

## Analysis of the Optical Spectra and Magnetic Susceptibilities of $\text{Li}_7\text{UO}_6$ , $\text{Ba}_3\text{NpO}_6$ , and $\text{Li}_5\text{PuO}_6$

YUKIO HINATSU

*Department of Chemistry, Japan Atomic Energy Research Institute,  
Tokai-mura, Ibaraki 319-11, Japan*

AND NORMAN EDELSTEIN

*Materials and Chemical Sciences Division, Lawrence Berkeley Laboratory,  
University of California, Berkeley, California 94720*

Received July 16, 1990; in revised form January 21, 1991

The absorption spectra and magnetic susceptibilities of  $\text{Li}_7\text{UO}_6$ ,  $\text{Ba}_3\text{NpO}_6$ , and  $\text{Li}_5\text{PuO}_6$  obtained from published data are reanalyzed on the basis of an octahedral crystal-field model with/without a small tetragonal distortion. © 1991 Academic Press, Inc.

### 1. Introduction

Contrary to the situation for the  $4f^n$  configurations of rare earths, it is usually impossible to treat the crystal field acting on  $5f^n$  configurations of actinide compounds as a small perturbation of atomic levels. In many cases, the crystal-field interaction, spin-orbit coupling interaction, and electronic repulsion interaction are of comparable magnitude, which makes the analysis of the experimental results complicated. However, for the actinide ions having the  $[\text{Rn}]5f^1$  electronic configuration, the situation is considerably simplified, because there is no electron-electron repulsion interaction.

Kanellakopoulos *et al.* (1) measured optical spectra and magnetic susceptibilities for a number of uranates ( $\text{U}^{5+}$ ), neptunates ( $\text{Np}^{6+}$ ), and one plutonate ( $\text{Pu}^{7+}$ ) and fitted their data to the theory developed by

Eisenstein and Pryce (2) with an additional parameter to account for a tetragonal distortion of the crystal field. However, an added empirical temperature-independent paramagnetic susceptibility was necessary in order to obtain a satisfactory fit of the magnetic susceptibility data as a function of temperature. Edelstein and Goffart (3) questioned the use of a spin-orbit coupling constant by Kanellakopoulos *et al.* which was almost constant for  $\text{U}^{5+}$ ,  $\text{Np}^{6+}$ , and  $\text{Pu}^{7+}$  ( $1800 \sim 1900 \text{ cm}^{-1}$ ), because the spin-orbit coupling constant is known to increase markedly with higher atomic number and higher charge on the ion (4). For some compounds, the orbital reduction factors obtained show the relation  $k < k'$ , which is an improbable result (5) because the orbital reduction factor  $k$  is associated with overlap of the  $t_{2u}$  orbitals of the  $5f$  electrons with  $p_\pi$  electrons on the oxygen; on the other hand the orbital reduction

factor  $k'$  is related to the antibonding of the  $5f\delta$  orbitals ( $t_{1u}$ ) with  $2p_\sigma$  electrons on the oxygen atoms. Eisenstein and Pryce pointed out that  $k$  should be very nearly unity (2).

We focus attention in this paper on the magnetic and optical properties of  $\text{Li}_7\text{UO}_6$ ,  $\text{Ba}_3\text{NpO}_6$ , and  $\text{Li}_5\text{UO}_6$ . Although electron paramagnetic resonance (EPR) measurements have not been carried out yet for  $\text{Ba}_3\text{NpO}_6$  and  $\text{Li}_5\text{PuO}_6$ , we may obtain the ground state  $g$ -values from their magnetic susceptibilities, because the susceptibilities are linear against reciprocal temperature over the temperature range from 4.2 to 300 K (1).

In this paper, we obtain the crystal-field parameters of  $\text{Li}_7\text{UO}_6$ ,  $\text{Ba}_3\text{NpO}_3$ , and  $\text{Li}_5\text{PuO}_6$  by fitting the calculations not only to the transition energies but also to the ground state  $g$ -values, and then calculate the magnetic susceptibilities (including the temperature-independent paramagnetism (TIP)) followed by comparison with the experimental data.

## 2. Calculation Method

The crystal structure of  $\text{Ba}_3\text{NpO}_6$  is a cubic perovskite (6). Although the optical absorption spectra measured by Kanellakopoulos *et al.* are not clear, it is evident that there is no splitting of the  $\Gamma_8$  quartet level (1) (see Fig. 1), which means that the symmetry of the crystal-field coordination around a neptunium ion is still octahedral.  $\text{Li}_5\text{PuO}_6$  is isomorphous with  $\text{Li}_5\text{ReO}_6$ ; this crystallizes in a hexagonal structure (space group:  $P3_112$ ). This compound contains isolated  $\text{PuO}_6$  octahedra (6). According to Kanellakopoulos *et al.* (1), the optical spectra of this compound show that the oxygen octahedron is tetragonally distorted. The crystal structure of  $\text{Li}_7\text{UO}_6$  is hexagonal and of the  $\text{Li}_7\text{SbO}_6$  type with the distorted octahedral coordination around a uranium ion (6). Kanellakopoulos *et al.* measured the optical

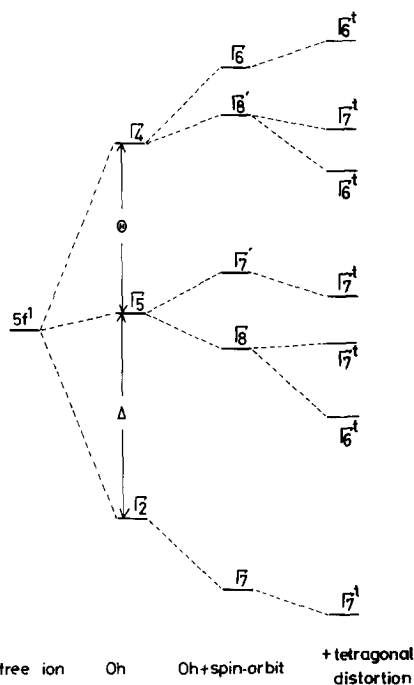


FIG. 1.  $f^1$  orbital energy splitting perturbed by octahedral crystal-field, spin-orbit coupling, and tetragonal crystal-field distortion.

spectra of this compound and reported that although the quartet  $\Gamma_8$  state splits into two doublets the energy separation of which is  $560\text{ cm}^{-1}$ , the splitting of the higher energy quartet  $\Gamma_8'$  state is not observed experimentally (Fig. 1). The distortion from octahedral symmetry seems to be small. We analyze the optical spectra and magnetic susceptibility data by assuming a tetragonal crystal-field distortion on the electronic states of a  $5f$  electron in octahedral coordination.

