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COMMENTS ON "A PERTURBATION-ITERATIVE METHOD FOR DETERMINING LIMIT CYCLES OF STRONGLY NON-LINEAR OSCILLATORS"

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The new idea of calculation of limit cycles of strongly non-linear systems and its several numerical examples were presented in [1]. It is interesting to study the calculation of limit cycles of non-linear systems further, however some defects have been found in [1]. They are discussed below:

1. Essentially, solving equations (2.8),† (2.9a) and (2.9b) can be considered as an approximate method for seeking the initial solution ($k = 0$) of the iterative scheme of equations (2.11), (2.12a) and (2.12b) (reference [2]). Therefore, the concept "perturbation" in the title of the paper [1] has not been developed and applied widely in [1] (reference [3]).

2. The operation of the iterative scheme in [1]. (1) The initial value of the iterative scheme in [1] are the solutions $\Phi_0(\phi)$, a_0, b_0 of the equations (2.8), (2.9a) and (2.9b). In paper [1], it is assumed that the equation

$$\ddot{x} + g(x) = f(x, \dot{x}) \qquad \text{(equation (1.1))}$$

possesses at least one limit cycle solution. But this condition does not ensure that the solution Φ_0, a_0, b_0 of equations (2.8), (2.9a) and (2.9b) can exist in the general cases of the equation (1.1) reference [4]. That is to say, the operation of the first step of the iterative scheme in [1] cannot be ensured in the general cases of the equation (1). (2) When $\mu = \mu_0$(μ_0 is small), one cannot ensure that the solutions a_k, b_k($k = 1, \ldots, M$) of equations (2.12a) and (2.12b) exist in the general cases of the equation (1.1), and therefore the operation of the iterative scheme in [1] cannot be ensured in the general cases of the equation (1.1).

3. The convergence of the iterative scheme. (1) The prerequisite of the demonstration of the convergence of the iterative scheme (2.11), (2.12a) and (2.12b) is the existence of the solutions Φ_k, a_k, b_k($k = 1, \ldots, M$) of equations (2.11), (2.12a) and (2.12b) in the general cases of the equation (1.1). Because of parts (1) and (2) in paragraph 2 of this paper, the demonstration of the convergence of the iterative scheme in [1] is not sufficiently strict. (2) The iterative scheme in [1] contains three parts: equations (2.11), (2.12a) and (2.12b). But equations (2.12a) and (2.12b) ($k = 1, \ldots, M$) are not considered in the demonstration of the convergence of the iterative scheme in [1]. Thus the demonstration of the convergence of the iteration of the equation (2.17) is not the demonstration of the convergence of the iterative scheme of equations (2.11), (2.12a) and (2.12b) ($k = 1, \ldots, M$). (3) The convergence of the iteration of the equation (2.17) is dependent on the smallness of the parameter. How big μ has to be to ensure the convergence of the iteration of the equation (2.17) when μ increases? This condition is unknown. The third step of the method in [1] is suggestion only.

† For the numbers of the equations and the meaning of the symbols in this paper please refer to [1].

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AUTHORS' REPLY

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The comments by Dr. Dai Decheng are mostly opinions in kind.

1. The concept of perturbation has been taken as constructing a near-by solution from a known initial state. However, the initial state was continuously updated instead of relying on higher order perturbation terms in μ.
2. The emphasis of our paper was on the constructive solutions of limit cycles starting from zero order perturbation. Existence of limit cycles and their relationship with this type of method has been discussed elsewhere, see for example [A1].
3. The iteration on (2.11) is performed with a_k, b_k held fixed, therefore it is not necessary to consider (2.11), (2.12) simultaneously in the proof of convergence. The third step of the iterative method is not only a suggestion but works well for values of μ up to 2 or 3. Higher values of μ will cause the iterative scheme to break down, which is dealt with by our companion paper [A2].

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