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OPTIMAL ENERGY ABSORPTION AS AN ACTIVE NOISE AND VIBRATION CONTROL STRATEGY

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The purpose of this research is to examine a particular non-application-specific active control strategy for reproducing noise and vibration. The mechanical power absorbed by a controller is taken as a cost function to be maximized. The models suggest that an adaptive scheme to maximize the power absorption is good provided that the adaptation is accomplished within the time taken for a wave emitted by the controller to come back to the control point; that interval is called the "sing around time". The validity of the strategy has been studied and explored through numerical simulations, the results of which are given in this paper and which describe the scope of the experimental studies to be reported later.

1. INTRODUCTION

Social requirements of energy-saving and resource-conservation are forcing mechanical systems, such as aircraft and automobiles, to be made ever lighter, causing more noise and vibration problems than before. In an attempt to overcome these problems while keeping the weight down, active control techniques have been extensively researched. The technique is now well established and seems to have entered the era of practical application (see the review by Ffowcs Williams [1]), although refinements of the basic strategy are still the subject of research. One of the greatest problems is that controllers tend to be specific to particular applications, and careful prior identification of an individual control objective is essential to success; hence the need for a versatile control strategy which is non-application-specific.

To implement active control, a target, or cost function, is usually defined. Since the objective here is to study control systems which are pre-programmed to adapt when attached to any mechanical device, the cost function must be based on information available within the controller. A possible cost function is the mechanical power handled by the controller. The power can of course be negative, and that negative power can be maximized by an optimal control strategy (see, e.g., references [2, 3]). The power absorbed by the controller is a possible cost function, the maximization of which might induce optimal damping on a sound or vibration field. Redman-White *et al*. [4] experimentally studied such a strategy on flexural waves in an infinite beam, concluding that "maximum power absorption" leads to good attenuation of propagating flexural waves. The author [5] confirmed that conclusion on a string model. Elliott *et al*. [6], however, pointed out that if the cost function is applied in general, the vibrational energy in the system can actually increase; the opposite of the desired effect. That negative remark is now widely accepted as a warning against relying on power absorption as a reliable cost function [7].

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The principal objective of the research to be described here is to re-examine this idea and devise a control method to realize its good points while minimizing its negative effects. In particular, the focus will be on the suppression of resonance in partly reverberant fields, which is a particularly important aspect of vibration (the word "reverberant" is used here for lightly damped sound and distributed vibration fields). The preferred control strategy will be based on a velocity feedback control adapted to maximize the mechanical power entering a monopole source. This has the virtue of simplicity and the advantage of being insensitive to wave direction, although admittedly it is less effective at absorbing energy than multiple sources [8]. The method will be developed and illustrated for the simplest possible mechanical configurations, though the ideas are not so restricted; they have potential for multi-dimensional and general application. It will be concluded that provided that the adaptation process is terminated within a definite time interval, the power absorbed becomes a useful cost function for an active controller and frees the strategy from the constraints that had previously been recognized by Elliott and Nelson [7].

2. MAXIMUM POWER ABSORPTION IN A STEADY STATE CONDITION

2.1. MASS–SPRING MODEL INVESTIGATION

Consider the simple mass–spring model depicted in Figure 1. The system is composed of a mass, M, spring, K, and damping, β, and is subject to an external force, F, the velocity of the mass being fed back to the mass by an actuator with a gain Z. As is widely known, the steady state response of the system to an external harmonic excitation, $F(t) = F_0\,e^{i\omega t}$, is given by

$$x(t) = \frac{F_0}{(K - M\omega^2) + i\omega(\beta + Z)}\,e^{i\omega t}. \quad (1)$$

Time-averaged power absorbed by the controller and time-averaged total energy in this system is then given as follows, Z being a complex number: (1) time-averaged power absorbed by the controller,

$$\overline{P} = \frac{1}{2}\frac{\mathrm{Re}\,[Z]\omega^2}{\{(K - M\omega^2) + i\omega(\beta + Z)\}\{(K - M\omega^2) - i\omega(\beta + Z^*)\}}\,|F_0|^2, \quad (2)$$

where the asterisk denotes a complex conjugate and an overbar (‾) denotes a time-averaged value; (2) time-averaged total energy in the system,

$$\overline{E_T} = \frac{1}{4}\frac{(M\omega^2 + K)}{\{(K - M\omega^2) + i\omega(\beta + Z)\}\{(K - M\omega^2) - i\omega(\beta + Z^*)\}}\,|F_0|^2. \quad (3)$$

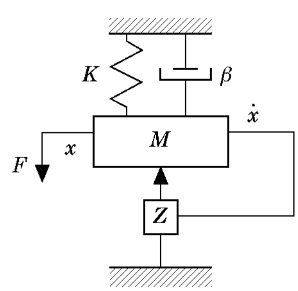

Figure 1. The mass–spring model.

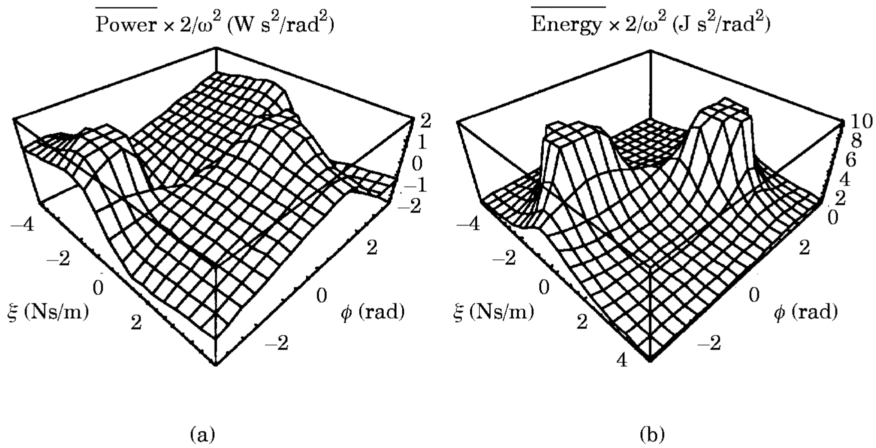

Figure 2. The time averaged power absorption and total energy. Peaks are infinite but truncated for plotting. (a) Power absorption; (b) the total energy in the system.

Consider the condition of maximum power absorption for this mass–spring model. In this study the time-averaged value of the power is maximized. For computational convenience, $F_0 = 1$ N, $M = 1$ kg and $K = 1$ N/m are assumed, where the natural angular frequency ω_n is 1 rad/s.