Figure 1 shows the effects of perturbing the  $f^1$  orbital energy levels successively by an octahedral crystal-field, spin-orbit coupling, and tetragonal crystal-field distortion. In an octahedral crystal field, the sevenfold degenerate energy state of the  $f$  orbitals is split into  $\Gamma_2$ ,  $\Gamma_5$ , and  $\Gamma_4$  states, where  $\Delta$  and  $\Theta$  represent the parameters

of the intensity of the crystal field. If spin-orbit coupling is taken into account, the  $\Gamma_2$  orbital state is transformed into  $\Gamma_7$ , whereas the  $\Gamma_5$  and  $\Gamma_4$  states are split into  $\Gamma_7^*$  and  $\Gamma_8$ , and  $\Gamma_8^*$  and  $\Gamma_6$ , respectively. The wave functions for these states are

$$\left\{ \begin{array}{l} |\Gamma_7^1\rangle = \sqrt{\frac{1}{2}}|2^+\rangle - \sqrt{\frac{1}{2}}|-\bar{2}^+\rangle \\ |\Gamma_7^2\rangle = \sqrt{\frac{1}{2}}|-\bar{2}^+\rangle - \sqrt{\frac{1}{2}}|2^+\rangle \\ |\Gamma_7^{*1}\rangle = \sqrt{\frac{1}{6}}|2^+\rangle + \sqrt{\frac{1}{6}}|-\bar{2}^+\rangle \\ \quad - \sqrt{\frac{5}{12}}|-\bar{1}^+\rangle + \frac{1}{2}|3^+\rangle \\ |\Gamma_7^{*2}\rangle = \sqrt{\frac{1}{6}}|2^+\rangle + \sqrt{\frac{1}{6}}|-\bar{2}^+\rangle \\ \quad - \sqrt{\frac{5}{12}}|1^+\rangle + \frac{1}{2}|-\bar{3}^+\rangle \\ |\Gamma_8^1\rangle = \sqrt{\frac{1}{3}}|2^+\rangle + \sqrt{\frac{1}{3}}|-\bar{2}^+\rangle \\ \quad + \sqrt{\frac{5}{24}}|-\bar{1}^+\rangle - \sqrt{\frac{1}{3}}|3^+\rangle \\ |\Gamma_8^2\rangle = \sqrt{\frac{1}{3}}|2^+\rangle + \sqrt{\frac{1}{3}}|-\bar{2}^+\rangle \\ \quad + \sqrt{\frac{5}{24}}|1^+\rangle - \sqrt{\frac{1}{3}}|-\bar{3}^+\rangle \quad (1) \\ |\Gamma_8^3\rangle = \sqrt{\frac{3}{8}}|3^+\rangle - \sqrt{\frac{5}{8}}|-\bar{1}^+\rangle \\ |\Gamma_8^4\rangle = \sqrt{\frac{3}{8}}|-\bar{3}^+\rangle - \sqrt{\frac{5}{8}}|1^+\rangle \\ |\Gamma_6^1\rangle = \sqrt{\frac{1}{3}}|0^+\rangle + \frac{1}{2}|-\bar{1}^+\rangle + \sqrt{\frac{5}{12}}|3^+\rangle \\ |\Gamma_6^2\rangle = \sqrt{\frac{1}{3}}|0^+\rangle + \frac{1}{2}|1^+\rangle + \sqrt{\frac{5}{12}}|-\bar{3}^+\rangle \\ |\Gamma_8^{*1}\rangle = \sqrt{\frac{3}{8}}|-\bar{1}^+\rangle + \sqrt{\frac{5}{8}}|3^+\rangle \\ |\Gamma_8^{*2}\rangle = \sqrt{\frac{3}{8}}|1^+\rangle + \sqrt{\frac{5}{8}}|-\bar{3}^+\rangle \\ |\Gamma_8^{*3}\rangle = -\sqrt{\frac{3}{8}}|0^+\rangle + \sqrt{\frac{1}{8}}|-\bar{1}^+\rangle + \sqrt{\frac{5}{24}}|3^+\rangle \\ |\Gamma_8^{*4}\rangle = -\sqrt{\frac{3}{8}}|0^+\rangle + \sqrt{\frac{1}{8}}|1^+\rangle + \sqrt{\frac{5}{24}}|-\bar{3}^+\rangle. \end{array} \right.$$

The ground state Kramers doublet is the  $\Gamma_7$  state and is coupled to the excited  $\Gamma_7^*$  state, arising from the  $\Gamma_5$  orbital, by spin-orbit coupling. The  $\Gamma_8$  state arising from the  $\Gamma_5$  orbital state is also coupled to the  $\Gamma_8^*$  state arising from the  $\Gamma_4$  orbital state by the same spin-orbit coupling interaction. The energy matrices for the  $\Gamma_7$ ,  $\Gamma_8$ , and  $\Gamma_6$  are

$$\Gamma_6: \left| \begin{array}{c} \Delta + \Theta + \frac{3}{2}k_{\Gamma_4\Gamma_4}\zeta \\ \Delta + \frac{1}{4}k_{\Gamma_5\Gamma_5}\zeta \quad \frac{3}{4}\sqrt{5}k_{\Gamma_4\Gamma_5}\zeta \\ \frac{3}{4}\sqrt{5}k_{\Gamma_3\Gamma_3}\zeta \quad \Delta + \Theta - \frac{3}{4}k_{\Gamma_5\Gamma_5}\zeta \end{array} \right| \quad (2)$$

$$\Gamma_7: \left| \begin{array}{c} 0 \quad \sqrt{3}k_{\Gamma_2\Gamma_5}\zeta \\ \sqrt{3}k_{\Gamma_2\Gamma_5}\zeta \quad \Delta - \frac{1}{2}k_{\Gamma_3\Gamma_3}\zeta \end{array} \right|.$$

