

Structures Related to the  $\beta$ -Tungsten or  $\text{Cr}_3\text{Si}$  Structure Type

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Using traditional crystallographic operations such as translation, rotation and reflection, and intergrowth, it is shown how several structures, among them the tetrahedrally close-packed, can be accurately derived from the  $\beta$ -tungsten structure type. Examples are  $\text{Zr}_4\text{Al}_3$ ,  $\text{Fe}_2\text{B}$ ,  $\text{Mo}_3\text{CoSi}$ ,  $\text{Ga}_2\text{Mg}_3$ ,  $\text{GaMg}$ ,  $\text{Ga}$ ,  $\alpha$ - and  $\beta$ - $\text{V}_3\text{S}$ ,  $\text{W}_2\text{CoB}_2$ ,  $\text{CeAl}$ ,  $\text{W}_3\text{Si}_3$ , the  $\sigma$ -phase, the Friauf-Laves phases, the  $\mu$ -phase, the  $P$  and  $M$  phases, the  $(\text{Co}_{0.37}\text{Si}_{0.43})_3\text{V}_2$  phase, the  $\chi$ -phase, and the  $\nu$ -phase.

When a crystal of a complex structure is formed, we like to think that nature starts by building a unit of a simple structure. We imagine, for various chemical reasons, that these units are then joined together at the atomic level in ways that can be described by (a) crystallographic operations (translation, rotation, reflection) and (b) intergrowth. The geometry of these operations was recently described in a review article (1). In the final structure, the simple unit often retains infinite extension in one or two dimensions. Considerable confidence in this theory has been accumulated through the years from indirect evidence; it is possible to describe and classify an increasing number of complex structures using these crystal building principles. For example, the structures of many transition-metal oxides and oxide fluorides could be described with the building-block principle (2, 3) and the intergrowth model (4). Such structures were once characterized as *predictable* (5), they were superficially very complex, but once the simple units were recognized, the structures could easily and accurately be described in simple terms. Experiments showed that some of the complex oxides

in the  $\text{MoO}_3$ - $\text{Nb}_2\text{O}_5$  system were formed very fast and easily (6), but direct evidence did not exist for the proposed general thought.

When a crystal grows, nature makes its faux pas in the form of mistakes or defects. The character of such faux pas is of course directly dependent on the structure building principle. Using electron microscopy, Allpress and co-workers (7) managed to resolve and determine the structure of these defects with the lattice image technique in several studies on niobium oxides. This work was further refined (8) with ultrahigh resolution microscopy and, from the detailed structure of disordered shear planes (Wadsley defects), it is now clear that much support exists for the building-block model of these transition-metal oxides (the translation operation).

The tetragonal tungsten bronze structure, and several others, were very elegantly derived by Hyde and Bursill (9, 10) by their rotation operation. Isolated rotation faults were also proposed by these authors (10) to exist in crystals, and they could also interpret certain observations by electron microscopy in this way later on (11).

Chemical twinning (reflection plus some-

times glide) was proposed as a structure building operation and many examples of structures have been classified according to this principle (12). Using electron microscopy it was possible to confirm this prediction, crystals of  $\text{Ru}_4\text{Si}_3$  and  $\text{V}_4\text{As}_3$  have been found to be rich in defects of the chemical twin type (13, 14). The twinning principle was recently extended to trillings (1) and furlings (15) and also to swinging twinning (16). An elegant and extended application of the twin principle was recently demonstrated (17) for certain alloys.

We wish now to report how some tetrahedrally close-packed structures can be derived from a simple structure type, the  $\beta$ -tungsten structure, using the operations referred to above. The  $\beta$ -tungsten structure, hereafter typified by  $\text{Cr}_3\text{Si}$ , cannot be accurately represented by regular polyhedra, and must be regarded as one of the densest of structures with an average coordination number (ACN) of 13.5 (hcp and ccp have  $\text{CN} = 12$ ). Regular tetrahedra cannot fill space, irregular ones can of course, and do so in  $\text{Cr}_3\text{Si}$  structure. In the bcc W structure, all atoms have the same coordination number,  $\text{CN} = 14$ , and the tetrahedra are distorted just to fill space. In  $\text{Cr}_3\text{Si}$  there are two different CN's, 14 and 12, and tetrahedra here are distorted in order to fill space as well as to accommodate atoms of two

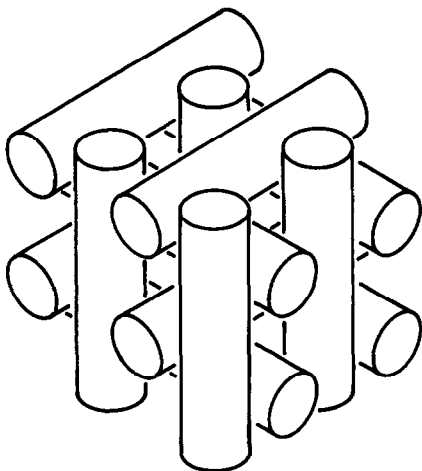


FIG. 1. Rod packing of  $\text{Cr}_3\text{Si}$ .

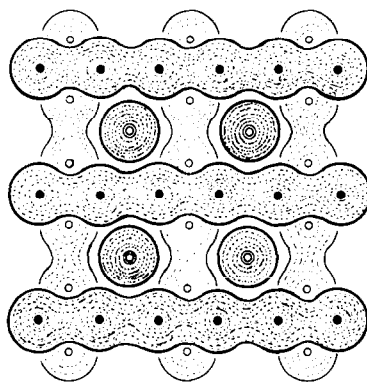


FIG. 2. Projection of rod packing in  $\text{Cr}_3\text{Si}$ . Filled and open circles are Cr on  $\frac{1}{2}$  and 0, respectively. Double circles two Cr on  $\frac{1}{4}$  and  $\frac{3}{4}$ .

different sizes. The  $\text{Cr}_3\text{Si}$  structure contains endless strings of chromium atoms strongly bonded to each other. This character is so pronounced that we have recently described the structure as a rod structure (18), with its packing given in Fig. 1. Each rod contains the chromium atoms, as demonstrated in Fig. 2. The Si atoms are situated in the interstices formed by the rod packing. These interstices can be called rod-octahedra, with each rod-octahedral corner occupied by two chromium atoms giving the well-known icosahedral configuration of chromium atoms around each silicon. With this description of the structure, it is easily related to other structures like a primitive cubic structure,  $\omega$ -phases, the garnet structure, etc., as was reported recently (18).

