Journal of Sound and Vibration (1997) **199**(4), 697–700

LETTERS TO THE EDITOR

COMMENTS ON "VIBRATION ANALYSIS OF BEAMS TRAVERSED BY UNIFORM PARTIALLY DISTRIBUTED MOVING MASSES"

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(*Received* 28 *October* 1995)

1. INTRODUCTION

In a recent article, a theory for dynamic analysis of beam structures traversed by uniform partially distributed moving masses is developed [1]. Bernoulli–Euler beam theory is applied. The solution is obtained by expressing the moving load as an infinite series and the beam response is computed using the modal expansion method. While the development is interesting, some statements, analysis results and conclusions are questionable and may be misleading.

2. ANALYSIS

First of all, the assumption statement concerning the use of the Bernoulli–Euler equation is inappropriate. Bernoulli–Euler beam theory cannot be applied simply because the beam is of constant cross-section and mass distribution. If the slenderness ratio, r/L, is large, where r is the radius of gyration and L is the beam length, or vibration of higher modes is concerned, the use of classical Bernoulli–Euler beam theory cannot ensure sufficient accuracy. In this case, Timoshenko beam theory, which takes into account the effects of shearing deformations and rotary inertia, must be applied for accurate analysis. On the other hand, the Bernoulli–Euler equation is still applicable for a beam with a non-uniform cross-section and mass distribution, as long as the beam is slender, i.e., r/L is sufficiently small, and only lower mode vibration analysis is involved.

The equation of motion, considering the effect of the moving mass, is incorrectly given. As shown in equation (2), the term $M\,\partial^2 y(x,t)/\partial t^2$ only partially describes the dynamic effect of the moving mass. Since the mass is moving along a vibrating path, the velocity of the moving mass is

$$\dot{y}(x,t) = (\partial y/\partial x)\dot{x} + \partial y/\partial t \tag{1}$$

and the acceleration of the moving mass can be written as

$$\ddot{y}(x,t) = \frac{\partial^2 y}{\partial x^2}\dot{x}^2 + 2\frac{\partial^2 y}{\partial x\,\partial t}\dot{x} + \frac{\partial y}{\partial x}\ddot{x} + \frac{\partial^2 y}{\partial t^2}. \tag{2}$$

The first term on the right side of equation (2) is the centripetal acceleration of the moving mass; the second term is the well-known Coriolis acceleration; the third term is the acceleration component in the vertical direction when the moving load speed is not a constant; and the last term is the support beam acceleration at the point of contact with the moving mass. All terms are absent in reference [1] except the last one. The dynamic force, in addition to the static load due to moving mass weight, must be described as $M\ddot{y}$, where $\ddot{y}$ is given in equation (2).

0022–460X/97/040697 + 04 $25.00/0/sv950675

It is claimed that the development in reference [1] is applicable to beams with various boundary conditions. However, it is known that for a beam with free boundary conditions, the out-flow term must be considered [2], which is absent in reference [1]. For instance, if analysis of cantilever beams, with the right end unsupported, traversed by distributed moving mass is considered, the term describing the out-flow effect, which can be written as $(M/\epsilon)V(\partial y/\partial t + V\,\partial y/\partial x)|_{x=L}$, must be taken into account.

The symmetry of the system response with respect to the mid-time of the motion does not seem to have any practical importance. The response profile is also dependent on the moving speed and the mass ratio M/mL. It was observed in reference [1] that increasing the number of modes in the computation resulted in insignificant changes in calculating the system response. The statement may be misleading and may yield erroneous results. If the weight of the moving mass is small in comparison with that of the support beam and the speed of movement is low, the conclusion may be applicable. Otherwise, more modes must be included for accurate analysis, especially when the computation of stress is required. Stress computation involves higher derivatives of the displacement. To calculate the bending stress requires the computation of bending moment, which involves the second derivative of displacement, while calculation of shear stress needs knowledge of the shear force, which is a function of the third derivative of the displacement. Therefore, convergence will be much slower for stress computation than for displacement calculation, and more modes will be required to ensure accuracy for stress analysis. Moreover, design of the support structure based on the information obtained during the forced response period, as was done in reference [1], is rather risky. As can be seen in Table 1, where D_{m_1} denotes the maximum dynamic deflection considering the forced response period only and D_{m_2} is obtained by considering further the free response region, where the moving mass has left the beam span, D_{m_2} may deviate significantly from D_{m_1} for higher moving speeds, indicating the inadequacy of considering the system response only for the forced vibration period when the moving mass is inside the beam span. For the present case, since the span of the distributed mass is very small when compared with that of the beam, D_{m_1} and D_{m_2} were computed using the moving concentrated mass model, which can be easily adapted from the work by Lin and Trethewey [3], in which a general treatment of various moving load problems was presented. D_m, as given in reference [1], is supposed to be approximately equal to D_{m_1}. However, as indicated earlier, the dynamic

