



ACTIVE VIBRATION CONTROL AND SUPPRESSION FOR INTELLIGENT STRUCTURES

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A finite element formulation for vibration control and suppression of intelligent structures with a new piezoelectric plate element is presented. On the basis of a negative velocity feedback control law, a general method of active vibration control and suppression for intelligent structures is put forth. Dynamic stability and the effect of vibration control for intelligent structures are investigated by introducing the state space equations of intelligent structures. The damped frequencies as well as the damping ratio are derived by state space analysis. The procedure is illustrated with the help of two numerical examples. The purpose of the first example is to check the accuracy of the present finite element solution with the analytical one. The second example is to study the problem of active vibration control and suppression for intelligent structures.

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1. INTRODUCTION

Space structures, aircraft, and so on are required to be light in weight due to the high cost of transportation. They are also lightly damped because of the low internal damping of the materials used in their construction, and the increased flexibility may allow large amplitude vibration, which may cause structural instability. These problems will reduce precision and affect operational performance. Since these structures are distributed parameter systems having an infinite set of vibration modes, distributed measurement and control are required. However, conventional control systems use discrete sensors and actuators (S/As) to control the vibration of distributed elastic systems. If the sensors are placed at nodal modes or on the lines of a vibration mode, that mode will be missed. Thus, it is essential to use an active control system in these structures to control and stabilize the structures during their operation.

The intelligent structure [1], which comprises the main structure and distributed piezoelectric S/As, is of integrated self-monitoring and self-controlling capabilities. It can detect and generate a number of vibration modes simultaneously. An active control system containing the intelligent structure has proven to be effective in controlling and suppressing the vibration of distributed light and flexible systems [2–6].

Finite element methods for analyzing dynamic measurement and controlling vibration of the intelligent plate structure are described in references [7] and [8]. In these methods the plate and thin S/As layer are generally modelled with isoparametric hexahedron solid elements. However, hexahedron solid elements are too thick for thin plate/shell applications, and will result in excessive strain energies and higher stiffness coefficients. To overcome these shortcomings, internal degrees of freedom were added to the formulation, which made the problem large and complex.

There are two essential ideas in this paper. The first is the formulation of dynamic equations for intelligent plate structures with a new piezoelectric plate element. The second is to develop a method of active vibration control and suppression for intelligent structures on the basis of a negative velocity feedback control law and derive the state space equation of motion for the intelligent structure to appraise the control effect as well as dynamic stability. Finally, two numerical examples are given to demonstrate the validity of the method presented in this paper.

2. THE FORMULATION OF THE FINITE ELEMENT EQUATION OF MOTION FOR THE INTELLIGENT STRUCTURE

2.1. CONSTITUTIVE EQUATION

The liner constitutive equation in a piezoelectric medium can be expressed by the direct and inverse piezoelectric equations respectively. These equations for the plate shape sensor and actuator are written as follows:

$$\{D\} = [e]\{\varepsilon\} + [\epsilon]\{E\}, \quad \{\sigma\} = [D_p]\{\varepsilon\} - [e]^T\{E\}, \quad (1, 2)$$

where $\{D\}$, $\{E\}$, $\{\varepsilon\}$ and $\{\sigma\}$ are the electric displacement, electric field, strain and stress vectors, and $[D_p]$, $[e]$ and $[\epsilon]$ are the elasticity, piezoelectric and dielectric constant matrices, respectively. $[e]^T$ is defined as the transpose of $[e]$.

Equation (1) describes the direct piezoelectric effect and equation (2) describes the inverse piezoelectric effect.

The constitutive equation in the elastic field is as follows:

$$[\sigma] = [D_e]\{\epsilon\}, \quad (3)$$

where $[D_e]$ is the elasticity constant matrix of the main structure in the intelligent structure.

2.2. FINITE ELEMENT DISCRETIZATION

The arbitrary quadrilateral bending element of plate [9] is a four-node, 12-degree-of-freedom isoparametric element for thin plates. The element nodal displacement variable $\{u^e\}$ is defined as

$$\{u^e\} = \{w_1 \ \theta_{x_1} \ \theta_{y_1} \ w_2 \ \theta_{x_2} \ \theta_{y_2} \ \dots \ w_4 \ \theta_{x_4} \ \theta_{y_4}\}^T, \quad (4)$$

where w is the normal displacement, $\theta_x = (\partial w / \partial y)$ and $\theta_y (= -\partial w / \partial x)$ are the rotation about the x - and y -axes. The normal displacement variables w is expressed in nodal variables by finite element interpolation functions as follows:

$$w = [N_u]\{u^e\}, \quad (5)$$

where $[N_u]$ is the displacement shape function matrix.