In order to relate  $\text{Cr}_3\text{Si}$  to other complex alloy structures we have chosen another description which is demonstrated in Fig. 3a. The shaded regions demonstrate what Schubert (19) calls *Tetraederstern*, and represented in this way it is easy to see how regular crystallographic shear (CS) can be applied to the structure. In  $\text{Cr}_3\text{Si}$  the "tetraedersterns" are joined by corner sharing and, if the structure is sheared so that "tetraedersterns" share edges, the  $\text{Zr}_4\text{Al}_3$  structure (20) is obtained. This of course involves the regular omission of atoms in the shear plane, and is exactly analogous to the construction of

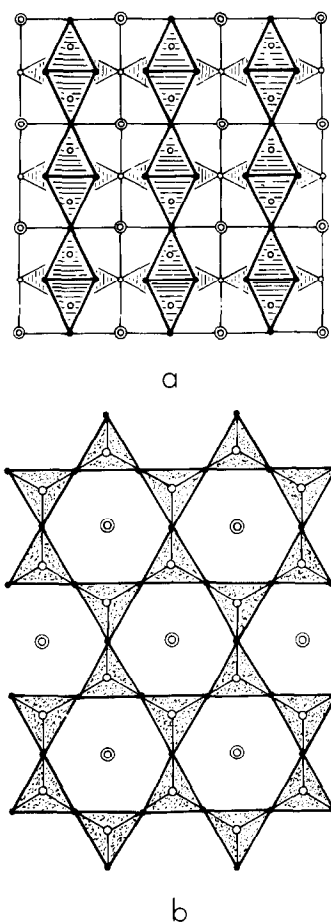


FIG. 3. (a)  $\text{Cr}_3\text{Si}$ . Within each square a tetraederstern. Cr as in Fig. 2. Si atoms on 0 or  $\frac{1}{2}$  where tetraedersterns share corners. (b)  $\text{Zr}_4\text{Al}_3$ , projection along  $c$ . The drawing after Ref. (20). Zr are double open circles, filled circles are Al.

$\text{Nb}_3\text{O}_7\text{F}$  and  $R\text{-Nb}_2\text{O}_5$  from the  $\text{ReO}_3$  structure type (26). The  $CS$  operation is shown in Fig. 4 with the idealized  $\text{Zr}_4\text{Al}_3$  in the upper right corner. If this idealized structure is projected along another axis, the  $c$ -axis of the real  $\text{Zr}_4\text{Al}_3$  structure, the structure given in Fig. 5 is obtained. The real structure of  $\text{Zr}_4\text{Al}_3$  is given in Fig. 3b, and if the two structures are compared it is easy to see that the difference between the two involves only a small distortion.

Figure 6 gives the real structure of  $\text{Zr}_4\text{Al}_3$  and it is easy to identify the  $\text{Cr}_3\text{Si}$  blocks.

Vertically in the drawing the  $\text{Cr}_3\text{Si}$  blocks are joined by tetraedersterns sharing corners. If a new, identical  $CS$  operation is carried out perpendicular to the first one, the  $\text{CuAl}_2$  or  $\text{Fe}_2\text{B}$  structure is obtained. This is shown in Fig. 7, with the shear plane indicated by dotted lines in the left-hand idealized  $\text{Zr}_4\text{Al}_3$  structure. Parameters calculated for the  $\text{Fe}_2\text{B}$  structure, using the  $\text{Cr}_3\text{Si}$  structure, become:

$$c/a = 2(2)^{1/2}/3 = 0.94,$$

$$8 \text{ Fe in } 8(h) \quad I4mcm, \quad x = 0.167.$$

The observed parameters from the structure (21) are  $c/a = 0.83$  and  $x = 0.167$ .

With these principles it is easy to design structures intermediate between  $\text{Cr}_3\text{Si}$  and  $\text{Fe}_2\text{B}$ . In terms of building blocks  $\text{Fe}_2\text{B}$  can be described as consisting of blocks of  $\text{Cr}_3\text{Si}$   $1 \times 1$ , the hypothetical structure of Fig. 8 is then  $2 \times 1$  and the structure of  $\text{Mo}_3\text{CoSi}$  (22) in Fig. 9 is clearly  $2 \times 2$ . One such  $2 \times 2$  column is shown in Fig. 9 by denser shading. The structure of  $\text{Mo}_3\text{CoSi}$  is thus built of  $\text{Cr}_3\text{Si}$  columns sharing tetraederstern edges across two mutually perpendicular shear planes. In the intersections, units of the  $\text{Fe}_2\text{B}$  structure can be traced, and the analogy with  $M\text{-Nb}_2\text{O}_5$  (23) or  $\text{W}_3\text{Nb}_{14}\text{O}_{44}$  (24) is obvious. The parameters for  $\text{Mo}_3\text{CoSi}$  are given in Table I.

The structures of  $\text{Mg}_5\text{Ga}_2$  (25) and  $\text{MgGa}$  (25) are very similar to  $\text{Cr}_3\text{Si}$ .  $\text{Mg}_5\text{Ga}_2$  in Fig. 10A can be described as infinite  $\text{Cr}_3\text{Si}$  layers, which, if they are joined, with omission of atoms, transform the structure to the  $\text{Cr}_3\text{Si}$  type. The transformation path is shown by arrows.  $\text{MgGa}$  and  $\text{Cr}_3\text{Si}$  are shown in Fig. 10B, and simple shear operations transform one structure into another.

### Rotation

The structure of  $\text{W}_5\text{Si}_3$  (30) can be accurately derived by means of the Hyde rotation operation (9, 10) on the  $\text{Cr}_3\text{Si}$  structure. In Fig. 11a the structure of  $\text{Cr}_3\text{Si}$  is drawn with the unit cell marked with broken lines. If cylinders of atoms, marked with

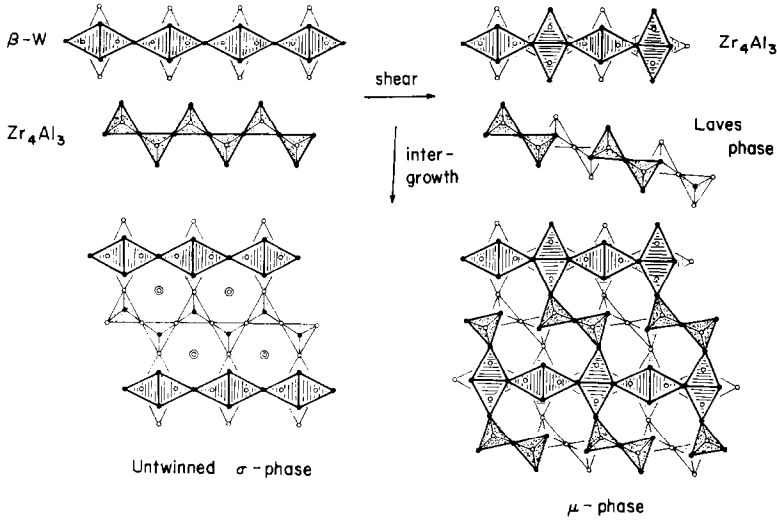


FIG. 4. The principles of shear and intergrowth in tetrahedrally close-packed structures. Structures to right idealized.