Suppose that the system is excited by a harmonic force of $\omega = 0{\cdot}5$ rad/s. In Figure 2 are shown the normalized power absorbed and total energy in the system with respect to $\omega^2/2$ based on equations (2) and (3). In the figure, ξ and ϕ denote the absolute value and the argument of the complex gain Z $(=\xi\,\mathrm{e}^{\mathrm{i}\phi})$. The three-dimensional plots with ξ and ϕ as variables clearly show that the condition of maximum power absorption is not the same as that of the minimization of energy, which is sometimes lowered by power extraction in the controller but is often increased. This suggests that if a controller is designed to maximize its time-averaged power absorption, the control action can also increase the total energy in the system. One can see that this undesirable response is caused by the feedback of components of the imaginary part of the velocity. This imaginary part changes the system's dynamic characteristics such that more power is supplied by the external force. This study reminds one of the importance of restricting the feedback gain to a real number, which means that only a velocity is fed back to the actuator. In this case, equations (2) and (3) become simpler:

$$\overline{P} = \frac{Z\omega^2|F_0|^2}{2\{(K - M\omega^2)^2 + \omega^2(\beta + Z)^2\}}, \qquad \overline{E_T} = \frac{(K + M\omega^2)|F_0|^2}{4\{(K - M\omega^2)^2 + \omega^2(\beta + Z)^2\}}. \quad (4, 5)$$

The maximum value of equation (4) is given when

$$Z = \sqrt{\{(K/\omega) - M\omega\}^2 + \beta^2}, \quad (6)$$

which is the mechanical impedance of the system. The power absorbed and the total energy in the system at a resonant condition with various levels of damping are illustrated in Figure 3. The values of gain Z for each power-maximizing impedance condition do not necessarily minimize the corresponding total energy in the system. This total energy can be derived from equation (5), and with control its value is a quarter of that without control. The total power output of the two sources is also reduced to a quarter of that without control. This agrees with the Elliott *et al*. [6] investigation mentioned earlier.

The foregoing example reveals that if the controller is adjusted so that the power absorbed becomes maximum, the feedback gain converges to the mechanical impedance of the system. Especially in the case of resonance, the value converges to the original damping value, doubling the damping coefficient. For instance, in the case of $\beta = 0{\cdot}5$ Ns/m, the value of gain Z for the power-maximizing condition is also 0·5 N s/m. Although this action attenuates the total energy in the system to a quarter of the original value, the attenuation performance of the maximum power absorption controller is not good enough. At first sight then, "maximum power absorption" does not seem to be such a good idea.

3. MAXIMUM POWER ABSORPTION AS A WAVE CONTROL STRATEGY

The above study was based on the idea of absorbing power from a very simple vibrating object; the controller affects the power input to the system as well as the power absorbed. Vibration in continuous systems is a superposition of travelling waves and here the equivalent power absorption model for the simplest case is to be developed. The finiteness of the wave region introduces a fundamental time parameter into the system and it will be shown that power optimization can have quite different effects if the optimization proceeds over a time that is short or long in comparison with that characteristic time.

Consider the simple string model depicted in Figure 4. The left-hand end of the string, at $x = 0$, is harmonically excited with a constant displacement $y = Y_0\,e^{i\omega t}$ and, at the right-hand end, a normal force is applied by the actuator. The force is actively created to be equal to the product of the velocity at that point and a transfer function $Z = \xi\,e^{i\phi}$. It

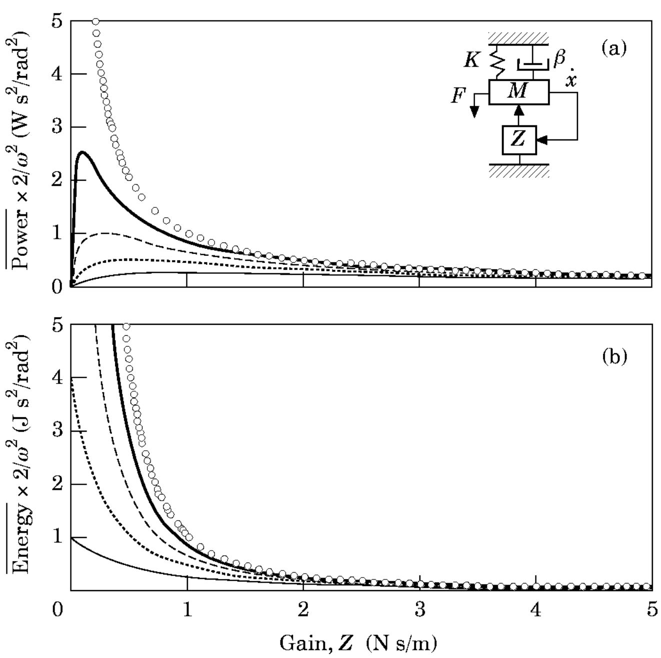

Figure 3. The power absorbed by the controller and total energy in a mass–spring system with a real-number-gain feedback control. (a) Time-averaged power absorbed; (b) time-averaged total energy in the system. β values (Ns/m): ○○○, 0; ▬▬, 0·1; – – –, 0·25; · · · ·, 0·5; ——, 1·0. $M = 1$ kg, $K = 1$ N/m, $F = \exp(i\omega t)$ N, $\omega = 1$ rad/s.

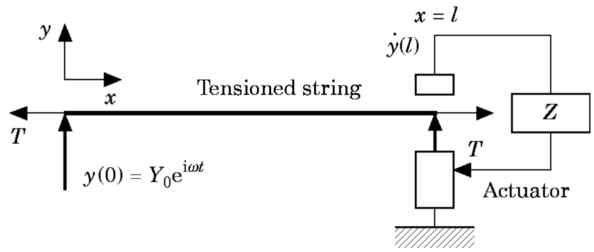

Figure 4. A finite string model.

is supposed that the system is linear, so that without loss of generality one can assume the following harmonic solution of the wave equation,

$$y = A\,\mathrm{e}^{\mathrm{i}\omega(t - x/c)} + B\,\mathrm{e}^{\mathrm{i}\omega(t + x/c)}, \tag{7}$$

where ω is the frequency and c is the speed of the wave in the string, $c = (T/\rho)^{1/2}$, T being the tension and ρ the mass per unit length.

The author [9] has shown that the time-averaged total energy in a string and the time-averaged power absorbed in a special case in which $l = 1$ m, $T = 1$ N and $\rho = 1$ kg/m with an excitation of unit amplitude are given by

$$\overline{E_{Total}} = \frac{\omega^2}{4}\frac{1 + \xi^2}{\sin^2 \omega\xi^2 \cos^2 \phi + (\sin \omega\xi \sin \phi - \cos \omega)^2}, \tag{8}$$

$$\overline{P_{absorbed}} = \frac{\omega^2}{2}\frac{\xi \cos \phi}{\sin^2 \omega\xi^2 \cos^2 \phi + (\sin \omega\xi \sin \phi - \cos \omega)^2}, \tag{9}$$

and that the condition of maximum power absorption tends to increase the total energy in the string, as was seen for the simple mass–spring model investigation. The power absorption and the total energy in the system when the feedback gain is restricted to a real number are shown in Figure 5. This figure shows that when the excitation is close to the first resonance frequency ($\omega = \pi/2$ rad/s), the total energy is attenuated to some extent, but for the other frequencies the condition of maximum power absorption increases the total energy in the string.