The strain variable $\{\varepsilon\}$ is expressed as a function of the nodal displacement variables. It is

$$\{\varepsilon\} = z \left\{ \begin{array}{c} -\frac{\partial^2 w}{\partial x^2} \\ -\frac{\partial^2 w}{\partial y^2} \\ -2\frac{\partial^2 w}{\partial x \partial y} \end{array} \right\} = z[B_u]\{u^e\}. \quad (6)$$

The element nodal electric potential variable $\{v^e\}$ is defined as

$$\{v^e\} = \{v_1 \ v_2 \ v_3 \ v_4\}^T. \quad (7)$$

The electric potential variable $\{v\}$ is expressed in terms of nodal electric potential variables via the shape functions as follows:

$$\{v\} = [N_e]\{v^e\}, \quad (8)$$

where $[N_e]$ is the electrical potential shape function matrix.

The electric field $\{E\}$ is defined by the electric potential $\{v\}$ by using a gradient operator ∇ , and is written in terms of nodal electrical potential variables:

$$\{E\} = -\nabla\{v\} = -[B_e]\{v^e\}. \quad (9)$$

2.3. FINITE ELEMENT FORMULATION

Using Hamilton's principle, we have

$$\delta \int_{t_1}^{t_2} (T_e - U_e - W_e) dt = 0, \quad (10)$$

where T_e is the element kinetic energy, U_e is the element potential energy and W_e is the element work done by the external forces.

The element kinetic energy is

$$T_e = \frac{1}{2} \int_{Q_{se}} \rho_s \{\dot{u}\}^T \{\dot{u}\} dQ + \frac{1}{2} \int_{Q_{pe}} \rho_p \{\dot{u}\}^T \{\dot{u}\} dQ. \quad (11)$$

Here, ρ_s and ρ_p are the mass density of the main structure material and the piezoelectric material respectively. $\{\dot{u}\}$ is the velocity vector. The subscripts s , p and e represent the main structure, the piezoelectric material field and the element, respectively. Q is the element volume.

The element potential energy is

$$V_e = \frac{1}{2} \int_{Q_{pe}} \{\varepsilon\}^T \{\sigma\} dQ + \frac{1}{2} \int_{Q_{se}} \{\varepsilon\}^T \{\sigma\} dQ - \frac{1}{2} \int_{Q_{pe}} \{E\}^T \{D\} dQ. \quad (12)$$

The work done by the surface force and the applied surface electrical charge density is

$$W_e = \int_{S_1} \{u\}^T \{f_s^e\} dS - \int_{S_2} \{v\}^T \sigma^e dS, \quad (13)$$

where S_1 and S_2 are the surface area applied forces and the electrical charge, respectively. $\{f_s^e\}$ and σ^e are the surface forces and the surface electrical charge density, respectively. Substituting equations (1)–(9) and equations (11)–(13) into equation (10) and not taking structural damping into consideration, equations of motion for the plate element with distributed piezoelectric S/As can be derived:

$$([m_p] + [m_s])\{\ddot{u}^e\} + [c_s]\{\dot{u}^e\} + ([k_{uss}] + [k_{uap}])\{u^e\} + [k_{uv}]\{v^e\} = \{F_s^e\}, \quad (14)$$

$$[k_{vu}]\{u^e\} - [k_{vv}]\{v^e\} = \{F_v^e\}, \quad (15)$$

where

$$\begin{aligned}
[m_p] &= \int_{Q_{pe}} [N_u]^T \rho_p [N_u] dQ, & [m_s] &= \int_{Q_{se}} [N_u]^T \rho_s [N_u] dQ, \\
[k_{mus}] &= z^2 \int_{Q_{se}} [B_u]^T [D_e] [B_u] dQ, & [k_{uup}] &= z^2 \int_{Q_{pe}} [B_u]^T [D_p] [B_u] dQ, \\
[k_{uw}] &= [k_{vu}]^T = z \int_{Q_{pe}} [B_u]^T [e]^T [B_v] d\Omega, \\
[k_{vv}] &= \int_{Q_{pe}} [B_v]^T [\epsilon] [B_v] dQ, \\
\{F_s^e\} &= \int_{S_1} [N_u]^T \{f_s^e\} dS, & \{F_c^e\} &= - \int_{S_2} [N_v]^T \sigma^e dS.
\end{aligned}$$