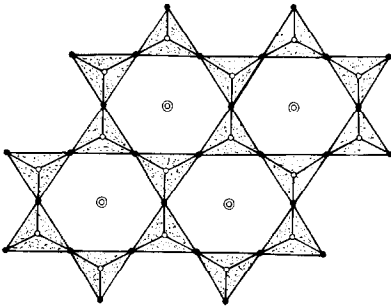


FIG. 5. Idealized  $\text{Zr}_4\text{Al}_3$ , projection along  $c$ .

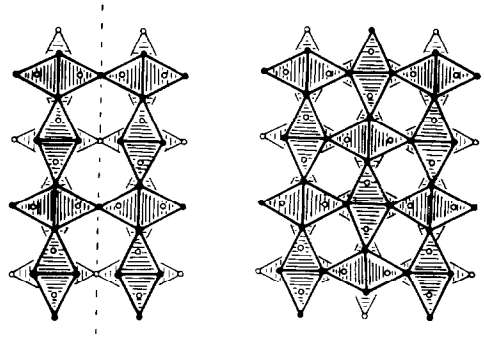


FIG. 7.  $\text{Zr}_4\text{Al}_3$  (idealized) to left,  $\text{Fe}_2\text{B}$  to right. Dotted line is shear plane.

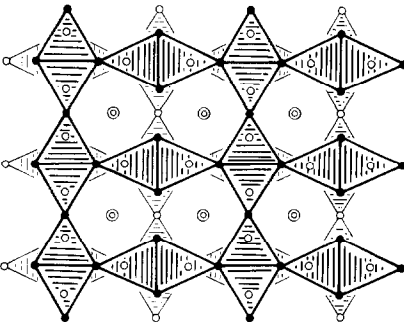


FIG. 6. The real structure of  $\text{Zr}_4\text{Al}_3$ , projection down  $[110]$ . Open and filled circles on 0 or  $\frac{1}{4}$ . Double circles, two Al on  $\frac{1}{4}$  and  $\frac{3}{4}$ . Al is also situated where tetrahedra share corners. The rest is Zr.

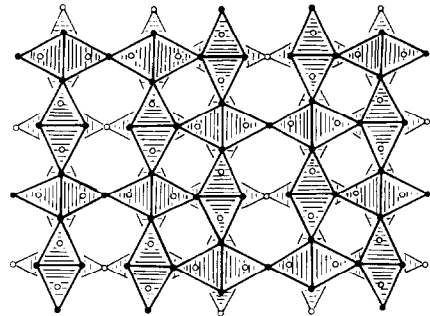


FIG. 8. Hypothetical structure, geometrically between  $\text{Fe}_2\text{B}$  and  $\text{Cr}_7\text{Si}$ .

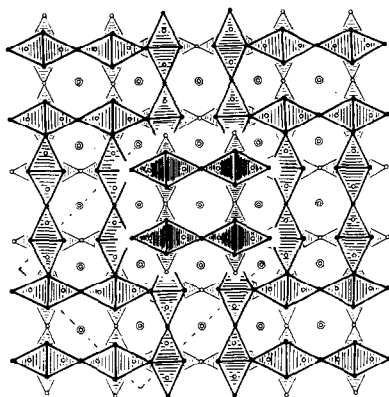


FIG. 9.  $\text{Mo}_3\text{CoSi}$ . Columns of rods of  $\text{Cr}_3\text{Si}$ , one such column heavily shaded in the center. Atoms on 0 or  $\frac{1}{2}$ , double circles are atoms on  $\frac{1}{4}$  and  $\frac{3}{4}$ . The figure is constructed from  $\text{Cr}_3\text{Si}$  units, but it is not possible to distinguish it from the real structure in the scale of this drawing.

TABLE I

Calculated	Observed (22)
$c/a = 2(2)^{1/2}/7 = 0.404$	$c/a = 0.387$
$x_1 = 0.143$	0.155
$y_1 = 0.071$	0.085
$x_2 = 0.071$	0.067
$y_2 = 0.286$	0.294
$x_3(8h) = 0.143$	0.138
$y_3 = 0.643$	0.638

circles, are rotated  $45^\circ$  so that the shaded regions obtain the orientations shown in Fig. 11b a structure very close to that of  $\text{W}_5\text{Si}_3$  is obtained. The structure in Fig. 11 is tetragonal as  $\text{W}_5\text{Si}_3$ , and calculated and observed parameters are given below:

$$\text{calc: } c/a = 0.500$$

$$16 \text{ W in } 16(k) \quad x = 2^{1/2}/16 = 0.088; \quad y = \frac{1}{2} - (3(2)^{1/2}/16) = 0.235 \quad x = 0.074, y = 0.223,$$

$$8 \text{ Si in } 8(h) \quad x = 1/4(2)^{1/2} = 0.177$$

During the operation atoms in 4(b) (double circles in Fig. 11) stay in position. A doubling of the unit cell of  $\text{Cr}_3\text{Si}$  in two directions is also the result of this clockwise and anticlockwise rotation.

Another Hyde rotation operation in  $\text{Cr}_3\text{Si}$

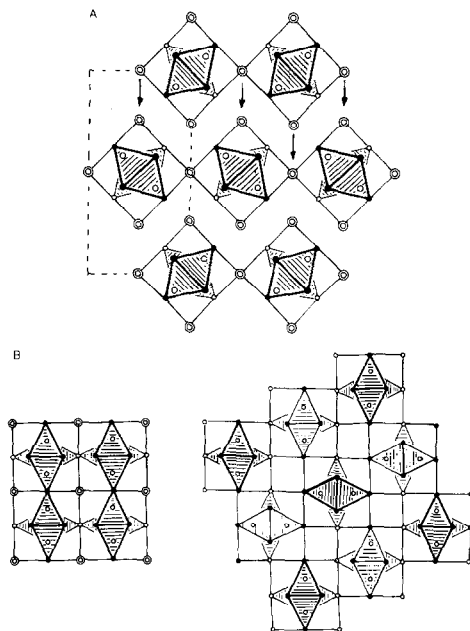


FIG. 10. (A)  $\text{Ga}_2\text{Mg}_3$ . Open and filled atoms on 0 and  $\frac{1}{2}$ . Double circles are two Mg on  $\frac{1}{4}$  and  $\frac{3}{4}$ . Larger circles are also Mg. If one sheet of  $\text{Cr}_3\text{Si}$ -type layers is shifted according to arrows, the  $\text{Cr}_3\text{Si}$ -type structure is obtained. (B)  $\text{GaMg}$ , idealized (to right) is compared with  $\text{Cr}_3\text{Si}$ . The slip operation is obvious.

