

## Solid Solutions and Phase Transitions in Langbeinites (I): $M_2^+Mn_2(SO_4)_3$ ( $M^+ = K, NH_4, Tl$ )

M. L. MARTINEZ SARRIÓN,\* A. RODRÍGUEZ CLEMENTE,  
 AND L. MESTRES VILA

*Facultad de Química, Departamento de Química Inorgánica, Diagonal 647,  
 08028 Barcelona, Spain*

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Solid solutions of general formula  $K_x(NH_4)_{2-x}Mn_2(SO_4)_3$  ( $0 < x < 2$ ) and  $K_yTl_{2-y}Mn_2(SO_4)_3$  ( $0 < y < 2$ ) have been prepared. All of the phases within the composition range studied have been established, and their cubic symmetry at room temperature has been confirmed. The cell parameters for each member of the solid solutions have been determined. The substitution has been found to be homogeneous. Differential scanning calorimetry experiments of solid solution with  $x = 2.00, 1.81, \text{ and } 1.49$ , and  $y = 2.00$  and  $1.79$  have been carried out in order to study the transition mechanism in the ferroelastic langbeinite  $K_2Mn_2(SO_4)_3$ . The  $T_c$  for the phase transition  $P2_13-P2_12_1$  is  $-75.9^\circ\text{C}$ . The mixed crystals of  $NH_4^+$  show phase transition up to 10% substitution, with a decrease in both  $T_c$  and  $\Delta H$ . On the other hand, the phase transition disappears in the mixed crystals of  $Tl^+$ . The size of the  $M^+$  ion plays an important role in the phase transition. © 1989 Academic Press, Inc.

### Introduction

Recently, the langbeinite crystals,  $M_2^+Mn_2^+(SO_4)_3$ , have attracted much attention, as some of them show ferroelectric and ferroelastic phase transitions (1-4).

The prototype of langbeinites is mineral  $K_2Mg_2(SO_4)_3$ . The structure of this compound has been determined by Zemann and Zemann (5). The double sulfate salts of langbeinites are cubic with space group  $P2_13$ .

Hikita classified the phase transitions in langbeinites into two groups. Group I represented by the prototype  $Tl_2Cd_2(SO_4)_3$  has the transition scheme  $P2_13-P2_1-P1-P2_12_1$  corresponding to paraelectric/elastic-ferroelectric-ferroelectric-ferroelastic transitions (6). Group II has the transition

scheme  $P2_13-P2_12_1$  (paraelastic-ferroelastic transition). The prototype is  $K_2Mn_2(SO_4)_3$  (KMS), with a cubic-orthorhombic transition at  $-82^\circ\text{C}$  (2). Langbeinites without any transitions at atmospheric pressures belong to group III.  $K_2Mg_2(SO_4)_3$  and examined  $(NH_4)_2Mn_2(SO_4)_3$ , (AMS) and  $Tl_2Mn_2(SO_4)_3$  (TMS) belong to this class (7).

The phase transition mechanisms of these crystals are far from clear despite detailed dielectric, optical, and elastic measurements available (8-11). In the langbeinite  $K_2Cd_2(SO_4)_3$ , X-ray structural studies revealed that the  $SO_4^{2-}$  ions rotate slowly with temperature (12). Yamada *et al.* (13) suggest that in the cubic phase there are two statistically equivalent positions for each oxygen atom. In the ordered

orthorhombic phase, one of the equivalent positions is chosen along with a small displacement of the  $\text{SO}_4^{2-}$  ion.

Kreske and Devarajan (3) studied the phase transitions in  $\text{K}_2\text{Mn}_2(\text{SO}_4)_3$  by means of vibrational spectroscopy. This group of crystals, if characterized by displacive type of phase transition, should show the presence of soft modes in the Raman spectra in both cubic and orthorhombic phases. The evidence for the phase transition is clearly seen in the splitting of the  $\text{SO}_4^{2-}$  internal vibration Raman bands in the cubic and orthorhombic phases. For a possible mechanism of the phase transition an order-disorder mechanism can be considered which is associated with the ordering of the  $\text{SO}_4^{2-}$  ions; but there are different possible types of ordering.