Here  $\zeta$  is the spin-orbit coupling constant

and  $k_{\Gamma_i\Gamma_j}$  are the orbital reduction factors. It does not appear profitable to attempt to interpret the experimental data with so many parameters. Therefore, we assume, as Eisenstein and Pryce (2) and Hecht *et al.* (5) did, that there are only two orbital reduction parameters  $k$  and  $k'$ , which are characteristic of the  $\Gamma_5$  and  $\Gamma_4$  states, respectively. This assumption sets  $k_{\Gamma_4\Gamma_4} = k'$ ,  $k_{\Gamma_5\Gamma_5} = k$ ,  $k_{\Gamma_2\Gamma_5} = k^{1/2}$ , and  $k_{\Gamma_4\Gamma_5} = k^{1/2}k'^{1/2}$  in the above energy matrices. Diagonalization of the  $\Gamma_7$  matrix produces the ground state  $\Gamma_7$  and the excited state  $\Gamma_7'$ , and the corresponding wavefunctions are written as

$$\begin{aligned} |\Gamma_7\rangle &= \cos\theta|^2F_{5/2}, \Gamma_7\rangle \\ &\quad - \sin\theta|^2F_{7/2}, \Gamma_7^*\rangle \quad (3) \\ |\Gamma_7'\rangle &= \sin\theta|^2F_{5/2}, \Gamma_7\rangle \\ &\quad + \sin\theta|^2F_{7/2}, \Gamma_7^*\rangle, \end{aligned}$$

where  $\theta$  is the parameter describing the admixture of the  $\Gamma_7$  levels in the ground state with the relation

$$\tan 2\theta = \frac{2\sqrt{3}k\zeta}{\Delta - \frac{1}{2}k\zeta}. \quad (4)$$

Similarly, diagonalization of the  $\Gamma_8$  matrix produces the two levels  $\Gamma_8$  and  $\Gamma_8'$ , and the corresponding wavefunctions are

$$\begin{aligned} |\Gamma_8\rangle &= \cos\varphi|^2F_{5/2}, \Gamma_8\rangle \\ &\quad - \sin\varphi|^2F_{7/2}, \Gamma_8^*\rangle \quad (5) \\ |\Gamma_8'\rangle &= \sin\varphi|^2F_{5/2}, \Gamma_8\rangle \\ &\quad + \cos\varphi|^2F_{7/2}, \Gamma_8^*\rangle, \end{aligned}$$

where  $\varphi$  is the parameter describing the admixture of the  $\Gamma_8$  levels in the excited state, and

$$\tan 2\varphi = \frac{\frac{3}{2}\sqrt{5}kk'\zeta}{\Theta - \frac{k+3k'}{4}\zeta}. \quad (6)$$

The energies for the  $\Gamma_7$ ,  $\Gamma_8$ ,  $\Gamma_7'$ ,  $\Gamma_8'$ ,  $\Gamma_6$  (in the order of ascending energies) are

$$\begin{aligned}
 E(\Gamma_6) &= \Delta + \Theta + \frac{3}{2}k'\zeta \\
 E(\Gamma'_8) &= \Delta + \frac{1}{4}\{k + 3\sqrt{5kk'} \cot \varphi\}\zeta \\
 E(\Gamma'_7) &= \sqrt{3k}\zeta \cot \theta \\
 E(\Gamma_8) &= \Delta + \Theta \\
 &\quad - \frac{3}{4}\{k' + \sqrt{5kk'} \cot \varphi\}\zeta
 \end{aligned} \tag{7}$$

$$E(\Gamma_7) = \Delta - \frac{1}{2}\{k + 2\sqrt{3k} \cot \theta\}\zeta.$$

The effect of the tetragonal distortion is that the ground state  $\Gamma_7$  is transformed into  $\Gamma'_7$ , whereas the excited states are split or transformed according to  $\Gamma_8 \rightarrow \Gamma'_6 + \Gamma'_7$ ,  $\Gamma'_7 \rightarrow \Gamma'_7$ ,  $\Gamma'_8 \rightarrow \Gamma'_6 + \Gamma'_7$ , and  $\Gamma_6 \rightarrow \Gamma'_6$  (Fig. 1). This effect introduces three additional terms into the Hamiltonian in addition to the octahedral terms,

$$V_{\text{tet}} = \tau V_2^0 + \gamma V_4^0 + \delta V_6^0. \tag{8}$$

Here  $\tau$ ,  $\gamma$ , and  $\delta$  are parameters which depend on the radial functions, and  $V_2^0$ ,  $V_4^0$ , and  $V_6^0$  transform like the spherical harmonics of order 2, 4, and 6, respectively. To reduce the number of parameters, and since the  $\tau V_2^0$  term is expected to be dominant, we take only this term into account, as do Selbin *et al.* (7) and Kanellakopoulos *et al.* (1). The complete energy matrices for the tetragonal  $\Gamma_7$  and  $\Gamma_6$ , and the corresponding wavefunctions, are

$$\Gamma_7: \begin{array}{cc}
 \Gamma_7^1 & \Gamma_7^{*1} \\
 \Gamma_7^2 & \Gamma_7^{*2} \\
 \left| \begin{array}{cc}
 0 & \sqrt{3k}\zeta \\
 \sqrt{3k}\zeta & \Delta - \frac{1}{2}k\zeta \\
 0 & 0 \\
 0 & \sqrt{10}\tau
 \end{array} \right. \\
 \Gamma_8^1 & \Gamma_8^{*1} \\
 \Gamma_8^2 & \Gamma_8^{*2} \\
 0 & 0 \\
 0 & \sqrt{10}\tau \\
 \Delta + \frac{1}{4}k\zeta & \frac{3}{4}\sqrt{5kk'}\zeta + \sqrt{5}\tau \\
 \frac{3}{4}\sqrt{5kk'}\zeta + \sqrt{5}\tau & \Delta + \Theta - \frac{3}{4}k'\zeta - 2\tau
 \end{array} \tag{9}$$

$$\Gamma_6: \begin{array}{cc}
 \Gamma_6^3 & \Gamma_6^{*3} \\
 \Gamma_6^4 & \Gamma_6^{*4} \\
 \left| \begin{array}{cc}
 \Delta + \frac{1}{4}k\zeta & \frac{3}{4}\sqrt{5kk'}\zeta - \sqrt{5}\tau \\
 \frac{3}{4}\sqrt{5kk'}\zeta - \sqrt{5}\tau & \Delta + \Theta - \frac{3}{4}k'\zeta + 2\tau \\
 \sqrt{10}\tau & 2\sqrt{2}\tau
 \end{array} \right. \\
 \Gamma_6^1 & \\
 \Gamma_6^2 & \\
 \sqrt{10}\tau & \\
 2\sqrt{2}\tau & \\
 \Delta + \Theta + \frac{1}{2}k'\zeta &
 \end{array} \tag{10}$$

Now we can calculate the transition energies by diagonalizing the energy matrices, Eqs. (9) and (10). Although the four  $\Gamma_7^i$  doublets interact with each other through this tetragonal crystal-field term, the transition energy between  $\Gamma_7^i(\Gamma_7) \rightarrow \Gamma_7^j(\Gamma_7)$  is nearly unaltered (7, 8).