produces the structures of  $\alpha$ - and  $\beta$ - $\text{V}_3\text{S}$  (27). The structure types are  $\text{Ni}_3\text{P}$  (28) and  $\beta$ - $\text{V}_3\text{S}$  and the structural chemistry of isostructural compounds have been discussed by Rundqvist (29). The two structures, given in Fig. 12, are very similar. Only a translation of  $\frac{1}{2}c$  of the atoms in the corners of the  $\text{Cr}_3\text{Si}$  squares transforms  $\alpha$ - into  $\beta$ - $\text{V}_3\text{S}$ . Another way to do this is by rotation; if the atom columns within

$$\text{obs: } c/a = 0.515$$

$$x = 0.17.$$

circles are rotated  $45^\circ$   $\alpha$ - $\text{V}_3\text{S}$  goes to  $\beta$ - $\text{V}_3\text{S}$ . On the other hand, if atom columns within circles in  $\beta$ - $\text{V}_3\text{S}$  are rotated  $45^\circ$ , and if the corner atoms of the tetraedersterns are translated  $\frac{1}{4}c$  and shifted somewhat so that they overlap, a structure which topologically is the

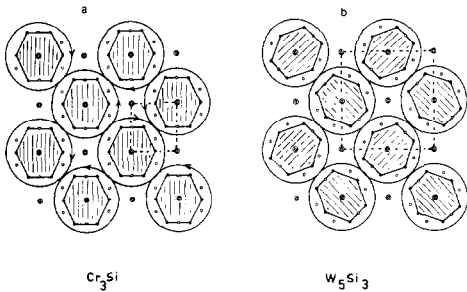


FIG. 11. Rotation mechanism for transforming the  $\text{Cr}_3\text{Si}$  structure type to the  $\text{W}_5\text{Si}_3$  structure type.

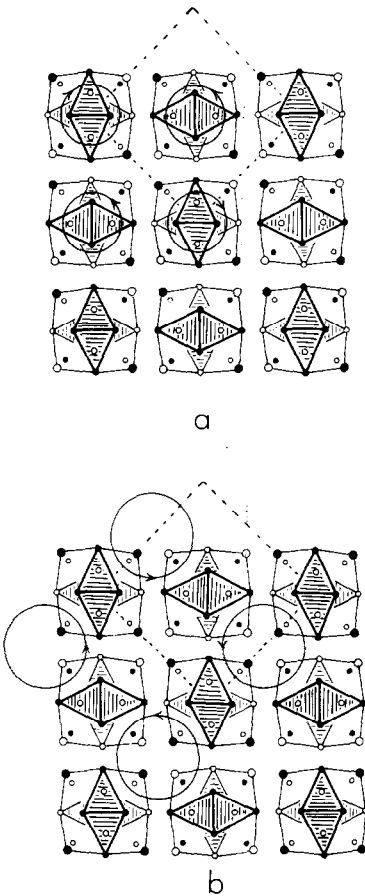


FIG. 12. (a)  $\beta\text{-V}_3\text{S}$ . Small and big, open and filled circles are S and V on 0 and  $\frac{1}{2}$ ; medium-sized circles, V on  $\frac{1}{4}$  and  $\frac{3}{4}$ . (b)  $\alpha\text{-V}_3\text{S}$ . Small and big, open and filled circles are S and V on 0 and  $\frac{1}{2}$ ; medium-sized circles, V on  $\frac{1}{4}$  and  $\frac{3}{4}$ . Circles show rotation mechanism for transforming these structures to  $\text{Cr}_3\text{Si}$ .

same as  $\text{Cr}_3\text{Si}$ , is obtained. This is indeed a simple mechanism and it is important to mention that the *A* and *B* atoms in  $A_3B$  ( $\text{V}_3\text{S}$  or  $\text{Cr}_3\text{Si}$ ) remain ordered during the transformation

### Tetrahedrally Close-Packed Structures

#### *Friauf-Laves Phases and Related Structures*

The basic geometry of some of the structures shown below was described by Frank and Kasper in a very elegant way (31, 32). However, analysis of some structures, notably the  $\nu$ - and  $\chi$ -phases seem to resist the classification, and we will describe how these and other structures can be derived with operations used above.

Going back to Fig. 4 in the upper left corner, a string of tetraedersterns share corners and represent the  $\text{Cr}_3\text{Si}$  structure. Underneath is a string of the real  $\text{Zr}_4\text{Al}_3$  structure (projected along *c*). If  $\text{Zr}_4\text{Al}_3$  is sheared again to give the string of edge-shared trigonal bipyramids, shown in the right part of Fig. 4, second from top, a part of the  $\text{Cu}_2\text{Mg}$  structure is obtained. This operation increases to 16 the coordination number for atoms in the shared edges of trigonal bipyramids, and, in the real structure of  $\text{Cu}_2\text{Mg}$ , these positions are also occupied by the larger Mg atoms. This causes the real structure, although topologically the same as shown in Fig. 13, to

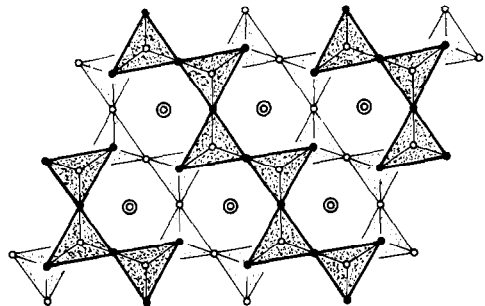


FIG. 13.  $\text{MgCu}_2$  (the real structure) projected along  $[1\bar{1}0]$ . Mg on 0 or  $\frac{1}{2}$ , situated in centers of truncated tetrahedra, or where trigonal bipyramids share edges. Cu, in 0 or  $\frac{1}{2}$ , where trigonal bipyramids share corners. Double circles, two overlapping Cu on  $\frac{1}{4}$  and  $\frac{3}{4}$ .

