Journal of Sound and Vibration (1997) **199**(5), 813–815

LETTERS TO THE EDITOR

COMMENTS ON "ON THE MODE SHAPES OF THE HELMHOLTZ EQUATION"

P. A. A. Laura, D. V. Bambill and V. Jederlinic
Institute of Applied Mechanics (CONICET)
AND
Department of Engineering, Universidad Nacional del Sur 8000—*Bahía Blanca, Argentina*

(*Received* 11 *March* 1996)

Professors Gladwell and Willms must be congratulated for their interesting and important contribution [1].

The writers agree with the author's conclusion: "the circular membrane is not a typical membrane" [1] but it may be of interest to mention that a basic relation exists between the fundamental frequency coefficient of a membrane of arbitrary shape and that of a membrane of circular shape.

As shown by Szego [2],

$$\lambda_{11} < \alpha_0/a_0, \tag{1}$$

where α_0 is the first root of Bessel's function of the first kind and order zero (the fundamental frequency coefficient of a circular membrane).

a_0 is the coefficient of the first term of the infinite series which maps a unit circle on to the arbitrary shape; see Figure 1.

For instance, in the case of a square membrane, the mapping function is given by [3]

$$z = x + \mathrm{i}y = 1{\cdot}078a_p[\xi - \tfrac{1}{10}\xi^5 + \tfrac{1}{24}\xi^9 - \tfrac{5}{208}\xi^{13} + \cdots], \tag{2}$$

where a_p is the apothem of the square. Consequently,

$$\lambda_{11} < \frac{2{\cdot}4048}{1{\cdot}078a_p} = \frac{2{\cdot}2308}{a_p}, \tag{3}$$

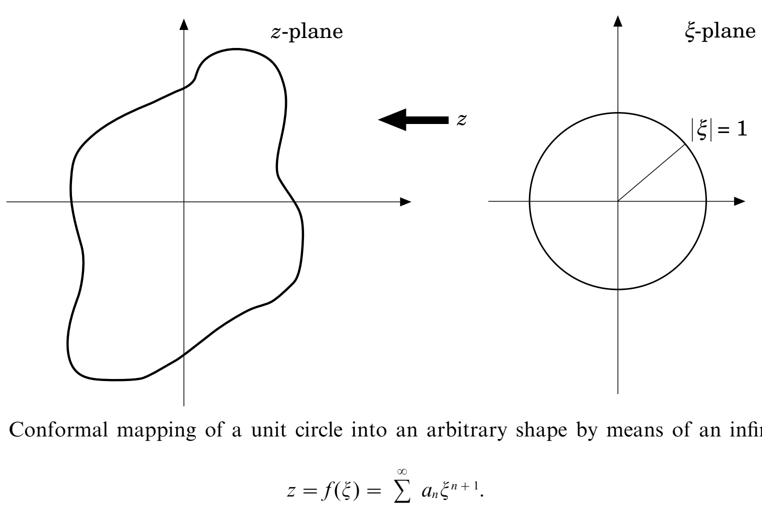

Figure 1. Conformal mapping of a unit circle into an arbitrary shape by means of an infinite series:

$$z = f(\xi) = \sum_{n=0}^{\infty} a_n \xi^{n+1}.$$

0022–460X/97/050813 + 03 $25.00/0/sv960639

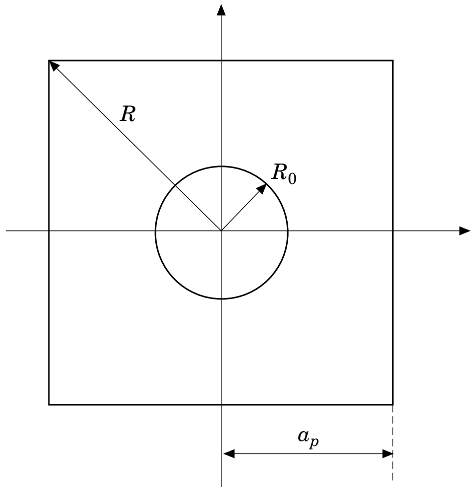

Figure 2. A square membrane with a concentric circular perforation [3].

the exact eigenvalue being

$$\lambda_{11} = \frac{2{\cdot}2214}{a_p}. \qquad (4)$$

The upper bound (3) is, in this case, less than 0·5% higher than the exact value.

In the case of a doubly connected membrane of fixed edges, it has recently been shown [4] that

$$\lambda_{11} \leqslant \alpha_{11}/a_0, \qquad (5)$$

where a_0 is the coefficient of the (ξ) term in the Laurent expansion which maps the given doubly connected membrane onto a circular annulus in the ξ-plane; namely,

$$z = x + \mathrm{i}y = \sum_{n=-\infty}^{+\infty} a_n \xi^{1+ns}, \qquad (6)$$

and where s is the number of axes of symmetry of the configuration ($s = 4$ in the case of equation (2) and Figure 2). The parameter α_{11} is the frequency coefficient of the corresponding circular, annular membrane.

ACKNOWLEDGMENTS

Research on applications of conformal mapping to non-classical solid mechanics problems is sponsored by the CONICET Research and Development Program and by the Secretaría General de Ciencía y Tecnologia of Universidad Nacional del Sur (Project Director Professor R. E. Rossi).

REFERENCES

1. G. M. L. Gladwell and N. B. Willms 1995 *Journal of Sound and Vibration* **188**, 419–434. On the mode shapes of the Helmholtz equation.
2. G. Szego 1952 in *Construction and Applications of Conformal Maps*, Proceedings of a Symposium held at the University of California, Los Angeles (E. F. Beckenbach, editor), 79–83. National Bureau of Standards, Applied Mathematics Series No. 18. Conformal mapping related to torsional rigidity, principal frequency and electrostatic capacity.

3. R. Schinzinger and P. A. A. Laura 1991 *Conformal Mapping: Methods and Applications*. Amsterdam: Elsevier.
4. V. H. Cortinez, P. A. A. Laura and H. C. Sanzi 1996 *Applied Acoustics* **48**, 301–309. Approximate determination of frequencies of vibration of doubly connected membranes of complicated boundary shape.

AUTHOR'S REPLY

G. M. L. Gladwell
Faculty of Engineering, University of Waterloo, Waterloo, Ontario, Canada N2L 3G1

(*Received* 10 *April* 1996)

What the authors have stated is true, but is completely irrelevant to our paper, which was concerned with mode shapes, and not with eigenfrequencies.