In the present compounds, no magnetic exchange interaction was found down to 4.2 K. In the paramagnetic temperature range, the magnetic susceptibility of the molecule is given by the equation

$$\chi = \frac{N\beta^2 \sum_i [(E_i^{(1)})^2/kT - 2E_i^{(2)}] \exp(-E_i^0/kT)}{\sum_i \exp(-E_i^0/kT)}, \tag{11}$$

where  $N$  is the Avogadro's number,  $\beta$  the Bohr magneton, and  $E_i$  the energy of the  $i$ th energy level, which can be expected as a power series in the magnetic field,  $H$ ,

$$E_i = E_i^0 + E_i^{(1)}H + E_i^{(2)}H^2 + \dots \tag{12}$$

Since the separation of levels within the ground state (when the magnetic field is applied) is much smaller and the energy of the next excited state is much larger compared to  $kT$ , the susceptibility is expressed by the form (9)

$$\chi = \frac{Ng^2\beta^2}{4kT} + \text{TIP}, \quad (13)$$

where

$$g = 2\langle\Gamma_7|\ell + 2s|\Gamma_7\rangle$$

$$\text{TIP} = 2N\beta^2 \sum_i \frac{|\langle\Gamma_i|\ell + 2s|\Gamma_7\rangle|^2}{E(\Gamma_i) - E(\Gamma_7)}. \quad (14)$$

When the symmetry of the crystal field is octahedral, the  $g$ -value of the ground  $\Gamma_7$  state is expressed by the following simple equation after diagonalizing energy matrix (9):

$$g = 2 \cos^2 \theta - 4 \sqrt{\frac{k}{3}} \sin 2\theta - \frac{2}{3}(1-k) \sin^2 \theta. \quad (15)$$

For the cases of  $\text{Li}_7\text{UO}_6$  and  $\text{Li}_3\text{PuO}_6$  in which the octahedral coordination around an actinide ion is tetragonally distorted, the magnetic susceptibility is no longer isotropic, i.e., both  $g$  and TIP are anisotropic. Since the wavefunctions of the ground  $\Gamma_7$  doublets are written as

$$|\Gamma_7\rangle = C_1|\Gamma_7^1\rangle + C_2|\Gamma_7^{*1}\rangle + C_3|\Gamma_8^1\rangle + C_4|\Gamma_8^{*1}\rangle \quad (16)$$

$$|\bar{\Gamma}_7\rangle = C_1|\Gamma_7^2\rangle + C_2|\Gamma_7^{*2}\rangle + C_3|\Gamma_8^2\rangle + C_4|\Gamma_8^{*2}\rangle,$$

the  $g_{\perp}$ -value is calculated to be

$$g_{\perp} = 2\langle\Gamma_7|\ell_z + 2s_z|\Gamma_7\rangle$$

$$= 2 \left\{ C_1^2 + \frac{1}{3}(k-1)C_2^2 + (\frac{1}{3}k + \frac{1}{3})C_3^2 + (\frac{2}{3}k' - 1)C_4^2 + \frac{4}{\sqrt{3}}\sqrt{k}C_1C_2 + 4\sqrt{\frac{2}{3}}\sqrt{k}C_1C_3 - \frac{\sqrt{2}}{3}(k-4)C_2C_3 + \sqrt{10kk'}C_2C_4 - \sqrt{5kk'}C_3C_4 \right\}. \quad (17)$$

Here  $|\bar{\Gamma}_7\rangle$  is the Kramers conjugate wavefunction for the ground  $\Gamma_7$  doublet. If we drop the  $C_3^2$ ,  $C_4^2$ , and  $C_3C_4$  terms because  $C_3, C_4 \ll C_1, C_2$ ,  $g_{\perp}$  is rewritten as

$$g_{\perp} = 2 \left\{ C_1^2 + 4 \sqrt{\frac{k}{3}}C_1C_2 - \frac{1}{3}(1-k)C_2^2 \right\} + 2 \left\{ 4 \sqrt{\frac{2k}{3}}C_1C_3 - \frac{\sqrt{2}}{3}(k-4)C_2C_3 + \sqrt{10kk'}C_2C_4 \right\} = g_0 + 2\gamma, \quad (18)$$

where

$$g_0 = 2 \left\{ C_1^2 + 4 \sqrt{\frac{k}{3}}C_1C_2 - \frac{1}{3}(1-k)C_2^2 \right\}$$

$$\gamma = 4 \sqrt{\frac{2k}{3}}C_1C_3 - \frac{\sqrt{2}}{3}(k-4)C_2C_3 + \sqrt{10kk'}C_2C_4. \quad (19)$$

The equation for  $g_0$  corresponds to the one for the ground  $\Gamma_7$  state of the  $5f^1$  electron in an octahedral symmetry (Eq. (15)). For  $g_{\perp}$ , we get the similar equation

$$g_{\perp} = 2\langle\Gamma_7|\ell_x + 2s_x|\bar{\Gamma}_7\rangle = g_0 - \gamma. \quad (20)$$

As predicted, the  $g$ -value becomes anisotropic when the crystal field around an actinide ion is tetragonally distorted. Equations (18) and (20) are the same with those of  $g_{\parallel}$  and  $g_{\perp}$  for which the octahedral coordination is trigonally distorted (10). The average  $g$ -value,  $\bar{g}$ , will be calculated as

$$|\bar{g}| = \sqrt{\frac{1}{3}g_{\parallel}^2 + \frac{2}{3}g_{\perp}^2}. \quad (21)$$

Since the  $g$ -values for  $\text{Cs}_2\text{PaCl}_6$  and  $\text{NpF}_6$  were empirically determined to be negative (11, 12), the signs of the  $g$ -values for the isoelectronic  $\text{U}^{5+}$ ,  $\text{Np}^{6+}$ , and  $\text{Pu}^{7+}$  are all assumed to be negative. Since EPR data are not available, we obtain the  $g$ -value for the ground  $\Gamma_7$  state from the temperature-

dependent part of the magnetic susceptibility, as shown in Eq. (13).