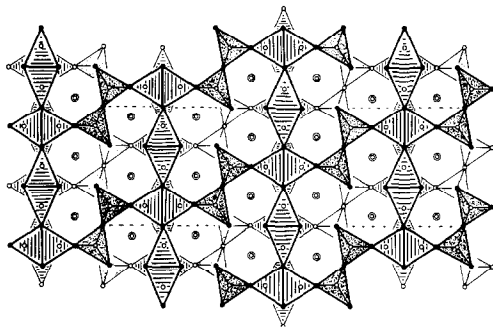


FIG. 14.  $\mu$ -phase,  $W_6Fe_7$ . Projection along  $[110]$ , filled and open circles atoms on 0 and  $\frac{1}{2}$ . Double circles are two Fe on  $\frac{1}{4}$  and  $\frac{3}{4}$ . Fe is also where trigonal bipyramids share corners, and where tetraedersterns share corners with trigonal bipyramids. W situated where trigonal bipyramids share edges, and in the corners of the central tetrahedron of the tetraederstern. The drawing is constructed from  $Zr_4Al_3$  and  $MgCu_2$ , but impossible to distinguish from the real structure.

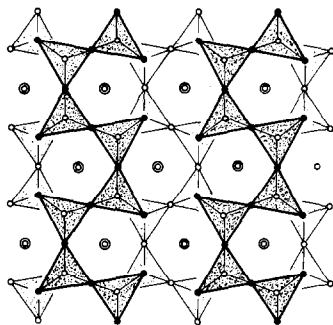


FIG. 15.  $MgZn_2$ , real structure. Ordering and heights of atoms as in  $MgCu_2$ , Fig. 13. Projection along  $[110]$ .

be slightly modified. The cubic  $Cu_2Mg$  can also be derived from ccp by the systematic substitution of two ccp atoms by one Mg. The coordination polyhedron around Mg is the truncated tetrahedron. Such truncated tetrahedra share faces in the Friauf-Laves phases and each Mg will have four other Mg atoms as next neighbors in the centers of four other truncated tetrahedra. This increases the CN of Mg to 16, and such four-capped truncated tetrahedra are called Friauf polyhedra.

In Fig. 4 is also shown how a part of  $Cu_2Mg$  (idealized) can grow together with idealized  $Zr_4Al_3$  to form the  $\mu$ -phase. In Fig. 14 the  $\mu$ -phase is constructed by letting sheets of the

real structures of  $Cu_2Mg$  and  $Zr_4Al_3$  intergrow. No parameters were calculated, but within the accuracy given by the drawing the constructed structure is identical with the real structure of the  $\mu$ -phase.

Another Laves phase, the  $MgZn_2$  compound, can analogously be derived from hcp and be described as the packing of truncated tetrahedra. It is derived equally well by twinning the  $Mg_2Cu$  structure, and the calculated parameters according to that operation are:

$$c/a = 2(2)^{1/2}/3^{1/2} = 1.633,$$

$Z_{Mg} = \frac{1}{16} = 0.0625$ , and  $x_{Zn} = \frac{1}{6} = 0.1667$ . Observed parameters are  $c/a = 1.648$ ,  $z = 0.062$ , and  $x = 0.17$ , respectively. The structure is given in Fig. 15.

It is easy to see from Figs. 13 and 15 that these two structures both can be sheared once more. The sheared structure of  $Cu_2Mg$  is shown in Fig. 16 and the "tunnels" are now relatively small. The coordination number of atoms are lowered, being 9 and 11 for the two independent atoms in the unit cell. The structure is not tetrahedrally close-packed, and the search for a representative has to be done in other fields of structures. The group of interstitial alloys that have trigonal prisms as hosts for interstitial atoms, have generally lower coordination numbers for atoms. The structure of  $W_2CoB_2$  (34) is topologically the same as this sheared version of  $Cu_2Mg$  with boron atoms entering the "tunnels." This structure, and consequently also  $W_2CoB_2$ , can be transformed to hcp by a new simple shear

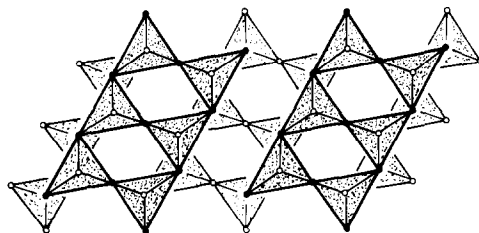


FIG. 16. Sheared structure of  $MgCu_2$ , topologically the same as  $W_2CoB_2$ .

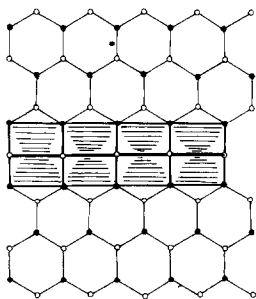


FIG. 17. Planar fault of rhombic prisms in hcp.

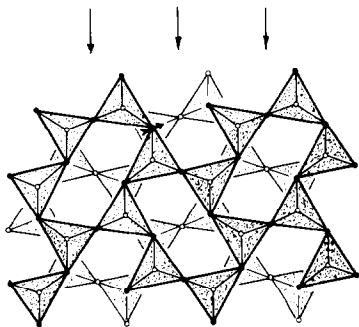


FIG. 18. Sheared structure of  $MgZn_2$ . Small arrow gives shift 0.13 Å to transform the structure of Ga. Larger arrows indicate twin planes for transforming the structure to CeAl.

operation, and in Fig. 17 we show how a chain of trigonal prisms, or, equally well, rhombic prisms (35), occurs as an extended planar fault in hcp.

The sheared structure of  $MgZn_2$  is shown in Fig. 18. It is interesting to find that the metal-atom positions in this structure agree very well with the positions of anions and lone pairs of electrons in  $L$ -SbOF (36). The two nets are compared in Fig. 19.

The space group for this sheared  $MgZn_2$  ( $AB_2$ ) structure is  $Cmca$ , with atoms 8  $A$  in  $8(f)$ , with  $y = \frac{1}{10}$  and  $z = \frac{1}{6}$ , and with 4  $B$  in  $4(b)$ . This structure is very similar to the structure of stable gallium. If the  $A$  atoms are shifted 0.13 Å in the direction of the arrow in Fig. 18 (this shifts the  $y$  and  $z$  parameters to 0.081 and 0.1549, respectively), the  $A$  atoms are in the positions of the gallium atoms in the metal structure. The  $a:b:c$  ratio for sheared

$MgZn_2$  is  $2^{1/2}:5(3)^{1/2}/6:3^{1/2}$  or 0.98:1:1.697 and for gallium it is 0.999:1:1.693. The only real difference between the two structures is the  $B$  atoms; if they are removed, the structures virtually become identical. Ordered metal-atom vacancies of this kind are to our knowledge unusual but do occur in the  $MgGa_2$  structure (39). Pearson (40) points out that if these vacancies are occupied by atoms, the structure of  $MgGa_2$  can be described as a regular network of pentagons and triangles. However, the X-ray work by Smith *et al.* (39) clearly indicates that no extra atoms are present. These observations need an explanation and it is convenient to assume localization of extra electrons, possible  $s_2$  pairs, to fill these holes in the structures of gallium and  $MgGa_2$ . Support for this is also given by the unusual behavior of gallium at its melting point. The volume and electrical resistivity increase in going from the melt to the solid phase, and according to Mott (41), the effective number of free electrons increases by a factor of about 10 when gallium melts. The structure of metastable gallium (42), prepared from a supercooled melt, has a more normal metal structure. It has the same structure as the chromium atoms in CrB, with