More recently, Speer and Salje (4) asserted that, whereas the  $\text{SO}_4^{2-}$  tetrahedra in these compounds are almost undistorted, the two symmetry-independent coordination polyhedra of  $M^{2+}$  are highly distorted octahedra with trigonal site symmetry  $P2_13$ . The deformation of the oxygen octahedra and the off centering of  $M^{2+}$  along the trigonal axis show systematic dependencies on the ionic radii and the electronegativities of the  $M^{2+}$  ions, indicating different  $M^{2+}$ -O bonding.

The octahedral deformations show also linear correlations with the phase transition temperatures ( $P2_13$ - $P2_12_12_1$ ) of the different compounds. This observation leads to a model for the phase transition mechanism which is based on thermal instabilities of the  $M^{2+}$ -O and  $M^+-O$  polyhedral distortions. The cubic, high-temperature phase is characterized by high symmetric oxygen coordinations around  $M^{2+}$  which show increasing distortion with decreasing temperature.

By studying the formation of solid solutions (replacing the metal  $M^+$  in langbeinite showing phase transition) we wish to contribute to the knowledge of the mechanism

by which the corresponding transition occurs.

### Experimental

The systems investigated are  $\text{K}_x(\text{NH}_4)_{2-x}\text{Mn}_2(\text{SO}_4)_3$  (KAMS) and  $\text{K}_y\text{Tl}_{2-y}\text{Mn}_2(\text{SO}_4)_3$  (KTMS) over the composition ranges  $2.00 > x > 0$  and  $2.00 > y > 0$ .

The starting langbeinites KMS, AMS, and TMS, were obtained by evaporation of the 1:2 molar ratio aqueous solution of analytical-grade  $\text{K}_2\text{SO}_4$ - $\text{MnSO}_4 \cdot \text{H}_2\text{O}$ ,  $(\text{NH}_4)_2\text{SO}_4$ - $\text{MnSO}_4 \cdot \text{H}_2\text{O}$ , and  $\text{Tl}_2\text{SO}_4$ - $\text{MnSO}_4 \cdot \text{H}_2\text{O}$ , at  $120^\circ\text{C}$  (14) (thermostated bath with ethyleneglycol).

The mixed crystals of general formula KAMS and KTMS were obtained mixing aqueous solution of the langbeinites KMS-AMS and KMS-TMS in varying proportions.

The cubic symmetry at room temperature for these crystals was confirmed by X-ray powder diffraction. The samples were mounted on a Siemens automatic diffractometer, D500-DACOMP. Ni-filtered,  $\text{CuK}\alpha$  radiation was used with a scan speed equal to  $\frac{1}{2}\theta \text{ min}^{-1}$  and a  $\theta$  range of  $5$ - $30^\circ$ .

The indexation was carried out from the values of  $\text{K}_2\text{Mn}_2(\text{SO}_4)_3$  (PDF 20-209). Cell parameters for each phase were refined by a full-matrix least-squares method from the cell parameters of  $\text{K}_2\text{Mn}_2(\text{SO}_4)_3$ . The computer program used was AFFMAIL (15). Atomic absorption analyses were carried out on a spectrophotometer PYE UNICAM S.P. 1900. Elemental analyses were carried out in a Perkin-Elmer 240 microanalyzer. The infrared spectra were recorded on a Perkin-Elmer 1330 spectrophotometer; samples prepared as KBr pellets or Nujol mulls gave the same spectra. The heat flows of the samples were measured with a Perkin-Elmer differential scanning calorimetric DSC 7 in the temperature range  $+30$  to  $-170^\circ\text{C}$  and at a rate of  $10.0^\circ\text{C}/\text{min}$ . The

TABLE I  
ANALYSES OF SOLID SOLUTIONS  
 $K_x(NH_4)_{2-x}Mn_2(SO_4)_3$

	% K	% NH <sub>4</sub>
KMS	16.39	—
KAMS-1	14.97	0.64
KAMS-2	14.32	0.95
KAMS-3	12.53	1.81
KAMS-4	9.90	3.18
KAMS-5	7.70	4.26
KAMS-6	6.19	5.05
KAMS-7	3.10	6.61
AMS	—	8.30

measurements were made on polycrystalline samples.

