega \simeq 1$ eV and then decreasing.

All the above properties are unlike those of any known metal. The very high resistivities and their increase with order have led to a suggestion that the ideal quasicrystal may be an insulator (10, 30). In this three dimensional system with high electron density, very weak correlation, and small disorder, known mechanisms for a metal–insulator transition are not applicable. It appears (31) that the reason here is the near coincidence of the Fermi surface and the Brillouin zone. Electronic states near the Fermi surface are mixtures of plane waves and their Bragg scattered parts. These are stationary states carrying very little current. Thermal excitation or random scattering mixes in states which can carry more current, and thus increases conductivity. The questions are whether this unusual semimetal can go insulating, how its properties can be explored, and how they are modified by disorder, quantum localization effects etc.

4. THEORETICAL IDEAS

4.1 Mott Transition

There has been very intense activity in the last six years or so aimed mainly at understanding strongly correlated electronic systems, such as those forming high temperature superconductors. As part of this, considerable work has focused on the Mott transition, and interesting new

results are now available. In one dimension, the problem of correlated lattice electrons can be solved exactly. For one electron per site, the ground state is always insulating, no matter how small U is. The problem is also amenable to solution in the limit of very high dimensions ($d = \infty$) or in the limit of very large coordination number. In this limit, a nonlinear selfconsistent equation for the local, time dependent potential on an electron is obtained (14) and solved numerically. It is possible to isolate the antiferromagnetic ordering effects from single site effects, and to look at the metal-to-paramagnetic-insulator transition.

The most surprising result obtained is that there is a *first order* transition to a Mott insulator at the critical $U_c = 3.6t^*$, where t^* is the effective bandwidth. A discontinuous transition in such a simple model with short range correlations is unexpected, and was not anticipated in earlier approximate theories. It was thought that the observed first order transition (e.g., in V_2O_3) is caused by coupling to the lattice. This result requires better understanding. It is also found that as $U \rightarrow U_c$, the correlated metal shows mass enhancement (and consequently enhancement in susceptibility χ, specific heat γ, and resistivity AT^2) which is of order five at U_C. In V_2O_3 larger enhancement values have been reported. It is not clear whether this is because of lower dimensionality or disorder effects.

4.2 Metal–Insulator Transition in Disordered Electron Gas

In the last decade or more, a perturbative theory of the disordered electron gas has been developed extensively, based on a scaling analysis of length and time dependent diffusion and electron interaction. These affect each other. An analysis of the coupled scaling equations, which takes into account only long wavelength fluctuations leads to several types of critical behaviour (13) depending on initial interactions and potential scattering processes. There are predictions for transport and equilibrium $T = 0$ properties. However, the approach has a number of known inadequacies. The model is a continnum one, where no local correlation or electronic commensuration effects are present; a half filled lattice system has the same behavior as a system away from half filling, whereas the former has a Mott transition and the latter is expected to be always metallic in the absence of disorder. The effects of Coulomb interaction are treated perturbatively. This may be qualitatively reasonable on the metallic side, where the interaction is screened, but for the insulator, the long range interaction leads to a soft energy gap and crucial low excitation energy effects discussed first by Efros and Shklovskii. There is at present no way of treating these effects around the transition regime, and in the presence of disorder.

4.3 Boson Localization

Much experimental and theoretical work on localization has dealt with fermions. There are, however, a number of experimental systems, such as He^4 or superconductors, where the question of boson localization naturally arises. For example, the superfluid-to-solid transition in He^4 (at $T = 0$) is a transition to a Mott insulator phase. Only a Mott insulator phase has a zero superfluid order parameter; this order is nonvanishing in a simple density wave solid. A superconductor with only phase disorder is very much like a charged Bose superfluid in a random potential. Such a system can become an insulator (Bose glass). He^4 loses its superfluidity in a sufficiently disordered medium. Because of such possibilities, the subject of boson localization has attracted considerable attention recently.

The simplest system to consider is bosons on a lattice. If there is, say, one boson per lattice site and the bosons have an on-site repulsion U, the system is a superfluid for small U, and is a Mott "insulator" for large U (the bosons stay at their sites). This transition and excitations have been explored recently (32). Away from integer filling, the ground state is always a superfluid. However, if there is disorder (e.g., site energies are random) the ground state may be a nonsuperfluid Bose glass (or an insulator for charged bosons). This transition, where phase stiffness goes to zero, is not well understood yet. There is considerable computer simulation work on the strongly correlated, disordered boson system, and it appears that there is a Bose glass transition for a $2d$ system at a finite disorder, with a dynamical exponent very different from scaling deductions (33). The same sort of scaling argument, applied to a charged interacting Bose system, suggests a universal conductance in $2d$ films at the superconductor insulator transition, but there is no detailed theory yet which takes serious account of disorder.

5. Summary

In the brief contribution I have tried to convey a sense of what some of the new physical phenomena and ideas are in the field of metal–insulator transitions. Some of the questions raised have been presented throughout the text, along with suggestions and speculations. Some of our contributions have also been mentioned.

The main experimental problem is the synthesis of materials which are well characterized (structurally, electronically, and chemically) and preferably exhibit continuous metal–insulator transitions. The problem theoretically is that this class of quantum phase transitions is not characterized by an order parameter. Neither phase has a simple broken symmetry. What goes singular is some response function. Because of the discovery of new phenomena as well as these major challenges, the subject is likely to continue to be significant and exciting.

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