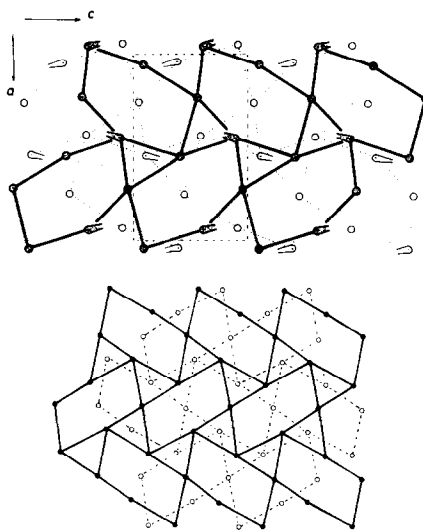


FIG. 19. Net of atoms of structure of Fig. 18, compared with net of anions and lone pairs in  $L$ -SbOF.



a CN of 10 compared with 7 for the gallium atoms in stable gallium. Naturally the metastable gallium reflects more the liquid state and the localization of electrons to ordered positions in stable gallium causes the unique electrical properties and the well-known volume decrease at melting. The lone pairs in ionic compounds of the  $s^2$  elements are attached to their cations (37, 38), but the analogy in description and classification of structure is obvious.

Sheared  $\text{MgZn}_2$  can be twinned at the planes indicated by arrows in Fig. 18. The polyhedra created in the twin plane by this operation are pentagonal antiprisms, and the structure is topologically the same as  $\text{CeAl}$  (43).

#### The $\chi$ , $\sigma$ , $P$ , $M$ ( $\text{Co}_{0.57}\text{Si}_{0.43}$ ) $_3\text{V}_2$ and $\nu$ -Phases

Fourling operations were recently suggested for the construction of structures, and some applications were demonstrated (15, 44). The complex structure of the  $\chi$ -phase (45, 46) is easily understood if it is dissected to fourling units. In Fig. 20 the structure is drawn as a

fourling of  $\text{MgZn}_2$  (compare Fig. 15) and such, somewhat elliptical, units are shaded and surrounded by thin lines in the figure. The fourling operation is easy and accurate to make, as the reader can convince himself by the use of transparent paper and Fig. 15. The operation creates empty space along the fourling axis; in the crystal this space is stuffed with extra atoms to form a tetraederstern. In a rather large drawing, fourling units of the exact structure of  $\text{MgZn}_2$  were used, and atomic coordinates for this constructed structure of the  $\chi$ -phase were measured directly on the drawing. The agreement between these coordinates and those observed (46) is surprisingly good as shown in Table II.

In the lower left part of Fig. 4 it is shown how  $\text{Cr}_3\text{Si}$  and  $\text{Zr}_4\text{Al}_3$  can intergrow and form a so far unknown structure. If this structure is twinned the well-known  $\sigma$ -phase structure (47) is obtained and given in Fig. 21, where also the twin planes are shown with arrows. In this case the atomic parameters have been calculated by the twin operation and in Table III they are compared with those actually observed. The twin block size can be varied, and an

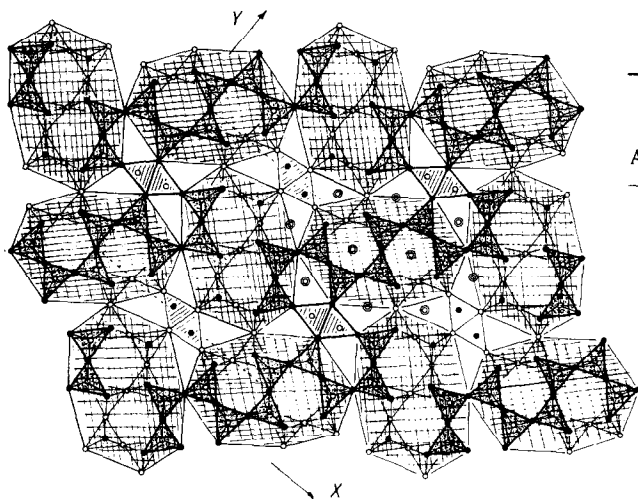


FIG. 20. The structure of the  $\chi$ -phase, constructed from fourling units (shaded and surrounded by thin lines) of  $\text{MgZn}_2$ . The drawing is not possible to distinguish from the real structure. Atoms on 0 and  $\frac{1}{2}$ , except double circles which are two atoms overlapping on  $\frac{1}{4}$  and  $\frac{3}{4}$ .

TABLE II  
THE  $\chi$ -PHASE<sup>a</sup>

Atom	Measured		Observed	
	<i>x</i>	<i>y</i>	<i>x</i>	<i>y</i>
1	0.113	0.093	0.111	0.086
2	0.179	0.400	0.176	0.398
3	0.394	0.219	0.398	0.210
4	0.057	0.276	0.055	0.278
5	0.232	0.216	0.236	0.213
6	0.214	0.00	0.216	0.992
7	0.358	0.061	0.361	0.057
8	0.500	0.310	0.499	0.311
9	0.338	0.403	0.339	0.399
10	0.500	0.500	$\frac{1}{2}$	$\frac{1}{2}$
14	0.074	0.534	0.070	0.529
15	0.230	0.591	0.235	0.587
16	0.350	0.749	0.355	0.742
17	0.206	0.808	0.209	0.802
18	0.051	0.909	0.051	0.903

<sup>a</sup>  $a/b$  (obs) = 1.24;  $a/b$  (meas) = 1.23.

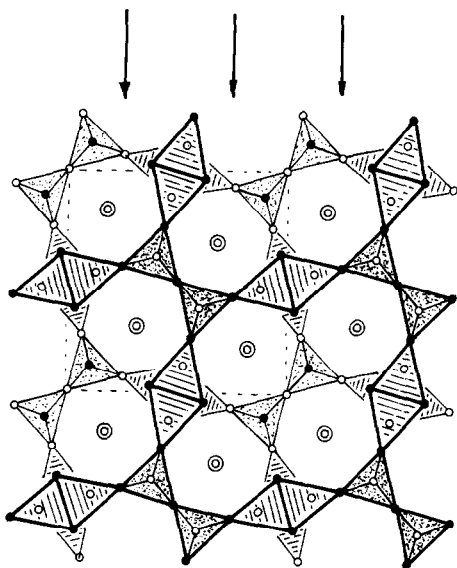


FIG. 21. The  $\sigma$ -phase structure. Arrows indicate twin planes. The drawing is constructed from a twin operation, but it is not possible to distinguish it from the real structure.