### Results

The results of the solid solution analyses are given in Tables I and II. The composition of the different phases has been established by chemical analyses. The results are given in Tables III and IV. The results of X-ray diffraction are shown in Tables V and VI.

The IR absorption spectra clearly show the internal vibrational modes ( $\nu_1$ ,  $\nu_2$ ,  $\nu_3$ , and  $\nu_4$ ) of the tetrahedral SO<sub>4</sub> group in all of

TABLE II  
ANALYSES OF SOLID SOLUTIONS  
 $K_yTl_{2-y}Mn_2(SO_4)_3$

	% K	% Tl
KMS	16.39	—
KTMS-1	13.81	7.66
KTMS-2	10.64	15.92
KTMS-3	8.66	20.62
KTMS-4	6.59	26.76
KTMS-5	5.15	29.90
KTMS-6	3.38	35.25
KTMS-7	1.78	40.11
TMS	—	50.67

TABLE III  
COMPOSITION OF MIXED CRYSTALS  
 $K_x(NH_4)_{2-x}Mn_2(SO_4)_3$

KMS	$K_2Mn_2(SO_4)_3$
KAMS-1	$K_{1.81}(NH_4)_{0.17}Mn_2(SO_4)_3$
KAMS-2	$K_{1.72}(NH_4)_{0.25}Mn_2(SO_4)_3$
KAMS-3	$K_{1.49}(NH_4)_{0.47}Mn_2(SO_4)_3$
KAMS-4	$K_{1.16}(NH_4)_{0.81}Mn_2(SO_4)_3$
KAMS-5	$K_{0.89}(NH_4)_{1.08}Mn_2(SO_4)_3$
KAMS-6	$K_{0.71}(NH_4)_{1.26}Mn_2(SO_4)_3$
KAMS-7	$K_{0.35}(NH_4)_{1.62}Mn_2(SO_4)_3$
AMS	$(NH_4)_2Mn_2(SO_4)_3$

the crystals:  $\nu_1$  above 1000 cm<sup>-1</sup>,  $\nu_2$  at 470 cm<sup>-1</sup>,  $\nu_3$  between 1170 and 1080 cm<sup>-1</sup>, and  $\nu_4$  between 640 and 605 cm<sup>-1</sup>. All samples with NH<sub>4</sub><sup>+</sup> show  $\nu_3$  and  $\nu_4$  vibrations in the region 1410–3200 cm<sup>-1</sup>.

Figure 1 shows the heat flow (mW)/temperature (°C) of  $K_2Mn_2(SO_4)_3$  (sample weight = 56.430 mg;  $T_c = -75.9^\circ\text{C}$ ;  $\Delta H = 1.626$  J/g). Figure 2 shows the heat flow (mW)/temperature (°C) of  $K_{1.81}(NH_4)_{0.17}Mn_2(SO_4)_3$  (sample weight = 54.070 mg;  $T_c = -110.9^\circ\text{C}$ ;  $\Delta H = 0.235$  J/g). Figure 3 shows the heat flow (mW)/temperature (°C) of  $K_{1.49}(NH_4)_{0.47}Mn_2(SO_4)_3$ . Figure 4 shows the heat flow (mW)/temperature (°C) of  $K_{1.79}Tl_{0.19}Mn_2(SO_4)_3$ . A small hump occurred reproducibly in all the DSC experiments at ca.  $-48^\circ\text{C}$ . Sample dependence of this small anomaly is yet to be studied. The Rb