Substituting $[m] = [m_p] + [m_s]$ and $[k_{uu}] = [k_{uus}] + [k_{uup}]$ into equation (14) yields

$$[m]\{\ddot{u}^e\} + [c_s]\{\dot{u}^e\} + [k_{uu}]\{u^e\} + [k_{uv}]\{v^e\} = \{F_s^e\}. \quad (16)$$

When the Guyan reduction method [10] is applied to equations (16) and (14), the equations are reduced to one equation as follows:

$$[m]\{\ddot{u}^e\} + [c_s]\{\dot{u}^e\} + [k]\{u^e\} = \{F_s^e\} + \{F_t^e\}, \quad (17)$$

where

$$[k] = [k_{uu}] + [k_{uv}][k_{vv}]^{-1}[k_{vu}], \quad \{F_t^e\} = -[k_{uv}][k_{vv}]^{-1}\{F_c^e\}.$$

If the sensing information is required, the electrical potential vectors can be recovered by

$$\{v^e\} = [k_{vv}]^{-1}([k_{vu}]\{u^e\} - \{F_c^e\}). \quad (18)$$

Note that $\{F_c^e\}$ is usually zero in the distributed piezoelectric sensor layer. Thus, the distributed piezoelectric sensor electrical potential output is estimated by

$$\{v^e\} = [k_{vv}]^{-1}[k_{vu}]\{u^e\}. \quad (19)$$

3. ACTIVE VIBRATION CONTROL AND SUPPRESSION FOR THE INTELLIGENT STRUCTURE

In equation (17) there are two load terms; i.e., the mechanical forces and the electrical forces. In the active vibration control application the electric force term in equation (17) can be regarded as the feedback control force. It can be written as

$$\{F_t^e\} = [k_{uv}][k_{vv}]^{-1}\{V^e\}. \quad (20)$$

$\{V^e\}$ is a function of the feedback voltage in terms of the output signal from the distributed piezoelectric sensor layer.

The negative velocity feedback control law on $\{V^e\}$ is implemented as

$$\{V^e\} = -G\{\dot{v}^e\} = -G[k_{vv}]^{-1}[k_{vu}]\{\dot{u}^e\}. \quad (21)$$

Hence, the feedback control forces can be written as

$$\{F_i^e\} = -[k_{uw}][k_{vv}]^{-1}G[k_{vv}]^{-1}[k_{vu}]\{\dot{u}^e\}, \quad (22)$$

where G is the feedback gain.

Substituting equation (22) into equation (17) and rearranging gives

$$[m]\{\ddot{u}^e\} + [c_i]\{\dot{u}^e\} + [k]\{u^e\} = \{F_s^e\}, \quad (23)$$

where

$$[c_i] = [c_s] + [k_{uw}][k_{vv}]^{-1}G[k_{vv}]^{-1}[k_{vu}]. \quad (24)$$

The equation of motion of the system can be written as follows:

$$[M]\{\ddot{u}\} + [C_T]\{\dot{u}\} + [K]\{u\} = \{F_s\}. \quad (25)$$

As shown in equation (24), the damping terms in equation (23) consist of two parts. One is the structural damping, and the other is the equivalent damping term induced by the feedback control force. The feedback control forces induced by the feedback voltage can effectively enhance the system damping and therefore suppress the vibration of the structure. Thus, we can conclude that under a negative velocity feedback control law, the intelligent structure has active vibration control and suppression capabilities.

From equation (24) it is known that equivalent damping is only relative to the feedback gain G when the intelligent structure is fixed. Thus, by adjusting the feedback gain we can change the damping of the intelligent structure so that the goal of controlling and suppressing the vibration of the intelligent structure can be achieved. An active vibration control system of the intelligent structure is shown in Figure 1.

4. THE STATE SPACE EQUATIONS OF MOTION FOR INTELLIGENT STRUCTURES

Here, only the feedback control forces are considered. According to equation (17), the system dynamic equation can be written as

$$[M]\{\ddot{u}\} + [C_S]\{\dot{u}\} + [K]\{u\} = [K_w][K_v]^{-1}\{C_F\}, \quad (26)$$

where $\{C_F\} = C_P\{V_A\}$. C_P is the capacitance of the piezoelectric actuators, and $\{V_A\}$ is the feedback voltage of the piezoelectric actuator.