### 3. Results and Discussion

#### 3.1. $\text{Li}_7\text{UO}_6$

We have to determine the crystal-field parameters  $\Delta$  and  $\Theta$ , the spin-orbit coupling constant  $\zeta$ , the orbital reduction factors  $k$  and  $k'$ , and the tetragonal distortion parameter  $\tau$ . Since the splitting of the quartet  $\Gamma'_8$  level is not experimentally observed, the degree of tetragonal distortion is considered to be fairly small, which is elucidated by the following energy level calculation. Thus we first determine approximately the crystal-field parameters by assuming octahedral symmetry around the central uranium ion. We can use Eq. (7) to calculate transition energies. The transition energy for  $\Gamma_7 \rightarrow \Gamma'_7$  is expressed by

$$\Delta E(\Gamma_7 \rightarrow \Gamma'_7) = \frac{2\sqrt{3}k\zeta}{\sin 2\theta}. \quad (22)$$

From Eqs. (15) and (22), for a given value of  $\zeta$ , the parameters  $k$  and  $\theta$  are determined (i.e.,  $\zeta$  and  $k$  are no longer independent). Through Eq. (4), the crystal-field parameter  $\Delta$  is obtained. From the transition energy for  $\Gamma_7 \rightarrow \Gamma_6$ , the crystal-field parameter  $\Theta$  is obtained for a given  $k'$  value. Then from Eq. (6), the admixture parameter of the  $\Gamma_8$  level,  $\varphi$ , is determined. The calculation to get the crystal-field parameters also can be fitted to the energies of the remaining two transitions,  $\Gamma_7 \rightarrow \Gamma_8$  and  $\Gamma_7 \rightarrow \Gamma'_8$ , so there are no independent parameters; i.e., all the crystal-field parameters are determined definitely by this procedure. Next, we took into account the tetragonal distortion effect, which resulted in the splitting of both quartets,  $\Gamma_8$  and  $\Gamma'_8$ , into Kramers doublets, respectively. The experimental results from the optical absorption spectrum show that the  $\Gamma_8$  state splits into  $\Gamma'_6$  and  $\Gamma'_7$ , the energy difference between them is  $560 \text{ cm}^{-1}$ , whereas the

TABLE I  
ELECTRONIC TRANSITION ENERGIES

Compound	Assignment	Transition energies ( $\text{cm}^{-1}$ )	
		Experiment	Calculation
$\text{Li}_7\text{UO}_6$	$\Gamma_7(\Gamma_7) \rightarrow \Gamma'_6(\Gamma_6)$	15,468	15,475
	$\Gamma_7(\Gamma_7) \rightarrow \Gamma'_7(\Gamma'_8)$	13,871	13,264
	$\Gamma_7(\Gamma_7) \rightarrow \Gamma'_8(\Gamma'_8)$		13,102
	$\Gamma_7(\Gamma_7) \rightarrow \Gamma'_7(\Gamma_7)$	7,439	7,435
	$\Gamma_7(\Gamma_7) \rightarrow \Gamma'_7(\Gamma_8)$	5,972	5,961
	$\Gamma_7(\Gamma_7) \rightarrow \Gamma'_8(\Gamma_8)$	5,416	5,409
$\text{Ba}_3\text{NpO}_6$	$\Gamma_7 \rightarrow \Gamma_6$	>16,500	19,077
	$\Gamma_7 \rightarrow \Gamma'_8$	>16,500	17,018
	$\Gamma_7 \rightarrow \Gamma'_7$	9,390	9,390
	$\Gamma_7 \rightarrow \Gamma_8$	7,690	7,580
$\text{Li}_5\text{PuO}_6$	$\Gamma_7(\Gamma_7) \rightarrow \Gamma'_6(\Gamma_6)$	18,519	18,511
	$\Gamma_7(\Gamma_7) \rightarrow \Gamma'_7(\Gamma'_8)$	15,873	15,758
	$\Gamma_7(\Gamma_7) \rightarrow \Gamma'_8(\Gamma'_8)$	14,350	14,347
	$\Gamma_7(\Gamma_7) \rightarrow \Gamma'_7(\Gamma_7)$	10,256	10,214
	$\Gamma_7(\Gamma_7) \rightarrow \Gamma'_7(\Gamma_8)$	8,818	8,772
	$\Gamma_7(\Gamma_7) \rightarrow \Gamma'_8(\Gamma_8)$	6,623	6,632

splitting of the  $\Gamma'_8$  state is not observed (1) (Table I). By diagonalizing the energy matrices, Eqs. (9) and (10), we obtained the energies for each level. For six transition energies, the parameters were adjusted. Unfortunately, not all the transitions could be fitted. Since the transition  $\Gamma_7 \rightarrow \Gamma'_8$  for octahedral symmetry is known to be broad and since this transition is furthermore broadened due to the tetragonal distortion, we have considered the  $\Gamma_7 \rightarrow \Gamma'_8$  transition energy to be the least reliable. The transition energies calculated here, and all the crystal-field parameters and the orbital reduction factors, are listed in Tables I and II, respectively. We obtained  $\Delta = 4604 \text{ cm}^{-1}$ ,  $\Theta = 6600 \text{ cm}^{-1}$ ,  $\zeta = 1868 \text{ cm}^{-1}$ , and  $\tau = 140 \text{ cm}^{-1}$ . The spin-orbit coupling constant ( $\zeta = 1868 \text{ cm}^{-1}$ ) is considered to be a reasonable value for  $\text{U}^{5+}$  in solids. This magnitude for the spin-orbit coupling constant has been obtained by others (1, 7, 13, 14) and the magnitude is intermediate between those of  $\text{Pa}^{4+}$  and  $\text{Np}^{6+}$  compounds (15).

