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An Ideal Model for Switching in Thin VO_2 Films*

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Analysis of an ideal model, where heat flow is restricted to one direction, results in good qualitative agreement with the current–voltage characteristics observed in materials that exhibit an abrupt increase in electrical conductivity at a critical temperature, such as VO_2.

Our present understanding of bias induced switching in VO_2 has emerged from a symbiosis between approximate calculations (*1*), numerical analysis (*2*), and experiment (*3*). In this note we outline the results of an exact calculation of an ideal one-dimensional model. For the slab geometry of Fig. 1, with the heat current, $\dot{Q}$, perpendicular to the transport current, I, the calculated current voltage, $I(V)$, characteristics are in harmony with the major experimental features of switching in VO_2 films (*3*).

We first derive a closed form expression for the S-shaped $I(V)$ characteristics and then outline the details of a full analysis of the stability of the sample when it is in series with a load resistor, R_L. The sample is characterized by an electrical conductivity σ_0 for $T < T_s$ and a larger value, σ_s, for $T > T_s$, where T_s is the transition temperature.

Assuming that the thermal conductivity, K, is a constant and the electric field is uniform in the sample, the heat equation is:

$$\rho c \frac{\partial T}{\partial t} = K \frac{\partial^2 T}{\partial X^2} + \sigma(T) \frac{V_B{}^2}{l^2} \frac{R_s{}^2}{(R_L + R_s)^2}, \quad (1)$$

where ρ is the density, c the specific heat and R_s, the nonlinear sample resistance, is given by:

$$l \Big/ h \int_{x=-(w/2)}^{w/2} \sigma(T(x))\, dX.$$

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We do not include a latent heat of transformation at T_s (*4*). The boundary condition is that $T(\pm(w/2)) = T_a$ (the ambient temperature). The electrodes, top and bottom of the sample are perfectly insulated (*5*). We also neglect all reactive components. Our analysis, which will appear in detail in a subsequent paper, is therefore valid in the regime when the negative differential resistance (NDR) points are always *circuit* stable (*6*).

We first seek steady state solutions of Eq. (1). As long as the maximum temperature is less than T_s, there is a solution

$$T(x) = \frac{\sigma_0 V^2}{l^2}\left(\frac{w^2}{8} - \frac{X^2}{2}\right) + T_a. \quad (2)$$

When V is increased to the point where Eq. (2) would yield a value greater than T_s at $x = 0$, the equation becomes invalid and we look for a "two-phase solution" with an internal hot filamentary region for which $T > T_s$ and $\sigma = \sigma_s$. The critical voltage, V_c, at which Eq. (2) becomes invalid is

$$V_c = l/w\,((8(T_s - T_a))/\sigma_0)^{1/2}. \quad (3)$$

A two-phase solution satisfies

$$K(\partial^2 T)/(\partial X^2) = -(\sigma_s V^2)/l^2, \qquad -f/2 \leqslant X \leqslant f/2 \quad (4a)$$

$$K(\partial^2 T)/(\partial X^2) = -(\sigma_0 V^2)/l^2, \qquad f/2 \leqslant X \leqslant w/2, \quad (4b)$$

with $T = T_s$ at $x = \pm f/2$. If these equations are

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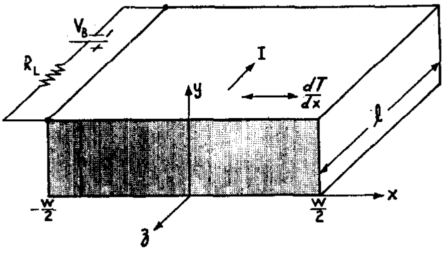

FIG. 1. Sample geometry. $V = V_B - IR_L$.

integrated with the additional conditions that the temperature and heat current are continuous at $f/2$, we obtain the width of the hot filamentary region as a function of V,

$$|f|/w = (1-\varepsilon)/(2-\varepsilon) \pm 1/(2-\varepsilon) \times (1 - (V_c^2/V^2)(2\varepsilon - \varepsilon^2))^{1/2}, \quad (5)$$

where $\varepsilon = \sigma_0/\sigma_s$. For $V > V_c$, only the plus sign in Eq. (5) applies and $T > T_s$ in all but a small strip near the surface; as $V \to \infty$, $|f| \to w$.

We see from Eq. (5) that there are also two-phase solutions for $V < V_c$. Defining

$$V_c' = (2\varepsilon - \varepsilon^2)^{1/2} V_c,$$

there are two values of f for voltages in the region $V_c' \leqslant V \leqslant V_c$. The current in the two-phase state is

$$I = (V/R_0)[1 + ((1-\varepsilon)/\varepsilon)(|f|/w)] \quad (6)$$

where $R_0 = l/\sigma_0 wh$.