The output equation of the piezoelectric sensors is

$$\{V_S\} = [K_v]^{-1}[K_{vu}]\{u\}. \quad (27)$$

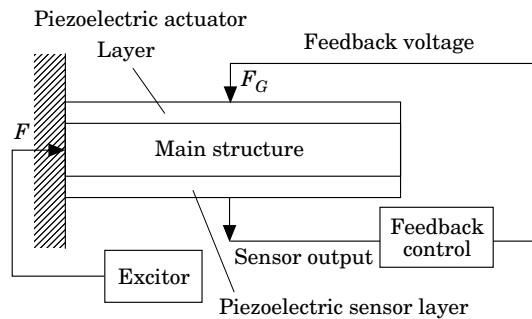


Figure 1. The active vibration control system for an intelligent plate.

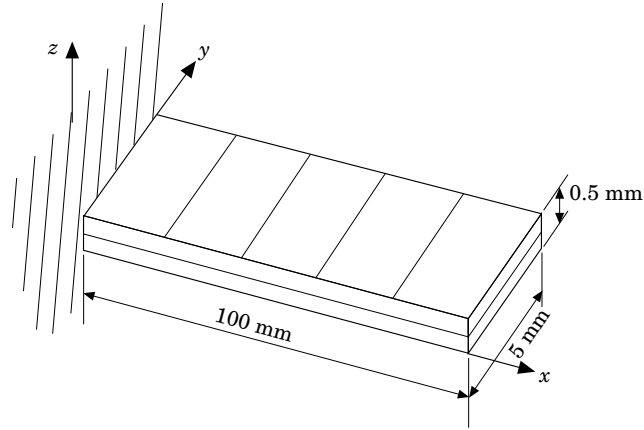


Figure 2. The piezoelectric PVDF bimorph beam.

When the state variables $\mathbf{x} = \{u \dot{u}\}^T$ and $\mathbf{u}_c = \{V_A\}$ are introduced, the system equation of motion can be written in standard state space form as

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}_c, \quad (28)$$

where

$$\mathbf{A} = \begin{bmatrix} [0] & [I] \\ -[M]^{-1}[K] & -[M]^{-1}[C_s] \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} [0] \\ [M]^{-1}[H] \end{bmatrix}.$$

Here

$$[K] = [K_{uw}] + [K_{uw}][K_{vv}]^{-1}[K_{vu}], \quad [H] = C_P[K_{uw}][K_{vv}]^{-1}.$$

The sensor output equation in state space form is

$$\mathbf{y} = \mathbf{C}\mathbf{x}, \quad (29)$$

where

$$\mathbf{y} = \{V_s\}, \quad \mathbf{C} = [[K_{uw}][K_{vv}]^{-1} \ 0]^T.$$

If the negative velocity feedback control law is applied, the sensor output voltage is multiplied by feedback gain and fed into the actuator. Then, the feedback control equation for the intelligent structure is

$$\mathbf{u}_c = G\mathbf{y} = G\mathbf{C}\mathbf{x}. \quad (30)$$

Substituting equation (30) into equation (28), we obtain the following equation:

$$\dot{\mathbf{x}} = (\mathbf{A} + \mathbf{B}\mathbf{G}\mathbf{C})\mathbf{x}. \quad (31)$$

From equation (31), a complex eigenvalue problem can be obtained as follows:

$$[\lambda\mathbf{E} - (\mathbf{A} + \mathbf{B}\mathbf{G}\mathbf{C})]\{\phi\} = 0. \quad (32)$$

The complex eigenvalue is

$$\lambda = \sigma + i\omega_d. \quad (33)$$

The damping ratio can be used to appraise the effect of active vibration control and

suppression for the intelligent structure. The damping ratio is defined as the negative of the normalized real part of the complex eigenvalue [12]; i.e.,

$$\zeta = -\frac{\sigma}{\sqrt{\sigma^2 + \omega_d^2}}. \quad (34)$$

The two measures of dynamic stability are the real part of the complex eigenvalue σ and the damping ratio ζ . A system is dynamically stable when, for each mode, the real parts of the complex eigenvalues are negative. In this case the damping ratios are positive for each mode.