A continuous system tends to respond to the action of the maximum power absorber in a similar way to that of a one-degree-of-freedom system. This indicates that modal behaviour dominates its response, the absorber not attenuating the vibration successfully. Therefore emphasis towards the control is now adjusted from the direct control of vibration to the control of travelling waves.

3.1. SEMI-INFINITE STRING MODEL

Consider the semi-infinite tensioned string depicted in Figure 6. A wave travels in from the left, and a normal force is applied at the right-hand end at $x = 0$. The force is actively created to be equal to the product of the velocity at that point and a transfer function $Z = \xi\,\mathrm{e}^{\mathrm{i}\phi}$. This is exactly the same as for the finite string model investigated earlier, there being only an origin change in the earlier boundary conditions. Considering the boundary conditions at $x = 0$, one obtains the following time-averaged power absorbed by the actuator [5],

$$|\overline{P}| = \tfrac{1}{2}k_p\omega^2|A|^2, \tag{10}$$

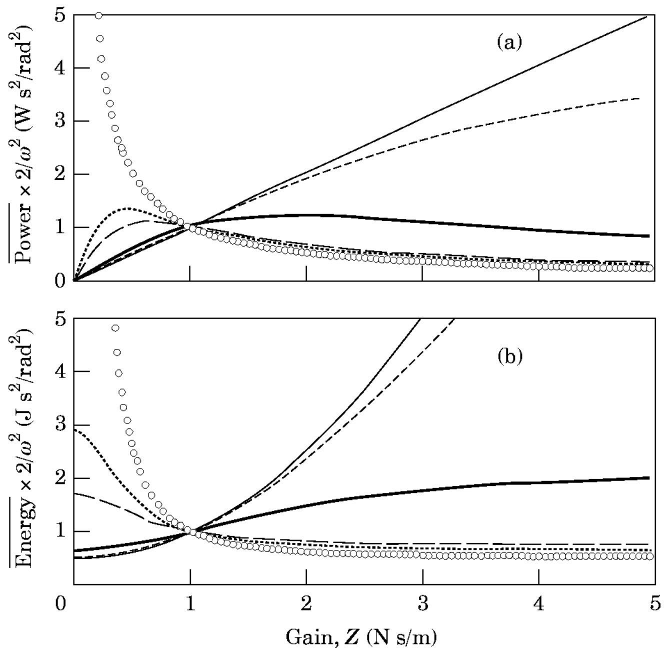

Figure 5. The power absorbed and total energy in a string with a real-number-gain feedback control. (a) Time-averaged power absorbed; (b) time-averaged total energy in a string. ω (rad/s): ——, 0·5; — — —, 1; ○○○, $\pi/2$; · · · ·, 2; - - - -, 3; ——, π.

where

$$k_p = 4\left(\frac{T}{c}\right)^2 \frac{Z_r}{\{Z_r + (T/c)\}^2 + Z_i^2}, \tag{11}$$

Z_r and Z_i being the real part and the imaginary part of the gain Z. The condition of maximum power absorption is given by

$$Z_r = T/c, \qquad Z_i = 0. \tag{12, 13}$$

This result indicates that when a controller is placed at the end of a string, an implementation of "maximum power absorption" is identical to adjusting the control gain to equal the system's characteristic impedance, avoiding reflection from the control end.

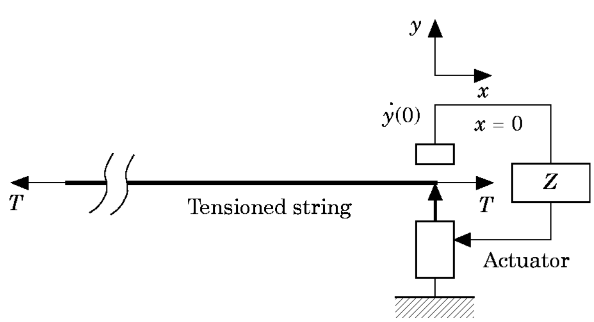

Figure 6. A semi-infinite string model.

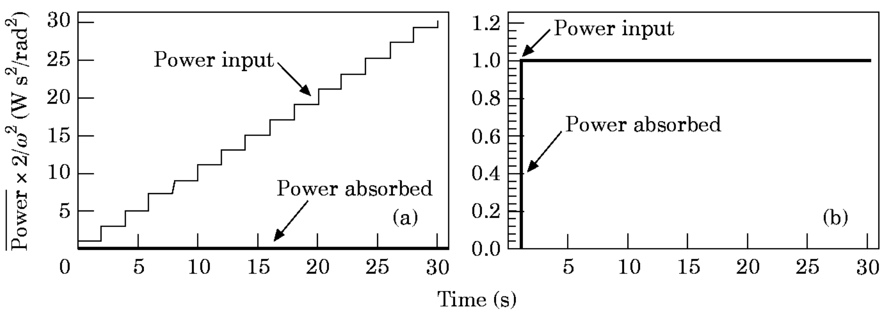

Figure 7. The time averaged power input and power absorbed. (a) $Z \approx 0$; (b) $Z = 1$.

3.2. DISCUSSION

In Figure 7 are shown the mean value of the power input and output for every two seconds in the finite string model shown in Figure 4 with the above two conditions of maximum power absorption, resonant cases, $\omega = (2n - 1)(\pi/2)$, $n = 1, 2, 3, \ldots$, being calculated. One is $Z \approx 0$, which is the condition for the "freely terminated" finite string as shown in Figure 5 with $\omega = \pi/2$; and the other is $Z = 1$, which is the impedance appropriate to an infinite string model; obtained from equation (12) with $T = c = 1$. These values are factored by $\omega^2/2$. When $Z \approx 0$, the controller appears to stimulate the input as soon as the primary source is subject to the reflected wave (it takes one second for a wave to travel from one end to the other), and the power input and output in the resonant state grows without bound as does the total energy in the system. The total energy in the system is the time integral of the difference between the power input and output. It is hardly visible that power absorbed is increasing from the figure, but it is actually as indicated in Figure 5, which shows the power absorbed in the steady state condition. On the other hand, when $Z = 1$, all the power supplied by the actuator is absorbed by the controller as soon as the wave reaches it, resulting in reasonably small energy in the system. Based on the foregoing consideration, one can see that the idea of maximum power absorption should be subdivided into two strategies. One, to maximize the amount of power absorbed by the controller in a steady state condition, is an unreliable strategy. The other, in which one absorbs as much power as possible from the *incident* wave, corresponds to the condition derived from a semi-infinite string investigation, namely $Z = 1$. The two are essentially different. The latter one, it is believed, is optimal.