A typical $I(V)$ curve is shown in Fig. 2. V_c' is the minimum voltage reached in the filamentary state. At V_c',

$$f/w = (1-\varepsilon)/(2-\varepsilon). \quad (7)$$

Furthermore,

$$I(V_c')\,V_c' = I(V_c)\,V_c;$$

the powers dissipated at V_c' and the low-current V_c are the same.

To investigate stability we consider a small perturbation about the time-independent solutions which we have found, and see whether Eq. (1) causes the perturbation to grow or decay. If $T_0(x)$ is a time-independent solution and $\eta(x,t)$ the perturbation, we write $T(x,t) = T_0(x) + \eta(x,t)$, insert it into Eq. (1), and linearize to terms of first order in $\eta(x,t)$. The equation for $\eta(x,t)$ is then

$$\rho C \frac{\partial \eta}{\partial t} = K \frac{\partial^2 \eta}{\partial X^2} + \frac{\sigma_s - \sigma_0}{\left|\frac{\partial T}{\partial X}\right|_{x=f/2}} \cdot \frac{V_B^2}{l^2} \frac{R_s^2}{(R_L + R_s)^2} \times \left[\delta(X - f/2) - \frac{2\sigma(T_0(x))}{\int_{-w/2}^{w/2} \sigma(T(x))\,dX} \cdot \frac{R_L}{R_L + R_s} \right] \eta \quad (8)$$

We seek a solution of the form

$$\eta(x,t) = e^{\alpha t}\,x(x). \quad (9)$$

If there is a solution of this form with a positive α, the perturbation grows and the time-independent $T_0(x)$ is unstable; if not, it is stable. Inserting Eq. (9) into (8), we find that there is no positive α (and hence stability) when

$$|f|/w \geqslant (\varepsilon(1-\varepsilon)R_0)/(R_L + \varepsilon(2-\varepsilon)R_0). \quad (10)$$

Comparing this result with Eq. (7), we see that all NDR points are unstable for the unloaded case. PDR points are always stable.

As R_L increases from zero, additional NDR states will stabilize and finally as $R_L \to \infty$, Eq. (10) shows us that $f/w \geqslant 0$. All NDR states can be stabilized with an infinite load. The

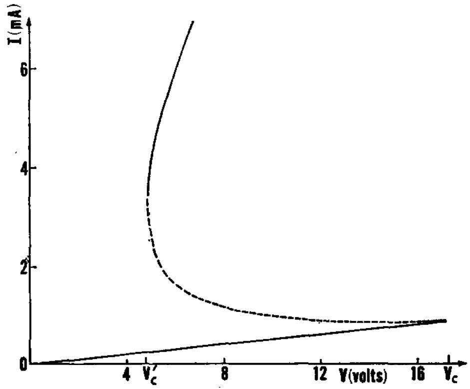

FIG. 2. A calculated current vs voltage curve for $\sigma_0 = 0.2$ $(\Omega\ \text{cm})^{-1}$, $T_a = 294°\text{K}$, $T_s - T_a = 47°\text{K}$, $lw = 0.4$ and $\varepsilon = 4 \times 10^{-2}$. Solid lines, stable. Dashed line, conditionally stable.

condition for stability can be shown to be $R_L + dV/dI > 0$. Inclusion of reactive components, however, must also be taken into account (*6*).

Although all NDR states can in principle be stabilized, the $I(V)$ characteristics approach the point (V_c, I_c) from the NDR region with zero slope (Fig. 2 and Eqs. (5) and 6)). Prohibitively large resistive loads are therefore required to stabilize the NDR points and an "open" region should always be present in the experimental characteristics. This behavior is in fact what is experimentally observed (*3*). The model also predicts (1) the details of the filament characteristic, including what region can be stabilized for different R_L, and (2) the narrow filaments that are observed even for relatively low ($\ll V_c$) voltages (*3*).

Acknowledgments

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References

1. See, e.g., C. N. Berglund, *IEEE Trans. Elec. Dev.*, **ED-16**, 432 (1969).
2. See, e.g., J. Duchene, G. Adam, and D. Augier, *Phys. Status Solidi* **A8**, 459 (1971).
3. See, e.g., J. Duchene, M. Terraillion, P. Pailly, and G. Adam, *Appl. Phys. Letters* **19**, 115 (1971).
4. As demonstrated to us by S. R. Coriell, its inclusion will not affect the results.
5. Analysis shows that $\dot{Q}$ in the z direction (toward the electrodes) will not contribute to the NDR.
6. M. P. Shaw, H. L. Grubin, and I. J. Gastman, *IEEE Trans. Elec. Dev.* **ED-20**, 169 (1973).