5. NUMERICAL EXAMPLES

5.1. PIEZOELECTRIC BIMORPH BEAM

A piezoelectric bimorph beam [11], shown in Figure 2, is considered in order to check the accuracy of the piezoelectric finite element method presented in this paper.

This beam consists of two identical PVDF uniaxial beams with opposite polarities. The cantilever beam is modelled with five identical elements. The material properties of PVDF are shown in Table 1.

A theoretical solution to the deflection of the beam is given by

$$w(x) = 0.375 \frac{e_{31} V_a}{E} \left(\frac{x}{l} \right)^2, \quad (35)$$

where E is Young's modulus, V_a applied voltage and l the thickness of the beam.

When a unit voltage is applied across the thickness, the deflections at the nodes are calculated by the finite element method presented in this paper. The deflection of the beam is calculated for various applied voltage between 0 and 200 V. The results are shown in Tables 2 and 3. The calculated deflections in the work of Tseng [12] and the theoretical solution are also listed in Tables 2 and 3. The results show the close agreement between the theoretical and the present finite element solutions, and that the accuracy of the present finite element solution is higher than that of Tseng [12]. The total number of degrees of freedom used in this analysis is compared in Table 4 with that in Tseng [12] showing that

TABLE 1

The material properties of the main structure and piezoelectric

Property	PVDF	Graphite/epoxy
E_1	0.2E+10 N/m ²	0.98E+11 N/m ²
E_2	0.2E+10 N/m ²	0.79E+10 N/m ²
G_{12}	0.775E+9 N/m ²	0.56E+10 N/m ²
ν_{12}	0.29	0.29
ν_{21}	0.28	0.28
ρ	1800 kg/m ³	1520 kg/m ³
e_{31}	0.046 C/m ²	0.0
e_{32}	0.046 C/m ²	0.0
e_{33}	0.0	0.0
ϵ_{11}	0.1062E-9 F/m	0.0
ϵ_{22}	0.1062E-9 F/m	0.0
ϵ_{33}	0.1062E-9 F/m	0.0

TABLE 2

The deflection (in m) of the PVDF bimorph beam (for a unit voltage)

Distance (mm)	Method		
	Theory	Tseng [12]	Present
20	0.0140E - 06	0.0150E - 06	0.0139E - 06
40	0.0552E - 06	0.0569E - 06	0.0547E - 06
60	0.1224E - 06	0.1371E - 06	0.1135E - 06
80	0.2208E - 06	0.2351E - 06	0.2198E - 06
100	0.3451E - 06	0.3598E - 06	0.3416E - 06

TABLE 3

The tip deflection (in m) of the PVDF bimorph beam (for various voltages)

Voltage V	Method		
	Theory	Tseng [12]	Present
50	0.1725E - 04	0.1570E - 04	0.1755E - 04
100	0.3451E - 04	0.3200E - 04	0.3409E - 04
150	0.5175E - 04	0.4897E - 04	0.5067E - 04
200	0.6900E - 04	0.6417E - 04	0.6819E - 04

TABLE 4

A comparison of problem sizes

Method	Node no.	D.O.F.		
		Structural	Electric	Total
Tseng [12]	36	108	36	144
Present	12	36	12	48

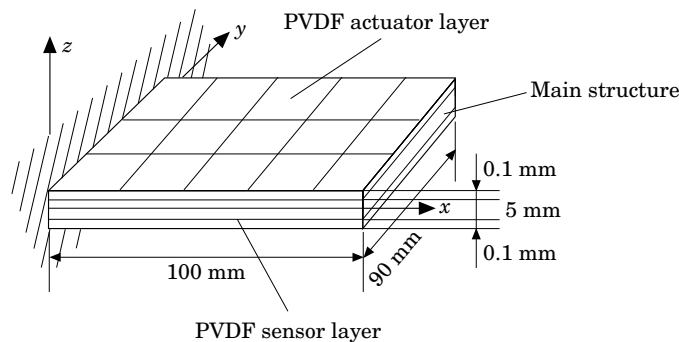


Figure 3. A plate with piezoelectric S/As.