4. ACTIVE CONTROL ALONG A STRING

A finite string model which is supported at both ends and controlled along the string is investigated here.

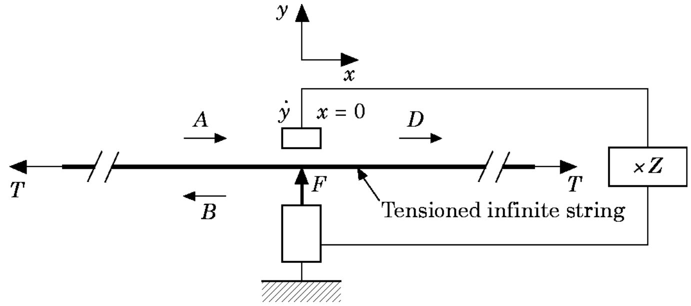

Figure 8. An infinite string model controlled in the middle.

4.1. CONDITION OF OPTIMAL ENERGY ABSORPTION

Consider the infinite string model shown in Figure 8. An anti-force, $F = \dot{y}Z$ $(=\dot{y}\xi\, e^{i\phi})$, is applied to the string at $x = 0$ to control wave A, which is coming from the left. A part of the wave will continue travelling to the right as wave D, and a part of the wave is reflected to the left as wave B. One has the following expressions:

$$y = A\, e^{i\omega(t - x/c)} + B\, e^{i\omega(t + x/c)}, \quad x \leqslant 0, \qquad y = D\, e^{i\omega(t - x/c)}, \quad x \geqslant 0. \tag{14, 15}$$

Considering the continuity of displacement and the equilibrium of forces at $x = 0$, one obtains the following displacement at $x = 0$:

$$y = (2T/c)/\{(2T/c) + Z\}A\, e^{i\omega t}. \tag{16}$$

Power output at $x = 0$ is given by

$$P = \mathrm{Re}\,[F(0)]\,\mathrm{Re}\,[\dot{y}(0)] = \{F(0) + F^*(0)\}\{\dot{y}(0) + \dot{y}^*(0)\}/4, \tag{17}$$

where $F(0) = -\dot{y}(0)\xi\, e^{i\phi}$. By substituting equation (16) into equation (17), one obtains

$$P = -\tfrac{1}{2}\xi\omega^2(|A|^2|K_m|^2)\{\cos\phi - \cos(\phi + 2\omega t + 2\phi_A + 2\phi_{K_m})\}. \tag{18}$$

where $K_m = (2T/c)/\{(2T/c) + Z\}$, and ϕ_A and ϕ_{K_m} are the phases of A and K_m, respectively. The time-averaged power output is then given by

$$\bar{P} = -\tfrac{1}{2}\xi\omega^2|A|^2|K_m|^2 \cos\phi, \tag{19}$$

which indicates that the controller always absorbs power as long as $\cos\phi$ is positive. A little algebra leads to

$$\bar{P} = -2\omega^2|A|^2\left(\frac{T}{c}\right)^2 \frac{Z_r}{\{(2T/c) + Z_r\}^2 + Z_i^2}, \tag{20}$$

where Z_r and Z_i are the real part and imaginary parts of Z, respectively. The maximum value of the power absorbed is defined by the last coefficient of equation (20),

$$k_{pr} = \frac{Z_r}{\{(2T/c) + Z_r\}^2 + Z_i^2}. \tag{21}$$

The maximum value of k_{pr} is obtained when Z_r and Z_i satisfy the conditions

$$Z_r = 2T/c, \qquad Z_i = 0, \tag{22, 23}$$

and the maximum value of the time-averaged power absorbed is

$$\bar{P} = \tfrac{1}{4}\omega^2|A|^2 T/c. \tag{24}$$

As will be seen later in equation (38), the total energy of the one-dimensional wave A is given by

$$\bar{E} = \tfrac{1}{2}\rho\omega^2|A|^2\eta, \tag{25}$$

where η is the length of the string, and the power it is conveying is

$$\bar{P} = \tfrac{1}{2}\omega^2|A|^2 T/c. \tag{26}$$

Therefore, one sees that the controller using the idea of maximum power absorption absorbs half the power carried by the travelling wave. This agrees with the results of similar investigations by Lighthill [2] and Crighton *et al*. [10]. A wave travelling from the right to the left gives exactly the same condition as above. The linearity of the wave equation

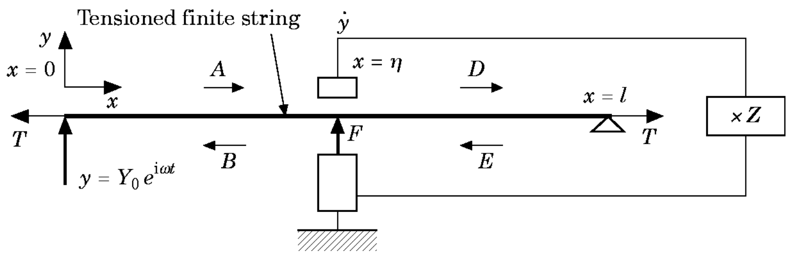

Figure 9. A finite string model controlled in the middle.

indicates that the optimal condition derived above for the controller is applicable to general waves, which are travelling in both directions on a string.

Interest here is to control vibration in a finite system. It has been shown in section 3 that an optimal control gain for this purpose is the one derived from the analysis of the semi-infinite system. Now that an optimal gain from a general infinite string model has been obtained, one can apply this optimal condition to a general finite string system in this section and study the effect of the control.