TABLE 1

Variation of maximum dynamic deflection with the velocity of the load

V (km/h)	D_m (cm) [1]	D_{m_1} (cm)	D_{m_2} (cm)
12	7·2	7·0	7·0
24	7·8	7·5	7·5
36	8·4	8·2	8·2
48	10·6	10·2	10·2
60	11·9	11·8	11·8
72	12·3	13·1	13·2
84	12·6	14·4	15·2
90	12·7	15·1	15·7
96	12·6	15·9	16·1
108	12·3	17·4	17·4
120	11·6	17·7	18·2
132	10·7	16·8	18·3
144	10·3	15·7	18·0

TABLE 2

Impact factors for the central displacement of a simply supported beam traversed by a moving mass load

M/mL	$T_{f/\tau}$	$(y_d/y_s)_1$	$(y_d/y_s)_2$
0·2	0·5	1·319	1·319
	1·0	1·824	1·824
	1·5	1·917	1·917
	2·0	1·731	1·797
2	0·5	1·844	1·860
	1·0	3·154	3·428
	1·5	1·923	3·734
	2·0	1·106	3·521
4	0·5	2·396	2·769
	1·0	2·934	4·931
	1·5	1·337	4·966
	2·0	0·702	4·466
8	0·5	4·367	4·451
	1·0	2·092	7·238
	1·5	0·809	6·309
	2·0	0·402	5·258

effect of the moving mass is incorrectly described in reference [1] and significant error can be found for higher moving speeds.

Another important issue to be addressed is that the mass ratio, M/mL, plays an important role in assessing the dynamic characteristics of the moving mass problems. In Table 2 are shown the impact factors, defined as the maximum dynamic displacement, y_d, at the beam center normalized by the maximum static displacement, y_s, with respect to various mass ratios and moving speeds. In Table 2, $(y_d/y_s)_1$ denotes the impact factor obtained by considering the forced response period only, i.e., when the moving mass is inside the beam span; whereas $(y_d/y_s)_2$ represents the impact factor computed from the overall response period, including free response when the moving mass has left the beam span; T_f is the fundamental period of the simply supported beam; and τ is the time required for the moving load to across the beam span. A total of 2τ time response was evaluated. As can be seen in Table 2, significant deviation of $(y_d/y_s)_2$ from $(y_d/y_s)_1$ can be observed as the mass ratios and moving speeds are increased. The case with a higher mass ratio results in a significantly larger dynamic impact on the support structure than that with a lower one. It is worth noting that the information given in Table 2 further demonstrates the need to include the free response region in examining the dynamic characteristics of the system. The free response is found to be of greater interest than the forced response for moving mass problems with a large mass ratio, M/mL, and a higher speed of movement. This peculiar phenomenon may be explained by noting that during the forced response region, the heavy moving mass prevents a large amplitude of vibration since the mass is part of the overall dynamic system. However, once the mass exits the beam span, the energy imparted by the heavy moving mass during the loaded period is taken by the support beam alone, which results in large vibration. Higher modes participation was found to be significant, contrary to the case of moving concentrated force problems, in which the fundamental mode predominates in computing the dynamic response.

The "critical speed" of the moving load is defined in reference [1] as the speed at which the maximum deflection occurs. In conventional dynamics notation, the critical speed refers to the speed at which resonance or a transition from a stability to an instability region occurs. For analysis of a simply supported beam traversed by a moving concentrated force, the critical speed, or resonance speed, occurs when the travel time of the moving load to across the beam span is half of the fundamental period of the support structure [4]. The corresponding maximum dynamic deflection was found to be 1·548 times the static deflection. However, although the mathematical description shows that resonance occurs, it is valid only in the forced response region. The true maximum dynamic deflection was found to be 1·732 times the static deflection when the travel time is 0·81 times the fundamental period of the beam. Care should be taken to differentiate between the critical speed, or resonance speed, and the maximum response speed. Similar concerns apply to the other types of moving load problems.

3. CONCLUSIONS

Bernoulli–Euler beam theory is applicable for the lower mode vibration of slender structures, but should be disregarded if the beam is of constant cross-section and mass distribution. In describing the dynamic effect of the moving mass load, the total differential must be considered since the mass is moving on a vibrating path. The dynamic response of elastic structures traversed by moving mass loads is a very complicated function of both the mass ratio between the moving mass and the support structure and the speed of the moving load. Any analysis of moving load problems must address fully the effects of these parameters to extract useful information for engineering design and to avoid possible misleading conclusions. For larger mass ratios, M/mL, and higher moving speeds, the free response can be more important than the forced response in assessing the dynamic behavior of the support beam, and higher mode participation is significant in computing the dynamic response. Also, the contribution of higher modes cannot be ignored when stress analysis is concerned, due to slower convergence characteristics.

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