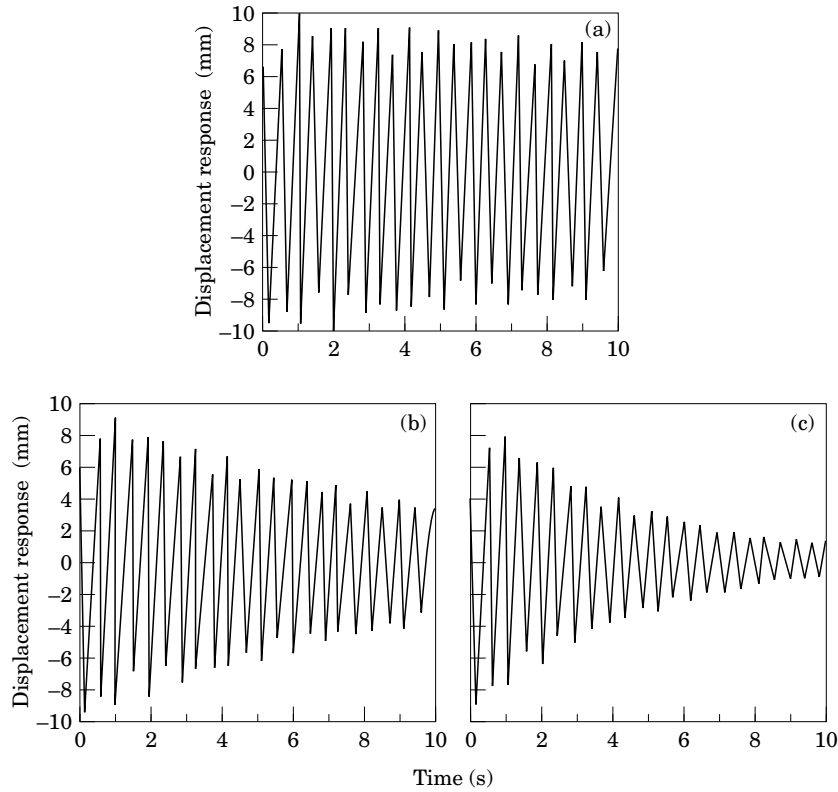


Figure 4. The tip transient displacement response for initial bending displacement. Gain = 0 (a), 100 (b) and 140 (c).

the present finite element formulation with a plate element for the intelligent structure saves a great deal of memory and computation time.

5.2. ACTIVE VIBRATION CONTROL OF INTELLIGENT STRUCTURES

An intelligent plate structure containing distributed piezoelectric S/As on both the top and bottom surfaces is shown in Figure 3. In this structure, the piezoelectric of the bottom layer is considered as a sensor to sense the strain and generate the electrical potential, and the piezoelectric of the top layer as an actuator to control the vibration of the structure. One edge of the plate is rigidly fixed and the others are free. The intelligent structure can be divided arbitrarily. All material properties used are shown in Table 1.

First the transient displacement for the plate is obtained by the Wilson- θ method. The transient displacement and displacement decay envelopes, for feedback gains of 0, 100 and 140, are shown in Figures 4, 5 and 6 for bending and torsional initial displacements respectively. From these figures, it can be seen that as the feedback gain increases, the displacement decays more rapidly.

Next, the damped frequencies and the damping ratios for the first two modes are obtained from complex eigenvalues. The damping ratio change versus the feedback gain is as shown in Figure 7.

From Figure 7, it can be seen that the damping ratio is positive and increases when the feedback gain increases. Thus, the vibration of the intelligent structure can be effectively suppressed by the increase of the feedback gain, and dynamic stability can be assured in active vibration control of intelligent structures.