4.2. THE STEADY STATE RESPONSE OF THE STRING

Consider the string model shown in Figure 9, in which the left end is excited harmonically with constant displacement, and the right end is fixed, with a control force at $x = \eta$. With reference to the figure, the wave field is given by

$$y = A\,\mathrm{e}^{\mathrm{i}\omega(t - x/c)} + B\,\mathrm{e}^{\mathrm{i}\omega(t + x/c)}, \quad 0 \leqslant x \leqslant \eta, \qquad y = D\,\mathrm{e}^{\mathrm{i}\omega(t - x/c)} + E\,\mathrm{e}^{\mathrm{i}\omega(t + x/c)}, \quad \eta \leqslant x \leqslant l. \tag{27, 28}$$

The left end is excited with a constant displacement, $y = Y_0\,\mathrm{e}^{\mathrm{i}\omega t}$. Considering the boundary conditions at both ends, and the continuity of displacement and the equilibrium of forces at $x = \eta$, one obtains four linear equations for the unknowns A, B, D and E:

$$M_E \times \begin{bmatrix} A \\ B \\ D \\ E \end{bmatrix} = \begin{bmatrix} Y_0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \tag{29}$$

where

$$M_E = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & \mathrm{e}^{2\mathrm{i}\omega l/c} \\ 1 & \mathrm{e}^{2\mathrm{i}\omega\eta/c} & -1 & -\mathrm{e}^{2\mathrm{i}\omega\eta/c} \\ Z - T/c & (Z + T/c)\,\mathrm{e}^{2\mathrm{i}\omega\eta/c} & T/c & -(\mathrm{T}/c)\,\mathrm{e}^{2\mathrm{i}\omega\eta/c} \end{bmatrix}. \tag{30}$$

The unknowns are easily found to be

$$A = (Y_0/\det M_E)\{(Z + 2T/c)\,\mathrm{e}^{2\mathrm{i}\omega(\eta + l)/c} - Z\,\mathrm{e}^{4\mathrm{i}\omega\eta/c}\}, \tag{31}$$

$$B = (Y_0/\det M_E)\{(Z - 2T/c)\,\mathrm{e}^{2\mathrm{i}\omega\eta/c} - Z\,\mathrm{e}^{2\mathrm{i}\omega l/c}\}, \tag{32}$$

$$D = (Y_0/\det M_E)2(T/c)\,\mathrm{e}^{2\mathrm{i}\omega(\eta + l)/c}, \qquad E = -(Y_0/\det M_E)2(T/c)\,\mathrm{e}^{2\mathrm{i}\omega\eta/c}, \tag{33, 34}$$

where

$$\det M_E = (Z + 2T/c)\, e^{2i\omega(\eta + l)/c} + (Z - 2T/c)\, e^{2i\omega\eta/c} - Z\, e^{4i\omega\eta/c} - Z\, e^{2i\omega l/c}. \tag{35}$$

The total energy in the string is

$$E_{Total} = \int_0^l \left\{ \tfrac{1}{2}\rho\dot{y}^2 + \frac{T}{2}\left(\frac{\partial y}{\partial x}\right)^2 \right\} dx. \tag{36}$$

Therefore, for the time average of the energy, one has

$$\overline{E_{Total}} = \int_0^\eta \left\{ \tfrac{1}{2}\rho\overline{\dot{y}^2} + \frac{T}{2}\overline{\left(\frac{\partial y}{\partial x}\right)^2} \right\} dx + \int_\eta^l \left\{ \tfrac{1}{2}\rho\overline{\dot{y}^2} + \frac{T}{2}\overline{\left(\frac{\partial y}{\partial x}\right)^2} \right\} dx. \tag{37}$$

By substituting equations (27) and (28) into equation (37), one obtains the following time-averaged energy in the string:

$$\overline{E_{Total}} = (\rho\omega^2/2)\{(|A|^2 + |B|^2)\eta + (|D|^2 + |E|^2)(l - \eta)\}. \tag{38}$$

4.3. THE EFFECTIVENESS OF CONTROL IN THE MIDDLE OF A STRING

Consider a simple string model, in which $T = 1$ N, $\rho = 1$ kg/m and $|Y_0| = 1$ m. The length of the string is 2 m and is controlled at $\eta = 1$ m. When no control force is fed back, the total energy is calculated with equation (38) to be

$$\overline{E_{Total}} = \frac{\omega^2}{1 - \cos(4\omega)} = \frac{4\pi^2}{1 - \cos(8\pi f)} f^2. \tag{39}$$

When the optimal feedback gain, $Z = 2T/c = 2$ Ns/m, is fed back to the string, the energy will be

$$\overline{E_{Total}} = \frac{2 - \cos(2\omega)}{1 - \cos(2\omega)}\frac{\omega^2}{4} = \frac{2 - \cos(4\pi f)}{1 - \cos(4\pi f)}\pi^2 f^2. \tag{40}$$

The total energies in a string (in decibels) for the two cases are compared in Figure 10. This figure indicates that the control force is effective in suppressing half of the resonance modes of the string, but is not effective for the other half of the modes. The phenomenon is explained by the fact that the control point is always at an anti-nodal point for the controllable modes, but at a nodal point for the uncontrollable modes. This means that the controller cannot absorb any power from the string for the uncontrollable modes since the control point is always stationary.

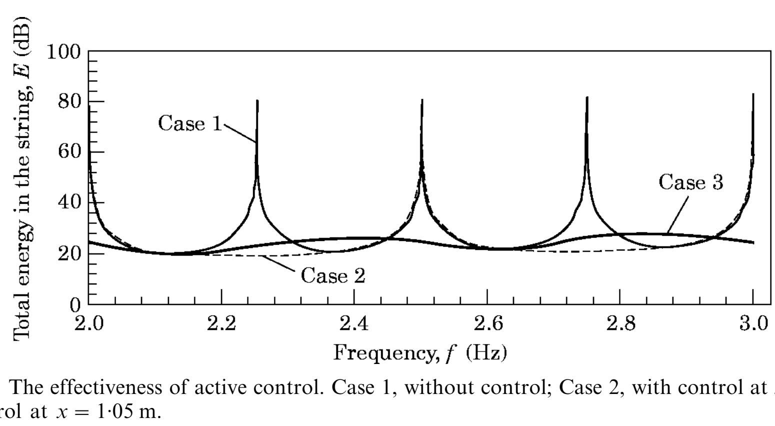

Figure 10. The effectiveness of active control. Case 1, without control; Case 2, with control at $x = 1$ m; Case 3, with control at $x = 1{\cdot}05$ m.

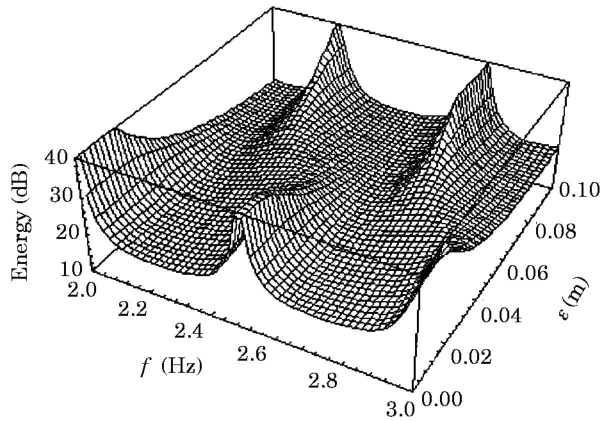

Figure 11. The effectiveness of offset.