TABLE IV  
COMPOSITION OF MIXED CRYSTALS  
 $K_yTl_{2-y}Mn_2(SO_4)_3$

KMS	$K_2Mn_2(SO_4)_3$
KTMS-1	$K_{1.79}Tl_{0.19}Mn_2(SO_4)_3$
KTMS-2	$K_{1.47}Tl_{0.51}Mn_2(SO_4)_3$
KTMS-3	$K_{1.27}Tl_{0.70}Mn_2(SO_4)_3$
KTMS-4	$K_{1.02}Tl_{0.96}Mn_2(SO_4)_3$
KTMS-5	$K_{0.84}Tl_{1.13}Mn_2(SO_4)_3$
KTMS-6	$K_{0.58}Tl_{1.40}Mn_2(SO_4)_3$
KTMS-7	$K_{0.30}Tl_{1.67}Mn_2(SO_4)_3$
TMS	$Tl_2Mn_2(SO_4)_3$

TABLE V  
CELL PARAMETERS AND VOLUME FOR EACH  
MEMBER OF THE SOLID SOLUTION  
 $K_x(NH_4)_{2-x}Mn_2(SO_4)_3$

	$a$ (Å)	$V$ (Å <sup>3</sup> )	$\epsilon$	NP
KMS	10.1194(9)	1036.2(3)	1.07	26
KAMS-1	10.125(1)	1037.9(3)	1.79	26
KAMS-2	10.130(1)	1039.6(3)	1.23	25
KAMS-3	10.137(2)	1041.7(6)	1.87	26
KAMS-4	10.151(1)	1046.1(3)	1.50	25
KAMS-5	10.163(2)	1049.8(6)	2.03	24
KAMS-6	10.169(1)	1051.6(3)	1.41	27
KAMS-7	10.182(1)	1055.6(3)	1.24	28
AMS	10.1908(6)	1058.3(2)	0.71	26

Note. NP, number of peaks used in the cell parameter determination.

$H_2PO_4$  and  $(NH_4)_{0.89}Rb_{0.11}H_2PO_4$  showed similar small humps at ca.  $-148$  and  $-175^\circ C$ , respectively (16).

### Discussion

The X-ray diffraction patterns of the samples indicated only reflections attributable to the pure phases. In order to determine the homogeneity of the substitution between the  $K^+$  and  $NH_4^+$  or  $K^+$  and  $Tl^+$  ions, a linear regression of cell parameters and cell volume has been carried out. The largest deviations from linearity are within three times the highest standard deviation,

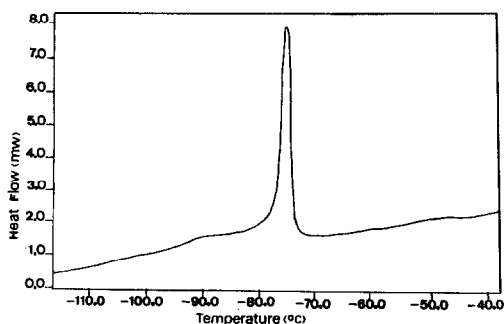


FIG. 1. Heat flow of  $K_2Mn_2(SO_4)_3$ .

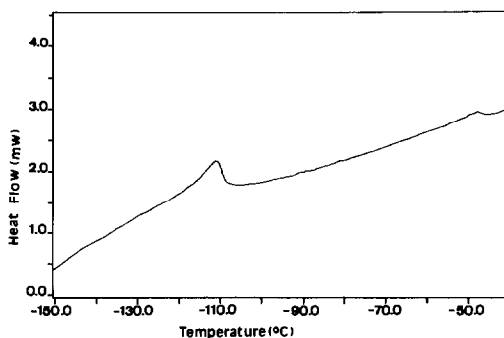


FIG. 2. Heat flow of  $K_{1.81}(NH_4)_{0.17}Mn_2(SO_4)_3$ .

to assume the substitution to be homogeneous.