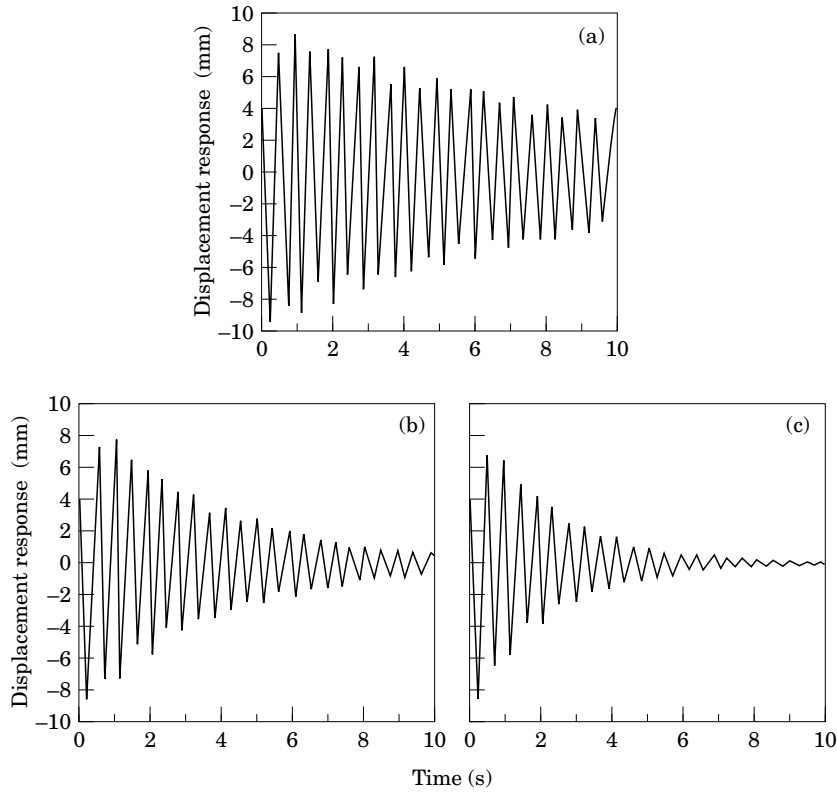


Figure 5. The tip transient displacement response for torsional initial displacement. Gain = 0 (a), 100 (b) and 140 (c).

6. CONCLUSIONS

An efficient finite element formulation for vibration control of the intelligent structure is presented. A new piezoelectric plate element, which saves memory and computation time, is developed. Active vibration control and suppression were also studied by using the negative velocity feedback control law. In order to investigate the effect and dynamic stability of active vibration control for the intelligent structure, the state space equation of the intelligent structure is introduced.

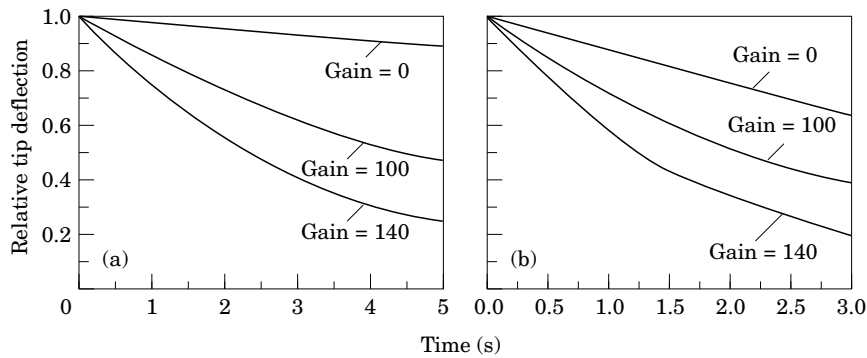


Figure 6. The tip deflection decay envelope. (a) given bending initial displacement; (b) given torsional initial displacement.

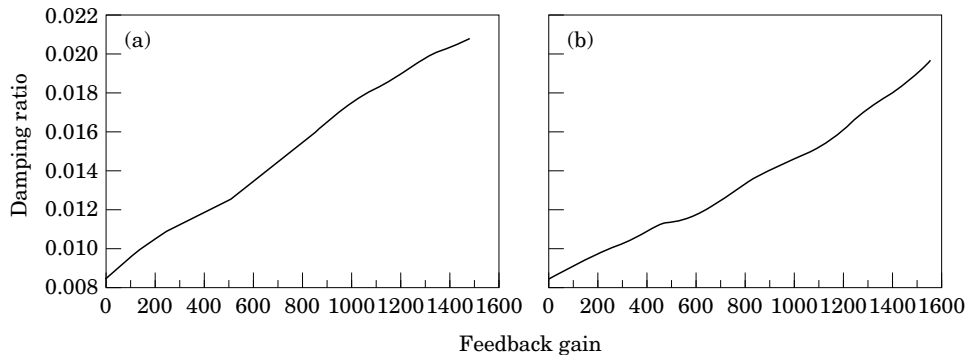


Figure 7. The damping ratio versus feedback gain. (a) First mode; (b) second mode.

By numerical simulation of the dynamic response and analysis of the state space equation of motion for intelligent structures, it is observed that the displacement decay amplitude and the damping ratio increase as the feedback gain increases. It is concluded that the vibration of the distributed parametric system can be effectively controlled and suppressed by using intelligent structures, and dynamic stability can be assured in active vibration control.

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