This result indicates that the active control with maximum power absorption is effective in suppressing resonance modes the nodal points of which do not coincide with the control point.

To increase the number of controllable modes, the control point was placed in the middle with an offset ε. The total energy for the non-dimensional model with the offset control is given by (refer to equation (38))

$$\overline{E_{Total}} = (\omega^2/2)\{(|A|^2 + |B|^2)(1+\varepsilon) + (|D|^2 + |E|^2)(1-\varepsilon)\}, \tag{41}$$

where

$$A = \frac{2\,e^{2(1+\varepsilon)i\omega} - e^{4\varepsilon i\omega}}{2\,e^{2(1+\varepsilon)i\omega} - e^{4\varepsilon i\omega} - 1}\,Y_0, \qquad B = \frac{-1}{2\,e^{2(1+\varepsilon)i\omega} - e^{4\varepsilon i\omega} - 1}\,Y_0, \tag{42, 43}$$

$$D = \frac{e^{2(1+\varepsilon)i\omega}}{2\,e^{2(1+\varepsilon)i\omega} - e^{4\varepsilon i\omega} - 1}\,Y_0, \qquad E = \frac{-e^{(\varepsilon-2)i\omega}}{2\,e^{2(1+\varepsilon)i\omega} - e^{4\varepsilon i\omega} - 1}\,Y_0. \tag{44, 45}$$

Substitution of equations (42)–(45) into equation (41) gives

$$\overline{E_{Total}} = \frac{4 + 2\varepsilon - 2(1+\varepsilon)\cos\{4\pi f(1-\varepsilon)\}}{6 - 4\cos\{4\pi f(1-\varepsilon)\} - 4\cos\{4\pi f(1+\varepsilon)\} + 2\cos(8\pi f\varepsilon)}\,4\pi^2 f^2. \tag{46}$$

The effectiveness of the offset ε is illustrated in Figure 11. This figure shows that the response of the string changes as the offset ε increases, and the response reads smallest when ε is about 0·05 m. When ε is 0·1 m, another clear resonance occurs. Again, this is because the control point coincides with the nodal points of the resonance modes. The response with $\varepsilon = 0{\cdot}05$ m is plotted in Figure 10 with the other conditions. This figure clearly indicates that the control with an offset, $\varepsilon = 0{\cdot}05$ m, suppresses resonance effectively within this frequency band. One should bear in mind that although this control condition is effective within the above frequency band, there is a possibility that a resonance mode exists whose nodal point coincides with the control point.

5. CONTROL LAW

5.1. ADAPTIVE FEEDBACK CONTROL

One promising control strategy is to try to absorb as much power as possible from the incident energy flux, as stated above. It should be possible to find an optimum feedback

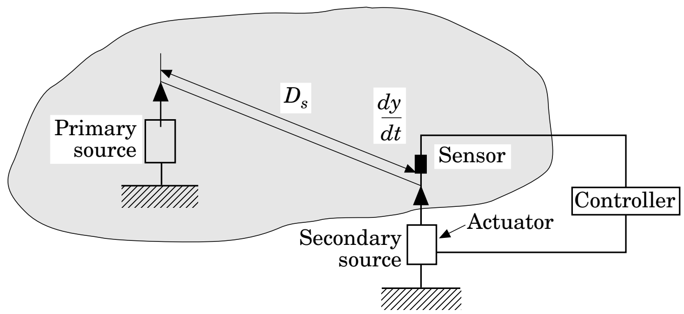

Figure 12. The vibrating structure.

gain by adaptively changing the gain while continuously monitoring the power absorbed. However, one has to remember that if the controller does not absorb all the power, some of the waves are reflected back from the control position. When the wave reaches the source of the vibration, the wave may stimulate the impedance of that point, letting more power enter the system. The optimal gain is the one obtained in an infinite system in which it is assumed that there is no reflection from the primary source; therefore, this effect makes it difficult for the controller to obtain the optimal feedback gain. To avoid this undesirable effect the controller has to find the optimum feedback gain in a short time and stop adapting before the reflected wave comes back to the control point, i.e., *within the sing-around time*. In Figure 12 is shown a general representation of a vibrating system which is subject to control. According to the above considerations, the sampling period, T_{SP}, must satisfy

$$T_{SP} < 2D_S/c, \tag{47}$$

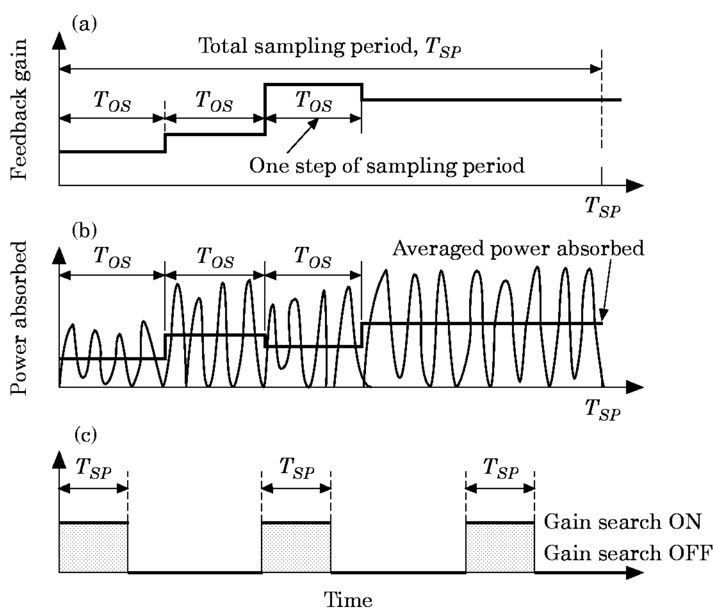

Figure 13. The feedback gain optimization process. (a) The feedback gain; (b) the power absorbed; (c) repetition of the gain search.

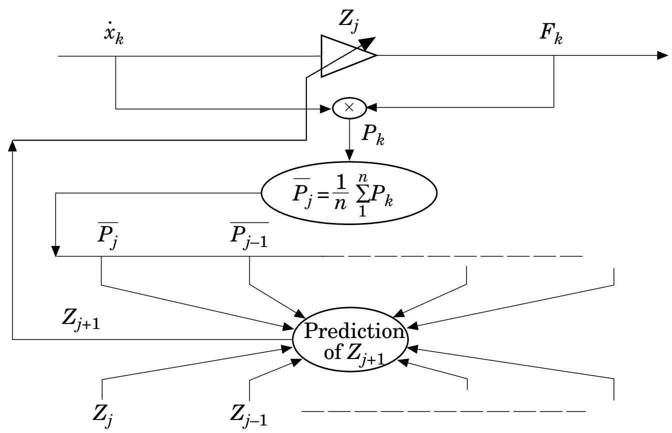

Figure 14. The algorithm for finding the optimum feedback gain.

where D_S is the distance from the control point to the source of vibration, and c is the speed of the wave concerned; it may differ with frequency depending on whether or not the wave system is dispersive.