TABLE III<sup>a</sup>  
THE  $\sigma$ -PHASE

Atom	calc.	obs.
B.	$x = 0.390$	0.398
C.	$x = 0.461$ $y = 0.125$	0.463 0.132
D.	$x = 0.750$ $y = 0.067$	0.738 0.065
E.	$x = 0.183$	0.182

$$^a (a/c)_{\text{calc}} = (2 + 3^{1/2})^{1/2} = 1.93; (a/c)_{\text{obs}} = 1.94.$$

example of this is given in Fig. 22 as a hypothetical structure.

The structure of the  $\sigma$ -phase can be used to derive the  $P$ -structure (48). In Fig. 23 a part of the  $\sigma$ -phase is drawn in the right part and if this part is twinned and sheared, the  $P$ -phase structure is obtained. The  $\sigma$ -phase blocks are indicated in the figure. Measured coordinates, obtained as in the  $\chi$ -phase case, are in good agreement with the observed ones and compared in Table IV. The structure can also be derived by twinning the  $\mu$ -phase structure.

The so-called  $M$ -phase (49) can be obtained

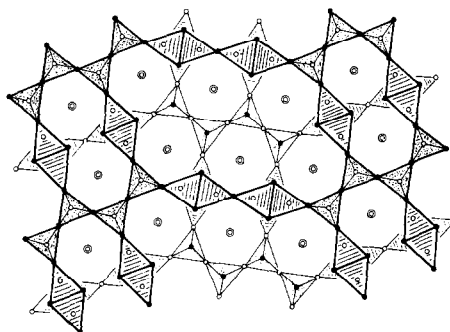


FIG. 22. The twin blocks of Fig. 21 have been expanded, so that larger units of the untwinned hypothetical structure of Fig. 4 are obtained. Hypothetical structure.

by a crystallographic shear operation from the  $P$ -phase. In Fig. 24, however, it has been derived by twinning plus shear of the  $\mu$ -phase structure. The extension and overlap (shear) of the  $\mu$ -phase blocks are shown with arrows in the figure. The measured and observed parameters are again very good and compared in Table V.

The relatively simple structure of  $(\text{Co}_{0.57}\text{Si}_{0.43})_3\text{V}_2$  (50) is easily derived through a perpendicular intergrowth of the  $\mu$ -phase structure and  $\text{MgZn}_2$ , as demonstrated in Fig. 25. The only atoms not generated by this construction are the two in the special position  $2d$  ( $0 \frac{1}{2} \frac{1}{2}$ ). A very accurate structure is derived as shown in Table VI where the measured and observed parameters are compared.

The complex structure of the so-called  $\nu$ -phase (51) is shown in Fig. 26. The structure contains large units of the  $\sigma$ -phase, marked with dotted trigonal bipyramids and shaded tetraedersterns. The  $\sigma$ -columns are intergrown along  $a$  with  $\text{Zr}_4\text{Al}_3$ , and such  $\sigma$ -phase- $\text{Zr}_4\text{Al}_3$  blocks are intergrown in the  $b$ -direction with a twinned  $\mu$ -phase structure.

## Conclusions

The observations made in this article call for experiments with high-resolution electron microscopy in order to reveal whether the

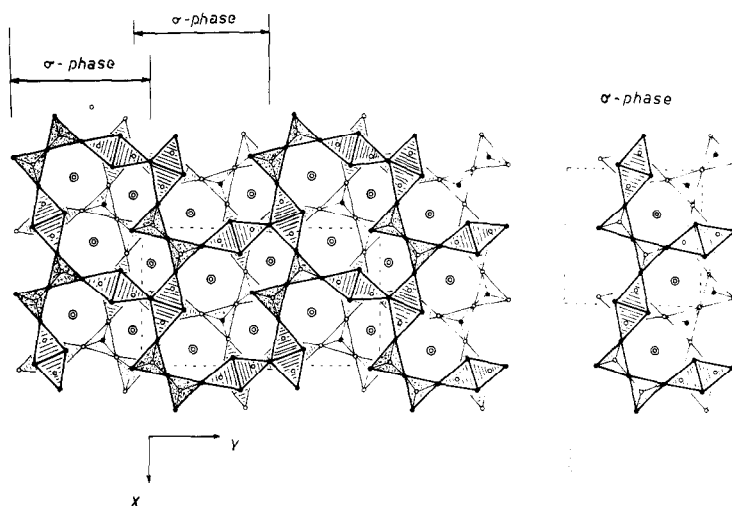


FIG. 23. The  $P$ -phase structure. To the right is shown the  $\sigma$ -phase structure, which is used in twin and shear operations to produce this  $P$ -phase drawing.

TABLE IV<sup>a</sup>  
THE  $P$ -PHASE

Atom	Measured		Observed	
	$x$	$y$	$x$	$y$
1	0.069	0.104	0.074	0.113
2	0.130	0.249	0.136	0.255
3	0.329	0.141	0.326	0.158
4	0.600	0.180	0.606	0.182
5	0.681	0.338	0.665	0.325
6	0.483	0.460	0.475	0.454
7	0.188	0.409	0.199	0.405
8	0.814	0.068	0.815	0.078
9	0.949	0.375	0.938	0.365
10	0.511	0.035	0.520	0.036
11	0.251	0.541	0.250	0.537
12	0.371	0.289	0.386	0.288

<sup>a</sup>  $(b/a)_{\text{meas}} = 1.76$ ;  $(b/a)_{\text{obs}} = 1.87$ .

ideas proposed are realistic or not. Such studies would also be fruitful when relating the nature of defect crystals with a physical property like conductivity.

The  $\text{Cr}_3\text{Si}$  structure type is common for many compounds which are good superconductors. The structural property of three-dimensional nonconnecting rods of atoms with strongly directional bonds is obviously one explanation for the superconductivity. In

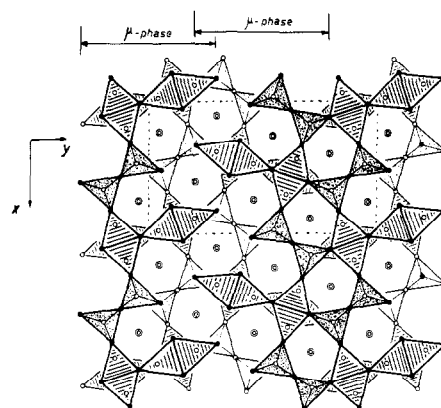


FIG. 24. The  $M$ -phase structure. It is generated by twinning and shear of the  $\mu$ -phase structure, blocks of which are shown between arrows. The structure of the drawing is very similar to the real structure.

several of the complex structures described in this article the rod-structure feature of  $\text{Cr}_3\text{Si}$  has been broken and exist only in two or one dimensions. Naturally we expect the defect structure of  $\text{Cr}_3\text{Si}$ -type crystals to be very complex, but in such studies we can be guided by the related structures discussed in this article, and which are derived by means of shear, rotation, twin, and intergrowth operations on  $\text{Cr}_3\text{Si}$ .