The infrared spectra are typical of the  $SO_4^{2-}$  ion, and the selection rules for  $C_1$  symmetry, approximately adhered to  $\nu_1$  and  $\nu_2$  regions (symmetric vibrations), are relatively weak;  $\nu_2$  invariably appears as only one component. The stronger absorption in the  $\nu_3$  and  $\nu_4$  (asymmetric vibrations) regions and the fact that they are triply degenerate agrees with the symmetry decrease of the  $SO_4^{2-}$  groups in the langbeinites.

The atomic arrangement proposed by Zemann and Zemann (5) implies the  $SO_4$  groups to be practically ideal tetrahedra in

TABLE VI  
CELL PARAMETERS AND VOLUME FOR EACH  
MEMBER OF THE SOLID SOLUTION  $K_yTl_{2-y}Mn_2(SO_4)_3$

	$a$ (Å)	$V$ (Å <sup>3</sup> )	$\epsilon$	NP
KMS	10.1194(9)	1036.2(3)	1.07	26
KTMS-1	10.136(1)	1041.4(3)	1.33	24
KTMS-2	10.145(1)	1044.1(3)	1.40	25
KTMS-3	10.163(1)	1049.7(3)	1.27	27
KTMS-4	10.175(1)	1053.3(3)	1.82	29
KTMS-5	10.182(1)	1055.7(3)	1.40	28
KTMS-6	10.199(1)	1060.9(3)	1.28	29
KTMS-7	10.2065(9)	1063.2(3)	1.09	29
TMS	10.2236(7)	1068.7(2)	0.81	29

Note. NP, number of peaks used in the cell parameter determination.

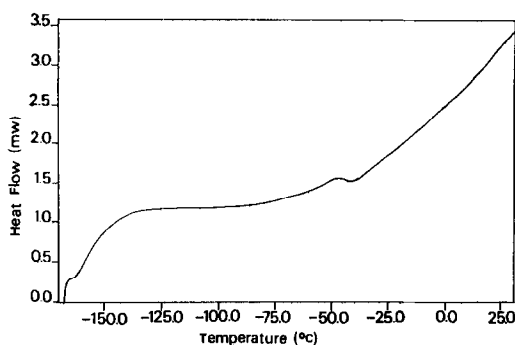


FIG. 3. Heat flow of  $K_{1.49}(NH_4)_{0.47}Mn_2(SO_4)_3$ .

$K_2Mg_2(SO_4)_3$ , which is isostructural with  $K_2Mn_2(SO_4)_3$ . This observation was confirmed by Speer and Salje (4). Distorted coordination polyhedra occur around the metal positions. The potassium atoms have 9 nearest oxygen neighbors in the cubic phase. In the orthorhombic phase there are 10 nearest neighbors for K(1) and 9 for K(2), which is similar to the orthorhombic crystal structure of  $K_2Cd_2(SO_4)_3$ .

The divalent metals are always coordinated by six oxygen atoms forming a distorted octahedron. The  $M^{2+}$  position is not at the midpoint between the two oxygen triangles around the trigonal axis. With increasing ionic radius, the octahedral central position  $M^{2+}$  is pushed from the octahedra midpoint toward one triangle of oxygen atoms. Salje shows that there is a linear correlation between the octahedral deformation and the ionic radius of  $M^{2+}$ .

In the  $P2_13-P2_12_12_1$  phase transition, there is a correlation between the critical temperature and the octahedral distortions. The structural phase transition between the cubic phase and the orthorhombic phase reduces the symmetry of the  $M^{2+}-O$  octahedra by destroying the trigonal axis. When the octahedron becomes sufficiently distorted the structural phase transition takes place.

At temperatures below  $T_c$ , the deformation of the rotation parameter, the distance

between the  $M^{2+}$  position and the geometrical midpoint of the octahedra, and the relative shrinking of one triangle face with respect to a second triangle face transform according to the orthorhombic symmetry.