By using a simple one dimensional tube, Guicking *et al*. [11] and Orduña-Bustamente and Nelson [12] demonstrated practical control systems capable of producing the desired impedance in the region of the secondary source. Their aim seems essentially the same as ours; however, in our system the power absorbed itself is taken as a cost function to be maximized by the method described below. Our strategy leads to a non-application specific controller which we believe holds promise for multi-dimensional and general application.

5.2. FEEDBACK GAIN OPTIMIZATION PROCESS

In Figure 13 is shown the algorithm of our control law, in which the feedback gain is altered successively in the direction that makes the controller absorb more power. In Figure 13(a) it is illustrated that a certain feedback gain is kept for T_{OS}, during which period power absorbed by the controller is measured and averaged as shown in Figure 13(b). By repeating this measurement with different feedback gains and comparing successive averaged power absorbed, optimal feedback gain is sought. One has to find the optimal feedback gain with the limitation of the total sampling period, T_{SP}. After the controller finds the optimal feedback gain, the controller holds the gain and continues controlling the vibration. Since the conditions of the vibrating structure might be subject to change and also because the optimal gain might not have been found in a single search, the control processing has to be continued after the first gain setting is found. In other words, the controller must be adaptive. For that purpose, the same optimization of the feedback gain will be carried out at regular intervals as illustrated in Figure 13(c), allowing for the settling of waves.

5.3. ALGORITHM FOR OPTIMIZATION

The general idea of the algorithm is shown in Figure 14. The velocity measured at the control point is multiplied by the feedback gain, Z_j, and the product will be fed back to the vibrating system as a force. In principle, the idea is identical to the classical velocity feedback control, but note that the feedback gain is attenuated automatically so that the active element absorbs maximum power as described in the figure. In order to find the optimum gain, the absorbed power is accumulated successively, and the time-averaged power absorbed, $\overline{P}_j$, is calculated. The power is stored with a corresponding feedback gain.

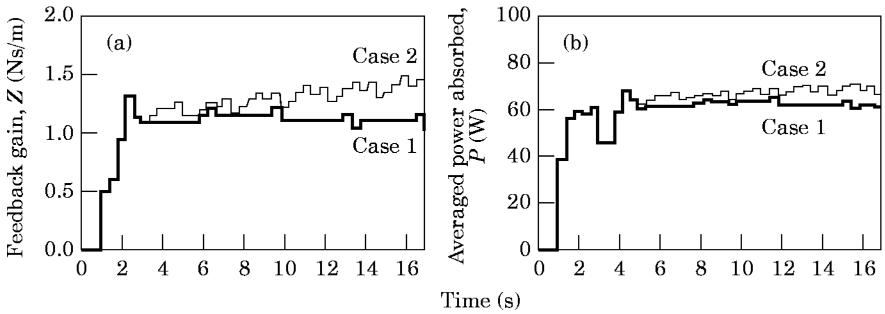

Figure 15. The effect of the gain search interval for the repetition of gain search. (a) Feedback gain; (b) averaged power absorbed. Case 1, gain search interval = 2·4; Case 2, gain search interval = 0.

The next feedback gain, Z_{j+1}, is then determined and replaces the previous gain. This problem can be considered as an optimization problem that searches for a maximum of a continuous function of a single variable, using function values only. This type of optimization problem can generally be solved by iterative methods (see the book by Box *et al*. [13]). For our work, the modified Davies, Swann and Campey algorithm has been used [14].

6. SIMULATION

To study the validity of the proposed control method a simulation program was coded and applied to two types of string models. One is the model controlled at the end and the other is controlled in the middle of the string.

6.1. CONTROL AT THE END OF A STRING

The left end of the string depicted in Figure 4 is excited harmonically with a constant displacement. The string is controlled by the controller placed at the other end with the proposed method.

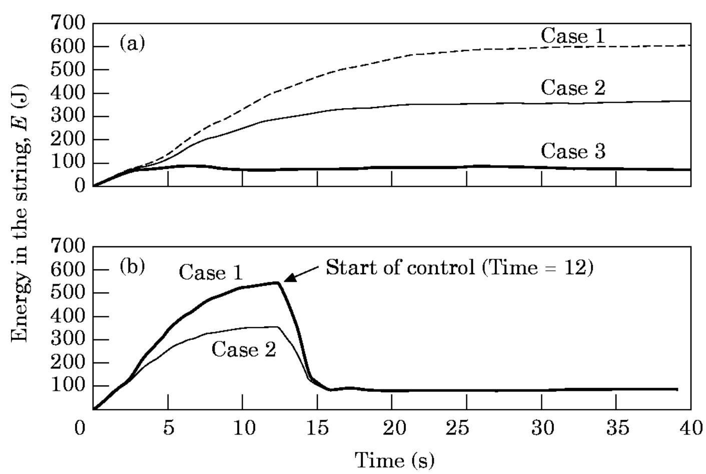

Figure 16. The effectiveness of active control when excited with a frequency $f = 2{\cdot}25$ Hz. (a) Control from the beginning: Case 1, $Z = 0$; Case 2, $Z = 0{\cdot}1$; Case 3; active control. (b) Control after storage of energy: Case 1, $Z = 0$; Case 2, $Z = 0{\cdot}1$.

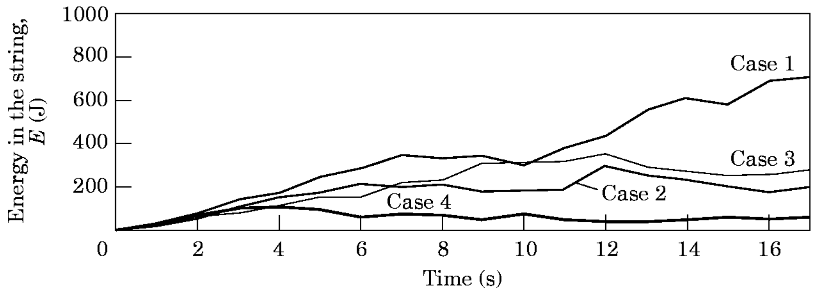

Figure 17. The effectiveness of active control with random excitation (3–5 Hz). Case 1, $Z = 0$; Case 2, $Z = 0{\cdot}1$; Case 3, Z is infinite (fixed); Case 4, optimal control applied.