Since we have already obtained the wavefunctions for the ground doublets and excited states, the magnetic susceptibility of  $\text{Li}_7\text{UO}_6$  is easily calculated by Eq. (13) as

TABLE II  
CRYSTAL-FIELD PARAMETERS AND ORBITAL  
REDUCTION FACTORS

Compound	$\zeta$ ( $\text{cm}^{-1}$ )	$\Delta$ ( $\text{cm}^{-1}$ )	$\Theta$ ( $\text{cm}^{-1}$ )	$\tau$ ( $\text{cm}^{-1}$ )	$k$	$k'$
$\text{Li}_7\text{UO}_6$	1868	4604	6600	140	1.0	0.8
$\text{Ba}_3\text{NpO}_6$	2378	5695	8800	0	1.0	0.6
$\text{Li}_3\text{PuO}_6$	2520	6700	5600	480	1.0	0.6

$$\begin{aligned}\chi &= \frac{1}{3}(\chi_{\parallel} + 2\chi_{\perp}) \\ &= 0.0256/T + 217 \times 10^{-6}.\end{aligned}\quad (23)$$

The magnetic susceptibility of  $\text{Li}_7\text{UO}_6$  was measured by several groups, and their reciprocal susceptibilities are drawn vs temperature in Fig. 2. However, the results are not consistent. Kemmler-Sack *et al.* (16) measured the susceptibilities at 195, 293, and 373 K and found that the susceptibilities did not obey the Curie-Weiss law but may be represented by  $\chi = 0.038/T + 300 \times 10^{-6}$ . Keller (6) and Kanellakopoulos *et al.* (1) extended the temperature range of the magnetic susceptibility measurements down to 4.2 K, and reported that the susceptibilities are linear vs  $1/T$  with  $\text{TIP} = 238 \times 10^{-6}$ . From their susceptibility data, the temperature dependence of susceptibility may be represented by  $\chi = 0.0253/T + 238 \times 10^{-6}$ . Miyake *et al.* (17) also reported that when the temperature-independent susceptibility is subtracted from the measured susceptibility, the reciprocal susceptibilities are linear vs temperature and the effective magnetic moment of  $\text{Li}_7\text{UO}_6$  is  $0.64 \mu_B$ . Thus, the susceptibilities measured by them would be represented by  $\chi = 0.051/T + \text{TIP}$  (the TIP value is not given in Ref. (17)). From the comparison of the susceptibility equation obtained by calculation, Eq. (23), with those obtained empirically, both the temperature-dependent and -independent parts of the susceptibility equation for the data measured by Kanellakopoulos *et al.* are close to those in Eq. (23). On the other hand, the susceptibility data measured by Kemmler-

Sack *et al.* and Miyake *et al.* are found to be far from the calculated values (also see Fig. 2). If we add extra  $\text{TIP} = 15 \times 10^{-6}$  to Eq. (23), the temperature dependence of magnetic susceptibility measured by Kanellakopoulos *et al.* can be well reproduced in the whole temperature range. We consider that some reactant impurities contribute to this excess temperature-independent susceptibility. If the  $\text{U}^{4+}$  ions of overreduced  $\text{Li}_7\text{UO}_{6-x}$  are octahedrally coordinated by six oxygens, the susceptibility would show temperature-independent paramagnetism over a wide temperature range (3) and its magnitude should be much larger than the temperature-independent paramagnetic

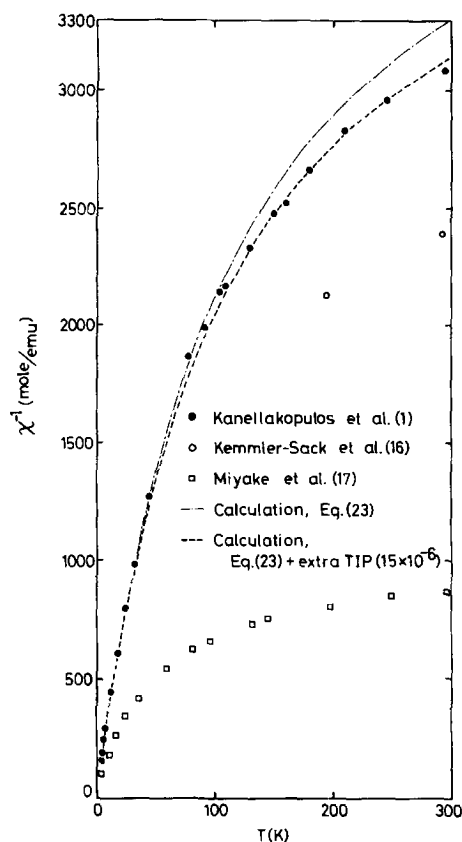


FIG. 2. Reciprocal magnetic susceptibilities of  $\text{Li}_7\text{UO}_6$  vs temperature.

susceptibility in Eq. (23). Since the temperature-independent part of the susceptibility of  $\text{UO}_2$  is large (18, 19), residual  $\text{UO}_2$  also would affect the susceptibility of  $\text{Li}_7\text{UO}_6$ .

Miyake *et al.* (17) claimed that for pure  $\text{Li}_7\text{UO}_6$ , an EPR spectrum was measured at liquid  $\text{N}_2$  temperature. The signal was extremely broad (the linewidth of which was 4000 G) and the  $g$ -value was  $\sim 0.6$ , although the center of the absorption was obscure. The  $g$ -value calculated from the temperature-dependent part of the susceptibility ( $|g| = 0.52$ ) is near the value measured by EPR.

### 3.2. $\text{Ba}_3\text{NpO}_6$

Kanellakopoulos *et al.* (1) measured the optical absorption spectra. Unfortunately, they are not clear. However, it is evident that the absorptions at  $7690$  and  $9390\text{ cm}^{-1}$  correspond to  $\Gamma_7 \rightarrow \Gamma_8$  and  $\Gamma_7 \rightarrow \Gamma_7'$  transitions, respectively, and that there is no splitting of the  $\Gamma_8$  quartet state, which is to be expected from the cubic crystal structure of  $\text{Ba}_3\text{NpO}_6$  (6). We do not assign the very weak absorption at  $11,760\text{ cm}^{-1}$  and consider that the absorption bands other than the  $\Gamma_7 \rightarrow \Gamma_8$  and  $\Gamma_7 \rightarrow \Gamma_7'$  transitions lie above  $16,500\text{ cm}^{-1}$  (Table I). Since we cannot use many transition energies to determine the crystal-field parameters, we assume the orbital reduction factor  $k$  is equal to one. The magnetic susceptibilities of  $\text{Ba}_3\text{NpO}_6$  have been measured by Kanellakopoulos *et al.* (1, 6) (Fig. 3) and they are linear vs  $1/T$  with  $\text{TIP} = 340 \times 10^{-6}\text{ emu/mole}$ . From their susceptibility data, the temperature dependence of the magnetic susceptibility may be represented by  $\chi = 0.0279/T + 340 \times 10^{-6}$ . Therefore, the ground state  $g$ -value is calculated to be 0.546. From the transition energies available and the ground state  $g$ -value, the crystal-field parameters were calculated as shown in Table II. The spin-orbit coupling constant  $\zeta$  is  $2378\text{ cm}^{-1}$ . This value is very close to  $2405\text{ cm}^{-1}$  obtained by Eisenstein and Pryce for  $\text{NpF}_6$  by assuming  $k = 1$  (2). Magnetic susceptibilities of  $\text{Ba}_3\text{NpO}_6$  are calculated and represented by