At temperatures far above  $T_c$ , the octahedra  $M^{2+}O_6$  are highly symmetric; this high symmetry coordination is only possible at the expense of larger  $M^+-O$  distances and a rather poor packing around  $M^+$ . With decreasing temperatures, the coordination of  $M^{2+}$  becomes less symmetric, and the structural phase transition is expected to take place. In the orthorhombic phase, the  $MO_6$  octahedra are heavily distorted, the average  $M^{2+}-O$  distances become larger, and the  $M^+-O$  distances become shorter. Therefore, the  $M^+O$  polyhedra show a closer packing at the expense of a wider  $MO_6$  packing.

The  $SO_4$  tetrahedra shift and rotate during the phase transition to achieve the better packing of the  $M^+-O$  polyhedron.

The mechanism by which the phase transition occurs can be explained as a result of a cooperative effect. The  $SO_4^{2-}$  tetrahedra shift and rotate, causing a decrease in the  $M^+-O$  distance, thereby making possible the distortion of the  $M^{2+}O_6$  octahedra. These three factors contribute to the phase transition.

For the langbeinites containing  $SO_4^{2-}$ , Mn as the bivalent cation, and  $K^+$ ,  $NH_4^+$ , or  $Tl^+$

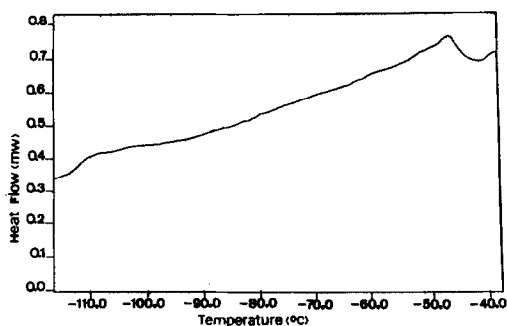


FIG. 4. Heat flow of  $K_{1.75}Tl_{0.19}Mn_2(SO_4)_3$ .

as the monovalent cation, the increase of the  $M^+$  radius complicates the packing of the  $MnO_6$  octahedra in such a way that either the transition does not take place or it takes place at a temperature too low to be detected by our equipment.

A critical size for  $M^{2+}$  should exist below which the transition does not occur, since an increase in the  $M^{2+}$  size favors the transition. In fact for  $M^{2+} = Mn^{2+}$  the transition is only observed when  $M^+$  is potassium, but for  $M^{2+} = Cd^{2+}$  the langbeinites of Tl, Rb,  $NH_4$ , and K do show transition (7).

For the langbeinite  $K_2Mn_2(SO_4)_3$ , the size of the potassium ion allows the operation of the cooperative mechanism and the occurrence of the phase transition. If the substitution of  $NH_4^+$  for  $K^+$  is less than ca. 10%, there is still transition; however, there is a decrease in both  $T_c$  and  $\Delta H$ .

The limit of substitution of  $K^+$  by  $Tl^+$  is less than 10% (agreeing with the larger radius of  $Tl^+$ ). No phase transition is detected even at the lowest  $Tl^+$  concentration. In fact, the only thallium langbeinite showing transition is that for which the bivalent metal is cadmium. The size of  $M^+$  is offset by the size increase of  $M^{2+}$ , in agreement with the mechanism proposed. We can assume that real ferroelastic crystals of  $Tl_2Cd_2(SO_4)_3$  as well as  $K_2Mn_2(SO_4)_3$  are made up of ferroelastic domains. It is known (6) that three different orientations of those domains exist. Application of a mechanical field could cause favorably oriented domains to grow at the expense of the disadvantageously oriented domains. The presence of  $M^+-O$  polyhedra other than  $K^+-O$  polyhedra probably complicates the domain ordering. The existence of  $NH_4^+-O$  or  $Tl^+-O$  polyhedra replacing  $K^+-O$  polyhedra

precludes a better packing and the distortion of the  $MnO_6$  octahedrons, as well as the decrease of symmetry accompanying the phase transition to the orthorhombic phase.

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