The time history of the feedback gain and its corresponding time-averaged power absorbed are shown in Figure 15. An anti-resonance condition was chosen for this simulation. Two cases were calculated. One is the case in which the gain search is repeated with a certain interval (gain search interval = 2·4) after the initial gain search, reflected waves not being considered in the optimization process. The other is the case in which the gain search is repeated continuously, reflected waves being considered (gain search interval = 0). The former result shows that the optimal feedback gain (unity in this model) is found within the first sing-around time and the gain is kept in the subsequent gain search, supporting the validity of the control method. The latter case shows that the gain increases gradually after the optimal gain is found and the power absorbed also increases. As is evident from Figure 5, the gain is expected to become infinite in the anti-resonance case. This simulation result agrees with this tendency, indicating that the gain search beyond the sing-around time gets the gain search direction wrong.

In order to examine the effectiveness of the control, the total energy in the string was calculated for different excitation conditions. In Figure 16(a) is shown the energy in a string with velocity feedback control and optimal energy absorption (indicated as "active control" in the figure). The string was shaken with the fifth mode frequency ($f = 2{\cdot}25$ Hz). This figure indicates that in the absence of feedback control resonance occurs, and that the energy level decreases as the value of the feedback gain increases. The condition of optimal power absorption, which is equivalent to $Z = 1$, is restraining the resonance.

To study the flexibility of our control strategy, the control of the wave was commenced after the system had stored plenty of energy. The result of the simulation is shown in Figure 16(b). Two cases were studied. One is the case in which no feedback is applied initially, and the other is the case in which the initial velocity feedback with $Z = 0{\cdot}1$ is applied. In both cases it is indicated that control after the storage of the energy is possible and effective in restraining the resonance.

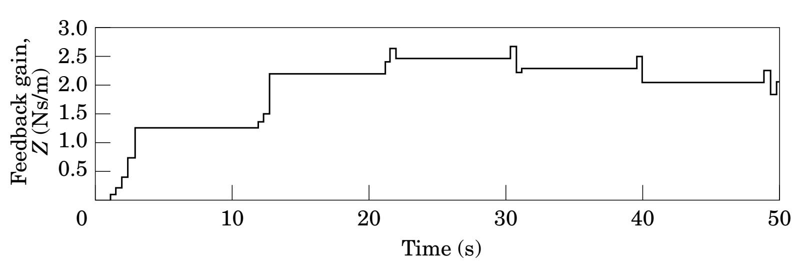

Figure 18. The convergence of the feedback gain search.

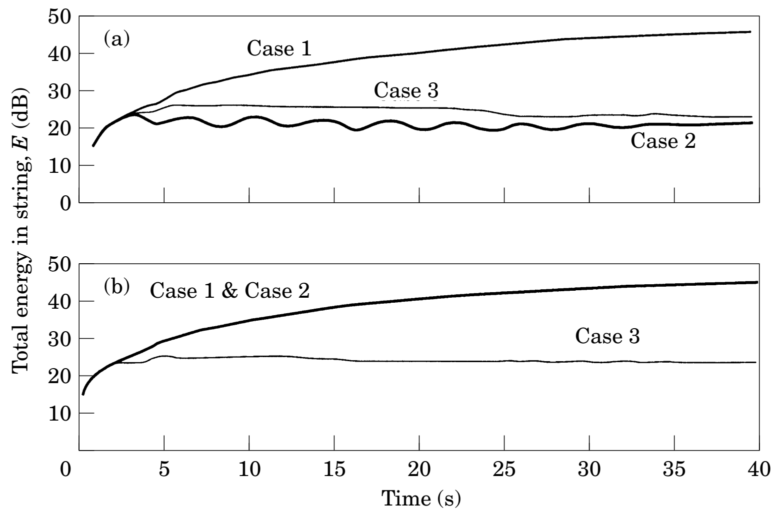

Figure 19. The effectiveness of active control. (a) Excited with frequency of 2·25 Hz; (b) excited with frequency of 2·5 Hz. Case 1, without control; Case 2, with control at $x = 1$ m; Case 3, with control at $x = 1{\cdot}05$ m.

The applicability of the proposed method to broadband random excitation is illustrated in Figure 17; the string model was excited by statistically stationary random noise (3–6 Hz). Four cases are illustrated; Case 1, which no feedback is applied ($Z = 0$), shows a steady energy increase. As the feedback gain increases (Case 2), the total energy tends to be suppressed. On the other hand, when the gain becomes infinite, which is the case with the right end fixed (Case 3), the energy becomes higher. Case 4, in which the active control is applied, appears to yield the best control result. This comparison indicates that the idea of maximum power absorption is also effective in controlling a vibrating system under random excitation.

6.2. CONTROL IN THE MIDDLE OF THE STRING

The response of the simple string model, studied in section 4.3, subjected to a harmonic excitation ($f = 2{\cdot}25$ Hz; resonance frequency) was simulated. In Figure 18 is shown a time history of feedback gain of a control point at $x = 1{\cdot}05$ m. This result indicates that the feedback gain converges to 2, which was previously found to be optimal (equation (22)), supporting the proposed control method.

The effectiveness of active control when the excitation frequency is 2·25 Hz is shown in Figure 19(a). When no control is applied, the total energy in the string grows steadily, whereas when a control force is applied at $x = 1$ m, the energy converges to about 20 dB. When a control is applied at $x = 1{\cdot}05$ m, the energy converges to just over that of $x = 1$ m. These results agree with the predicted responses of the energy shown in Figure 10. The same results are shown in Figure 19(b), but with an excitation frequency of 2·5 Hz. This illustrates that a control at $x = 1$ m is ineffective; however, a control at $x = 1{\cdot}05$ m suppresses the resonance. These results also agree with the predicted responses shown in Figure 10.

7. CONCLUSIONS

In an attempt to realize non-application-specific active control, a new exploitable control principle was sought. In analogy with the good effect of damping, which extracts energy from a vibrating field, we have studied the possible use of active damping as a general principle, optimal damping being possibly associated with the maximum extraction of power from the vibration. It was soon realized that such a notion was a gross oversimplification; control can change both the energy entering and leaving the system. None the less, a usable strategy has evolved during the reported research to optimize maximally the power extraction without affecting the power entering the system. This is achieved by adapting the controller over a definite and limited learning time. This strategy was then supported by numerical simulations. The research has shown that the active control of vibration without any prior identification of the control object, or without any measurement of the incident wave, is possible by a scheme that takes the power absorbed by the control device as the cost function. This cost function, the power absorbed, has in the past been considered ineffective and potentially dangerous; however, it has been possible to demonstrate that if the power is measured and the adaptation of the controlling gain is made only within a sing-around time, the power absorbed becomes a good cost function.

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