TABLE V<sup>a</sup>  
THE *M*-PHASE

Atom	Measured		Observed	
	<i>x</i>	<i>y</i>	<i>x</i>	<i>y</i>
1	0.062	0.147	0.059	0.149
2	0.294	0.398	0.299	0.389
3	0.521	0.539	0.524	0.541
4	0.613	0.696	0.616	0.707
5	0.027	0.446	0.014	0.448
6	0.829	0.296	0.839	0.294
7	0.070	0.626	0.071	0.623
8	0.329	0.672	0.326	0.670
9	0.828	0.579	0.817	0.578
10	0.118	0.295	0.112	0.295
11	0.264	0.541	0.255	0.545

<sup>a</sup>  $(b/a)_{\text{meas}} = 1.75$ ;  $(b/a)_{\text{obs}} = 1.75$ .

With the classification system used it is easy to derive many new hypothetical structures. Whether it is possible to make these is a matter of speculation at the moment. We know that atomic size is one important parameter and it is interesting to calculate average coordination number (ACN) for various phases (compare Ref. (53)):

#### Cr<sub>3</sub>Si, ACN 13.50

<i>Shear phases</i>	<i>ACN</i>	<i>Twin phases</i>	<i>ACN</i>
Zr <sub>4</sub> Al <sub>3</sub>	13.43	$\sigma$ -phase	13.47
Mo <sub>3</sub> CoSi	13.43	$\chi$ -phase	13.35
Fe <sub>2</sub> B	13.33		
Laves phase	13.33		
<i>Intergrowth</i>	<i>ACN</i>	<i>Twin + shear phases</i>	<i>ACN</i>
untwinned $\sigma$ (hypothetical)	13.38	<i>M</i> phase	13.38
$\mu$ -phase	13.38	<i>P</i> -phase	13.43
(Co <sub>0.57</sub> Si <sub>0.43</sub> ) <sub>3</sub> V <sub>2</sub>	13.36		
$\nu$ -phase	13.44	<i>Rotation</i>	<i>ACN</i>
		W <sub>5</sub> Si <sub>3</sub>	13.0

The CN for atoms in these complex structures is easily found using the coding system of Pearson and Shoemaker (52). The general trend in ACN is that shear and rotation operations cause lower ACN while twinning

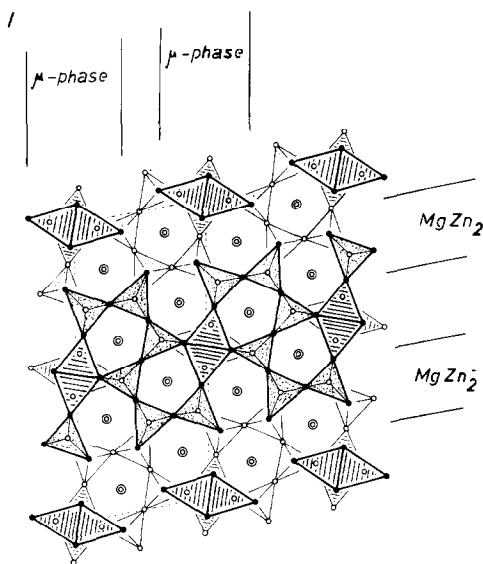


FIG. 25. The (Co<sub>0.57</sub>Si<sub>0.43</sub>)<sub>3</sub>V<sub>2</sub> structure. The perpendicular intergrowth of blocks of  $\mu$ -phase and MgZn<sub>2</sub> shown with arrows. The drawing constructed, but not possible to distinguish from the real structure.

has the opposite effect. Intergrowth gives an average in ACN of the two structures intergrowing. The  $\nu$ -phase has a rather high ACN, which of course reflects the very big

units of  $\sigma$ -phase in the structure. The  $\mu$ -phase and Laves-phase components in *M* and *P* are rather large and would give a low ACN, but ACN is increased by twin operations. It is probable that high temperature would favor

TABLE VI  
( $\text{Co}_{0.57}\text{Si}_{0.43}$ ) $_3\text{V}_2$

Atom	Measured		Observed	
	x	y	x	y
1	0.054	0.356	0.048	0.367
2	0.208	0.550	0.208	0.562
3	0.208	0.929	0.208	0.932
4	0.470	0.818	0.473	0.817
5	0.067	0.979	0.069	0.981
6	0.291	0.265	0.287	0.267
7	0.371	0.549	0.374	0.543
8	0.378	0.054	0.375	0.042
9	0.167	0.227	0.160	0.226
10	0.085	0.700	0.088	0.695

$$^a (a/b)_{\text{meas}} = 2.18; (a/b)_{\text{obs}} = 2.27.$$

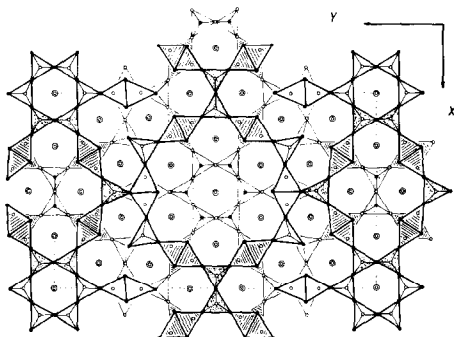


FIG. 26. The real structure of the  $v$ -phase. The large elliptical units of  $\sigma$ -phase structure are shaded and dotted.

shearing while low temperature and high pressure would favor twinning. The Hyde rotation mechanism for transforming  $\alpha$ - and  $\beta$ - $\text{V}_3\text{S}$  to the  $\text{Cr}_3\text{Si}$  type is of special interest. Surely such a phase transformation would be favored by high pressure, and new superconducting material could be prepared. For example,  $\text{Mo}_3\text{P}$  of the  $\text{Ni}_3\text{P}$  type is already a good superconductor with  $T_c = 7.0^\circ\text{K}$  and if it could be transformed to a  $\text{Cr}_3\text{Si}$  structure type it might have a considerably higher  $T_c$ .

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