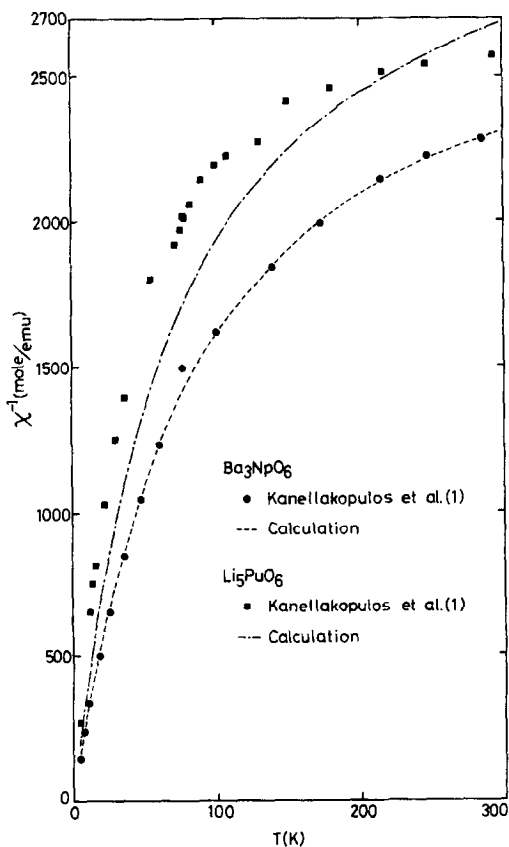


FIG. 3. Reciprocal magnetic susceptibilities of  $\text{Ba}_3\text{NpO}_6$  and  $\text{Li}_3\text{PuO}_6$  vs temperature.

$\chi = 0.0279/T + 155 \times 10^{-6}$ . The TIP obtained experimentally is much larger than that calculated. This result suggests that some of the neptunium ions are in the pentavalent state. The electronic configuration of the  $\text{Np}^{5+}$  ion is  $[\text{Rn}]5f^2$ . If the  $5f^2$  ion is octahedrally coordinated by six oxygens, its susceptibility shows a large temperature-independent paramagnetism over a wide temperature range (3). The temperature-independent susceptibility needed to obtain a satisfactory fit of the magnetic susceptibility data as a function of temperature is  $185 \times 10^{-6}\text{ emu/mole}$ .

### 3.3. $\text{Li}_3\text{PuO}_6$

$\text{Li}_3\text{PuO}_6$  crystallizes in a hexagonal structure. This solid compound contains isolated



$\text{PuO}_6$  octahedra (6). Kanellakopoulos *et al.* (1) measured the optical spectra of this compound and reported that the oxygen octahedron is tetragonally distorted. Our calculation model, which indicates that the octahedral crystal-field around the  $\text{Pu}^{7+}$  ion is a little tetragonally distorted, is based on this result. Kanellakopoulos *et al.* have presented the solution spectra measured in 1 *N*  $\text{LiOD}/\text{D}_2\text{O}$  as a more reliable spectra than the solid state spectra. Both the  $\Gamma_8$  and the  $\Gamma'_8$  quartets split greatly due to the tetragonal component of the crystal field around a plutonium ion. Kanellakopoulos *et al.* considered that the very weak shoulder absorption at  $13,158\text{ cm}^{-1}$  was the transition from  $\Gamma'_7(\Gamma_7)$  to  $\Gamma'_6(\Gamma'_8)$ . If this is the case, the splitting of the  $\Gamma'_8$  quartet is greater than that of the  $\Gamma_8$  quartet, which contradicts the above-mentioned theoretical consideration that the octahedral coordination around a  $\text{Pu}^{7+}$  ion is tetragonally distorted. The weak absorptions similar to the one found at  $13,158\text{ cm}^{-1}$  are also found at ca.  $14,000$  and  $14,350\text{ cm}^{-1}$ . Considering the  $\Gamma'_7(\Gamma_7) \rightarrow \Gamma'_6(\Gamma'_8)$  transition to be the least reliable one, we tried to obtain the crystal-field parameters by fitting the calculations to the other five transitions. The transition energies and crystal-field parameters are listed in Tables I and II, respectively. Table I shows that the transition energies calculated are in good agreement with those obtained experimentally. The transition energy calculated for  $\Gamma'_7(\Gamma_7) \rightarrow \Gamma'_6(\Gamma'_8)$  is  $14,350\text{ cm}^{-1}$ , which also has been observed experimentally. The spin-orbit coupling constant calculated for a  $5f$  electron of  $\text{Pu}^{7+}$  is  $2520\text{ cm}^{-1}$ , which is larger than the value for  $\text{Np}^{6+}$ . This trend is constant with the results of the theoretical calculation (4). The tetragonal distortion in this compound ( $\tau = 480\text{ cm}^{-1}$ ) is much larger than that in  $\text{Li}_7\text{UO}_6$ , as expected from the optical absorption spectra which show the splittings of both quartets  $\Gamma_8$  and  $\Gamma'_8$  are much larger than those for  $\text{Li}_7\text{UO}_6$ .

From the wavefunctions, the magnetic susceptibility of  $\text{Li}_5\text{PuO}_6$  is calculated to be

$\chi = 0.0218/T + 112 \times 10^{-6}$ . Kanellakopoulos *et al.* (1) measured the magnetic susceptibilities of this compound (Fig. 3) and reported that they are linear vs  $1/T$  with  $\text{TIP} = 300 \times 10^{-6}$ . From their magnetic susceptibility data, the temperature dependence of the magnetic susceptibility may be represented by  $\chi = 0.0158/T + 300 \times 10^{-6}$ , although this equation is not valid in the whole temperature range. Neither the temperature-dependent nor the temperature-independent part of the susceptibility agrees between experiment and calculation. Even if an extra TIP ( $188 \times 10^{-6}\text{ emu/mole}$ ) were added to the susceptibility equation obtained from calculation, it could not reproduce the experimental data (Fig. 3). We believe the reason for this is that some of the  $\text{Pu}^{7+}$  ions are reduced to the  $\text{Pu}^{6+}$  state which is much more stable than the  $\text{Pu}^{7+}$  state. When the  $\text{Pu}^{6+}$  ion is octahedrally coordinated by six oxygens, its susceptibility shows the large temperature-independent paramagnetism over a wide temperature range (3), and if its coordination is tetragonally distorted, the susceptibility becomes temperature-dependent (20). The temperature-independent susceptibility obtained experimentally is much larger than that calculated, which is due to the existence of the  $\text{Pu}^{6+}$  ions in octahedral symmetry. If the number of the  $\text{Pu}^{6+}$  ions in  $\text{Li}_5\text{PuO}_6$  were comparable to that of the  $\text{Np}^{5+}$  ions in  $\text{Ba}_3\text{NpO}_6$ , the TIP value for the former compound should be smaller than that for the latter, because the energy difference between the ground state and the excited state for the former compound is larger than that for the latter. The experimental results show the contrary, i.e., the TIP difference between experiment and calculation for  $\text{Li}_5\text{PuO}_6$  is comparable to that for  $\text{Ba}_3\text{NpO}_6$ , indicating that the number of  $5f^2$  ions in  $\text{Li}_5\text{PuO}_6$  is much larger than that in  $\text{Ba}_3\text{NpO}_6$ .

The spin-orbit coupling constants obtained for  $\text{Li}_7\text{UO}_6$ ,  $\text{Ba}_3\text{NpO}_6$ , and  $\text{Li}_5\text{PuO}_6$  are all reasonable values for  $\text{U}^{5+}$ ,  $\text{Np}^{6+}$ , and

$\text{Pu}^{7+}$ , and show an increase with atomic number and/or oxidation state, which is consistent with the results of the theoretical calculations (4). In this study, we have taken into account the covalency effect in chemical bonding, i.e., introduced the orbital reduction factors in the calculation of energy levels and magnetic susceptibilities. Although the orbital reduction factors determined here ( $k$  and  $k'$ ) may not be the unique values for each compound, the results indicate that  $k'$  should be much smaller than  $k$ , which is consistent with the theoretical view (2, 5). To obtain a satisfactory fit of the magnetic susceptibility data as a function of temperature, an added empirical temperature-independent paramagnetic susceptibility was needed for all of the compounds. We suggest the reduced actinide ions with  $[\text{Rn}]5f^2$  configuration, i.e.,  $\text{U}^{4+}$ ,  $\text{Np}^{5+}$ , and  $\text{Pu}^{6+}$ , are reasonable for this extra TIP. These ions in an octahedral coordination are known to show a temperature-independent paramagnetism over a wide temperature range. With this hypothesis and from the comparison of the extra TIP between compounds, it was found that a considerable amount of  $\text{Pu}^{6+}$  ions which are much more stable than  $\text{Pu}^{7+}$  ions exist in  $\text{Li}_5\text{PuO}_6$ . The crystal-field splitting  $\Delta$ , which is very sensitive to the magnetic properties of the compounds, increases with atomic number (from  $\text{U}^{5+}$  to  $\text{Np}^{6+}$  to  $\text{Pu}^{7+}$ ), as expected.

## References

1. B. KANELLAKOPOULOS, E. HENRICH, C. KELLER, F. BAUMGÄRTNER, E. KÖNIG, AND V. P. DESAI, *Chem. Phys.* **53**, 197 (1980).
2. J. C. EISENSTEIN AND M. H. L. PRYCE, *Proc. R. Soc. Ser A* **255**, 181 (1960).
3. N. M. EDELSTEIN AND J. GOFFART, in "The Chemistry of the Actinide Elements" (J. J. Katz, G. T. Seaborg, and L. R. Morss, Eds.), 2nd ed., Chap. 18, Chapman & Hall, London/New York (1986).
4. W. B. LEWIS, J. B. MANN, D. A. LIBERMAN, AND D. T. CROMER, *J. Chem. Phys.* **53**, 809 (1970).
5. H. G. HECHT, W. B. LEWIS, AND M. P. EASTMAN, *Adv. Chem. Phys.* **21**, 351 (1971).
6. C. KELLER, in "MTP International Review of Science," Vol. 7, "Lanthanides and Actinides" (K. W. Bagnall, Ed.), Series I, p. 47, Butterworths, London (1972).
7. J. SELBIN, C. J. BALLHAUSEN, AND D. G. DURRETT, *Inorg. Chem.* **11**, 510 (1972).
8. Y. HINATSU AND N. EDELSTEIN, submitted for publication.
9. J. H. VAN VLECK, "The Theory of Electronic and Magnetic Susceptibilities," Oxford Univ. Press, London (1932).
10. P. RIGNY AND P. PLURIEN, *J. Phys. Chem. Solids* **28**, 2589 (1967).
11. J. D. AXE, M. J. STAPLETON, AND C. D. JEFFERIES, *Phys. Rev.* **121**, 1630 (1961).
12. C. A. HUTCHISON, JR., AND B. WEINSTOCK, *J. Chem. Phys.* **32**, 56 (1960).
13. N. EDELSTEIN, D. BROWN, AND B. WHITTAKER, *Inorg. Chem.* **13**, 563 (1974).
14. J. SELBIN AND H. J. SHERRILL, *Inorg. Chem.* **13**, 1235 (1974).
15. J. SELBIN AND J. D. ORTEGO, *Chem. Rev.* **69**, 657 (1969).
16. S. KEMMLER-SACK, E. STUMPP, W. RÜDORFF, AND H. ERFURTH, *Z. Anorg. Allg. Chem.* **292**, 287 (1967).
17. C. MIYAKE, H. TANIGUCHI, H. OHYA-NISHIGUCHI, AND S. IMOTO, *Phys. Status Solidi A* **74**, 173 (1982).
18. J. K. DAWSON AND M. W. LISTER, *J. Chem. Soc.*, 5041 (1952).
19. S. NASU, *Jpn. J. Appl. Phys.* **5**, 1001 (1966).
20. T. H. SIDDALL, III, in "Theory and Applications of Molecular Paramagnetism" (E. A. Boudreaux and L. N. Mulay, Eds.), Chap. 6, Wiley-Interscience, New York (1976).