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LETTERS TO THE EDITOR

FURTHER OBSERVATIONS ON WAVE PROPAGATION IN ONE-DIMENSIONAL MULTI-BAY PERIODIC PANELS UNDER SUPERSONIC FLUID FLOW

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1. Introduction

Supersonic flutter of finite periodically supported panels subjected to lengthwise fluid flow has been investigated by Dowell [1] using classical methods. Recently, Mukherjee and Parthan [2] showed that using the wave-progagation approach, proposed in earlier works on free vibration of periodic beams presented by Mead [3] and Sengupta [4], the analysis of flutter of periodic panels on simple supports (both ends simply supported) can be carried out with much less computational effort than is necessary in other methods. The results of reference [2] are in excellent agreement with those presented by Dowell [1].

In section 6 of reference [2], the authors commented that, unlike the case of free vibration of periodic panels, supersonic flutter analysis of periodic panels with clamped end supports cannot be carried out from the dispersion relationships. Subsequent research conducted by the authors has shown that this is not true. The authors regret the error made inadvertently and withdraw this particular erroneous conlcusion of [2]. In fact, the method proposed in reference [2] *can also* predict the frequencies and critical flutter conditions for periodic panels with clamped ends *as easily as* for those with simply supported ends. The present communication is made to clarify this fact. For the purpose of completeness, some of the salient features of reference [2] are repeated here. Furthermore, the standing wave characteristics of finite periodic structures have been discussed here in greater detail.

2. Wave propagation in the periodic panel

In Figure 1 are represented two finite periodic panels, each of five identical bays, with both ends simply supported or both ends clamped, subjected to a supersonic fluid flow of speed U. The differential equation [1, 2] of a periodic unit of length l, with left support as the origin, is given by

$$(\mathrm{d}^4W/\mathrm{d}\xi^4) + \lambda(\mathrm{d}W/\mathrm{d}\xi) - \pi^4 Z \cdot W = 0 \tag{1}$$

where $\xi = x/l$, $\lambda = 2ql^3/\beta D$, $\pi^4 Z = \Omega^2 - \mathrm{i}\pi^2 g_t \Omega (\mathrm{i} = \sqrt{-1})$, $\Omega^2 = m\omega^2 l^4/D$, $g_t = (\rho U/\beta + c)/(m\omega_0)$ and $\omega_0 = (\pi/l)^2\sqrt{(D/m)}$. Here m is the mass per unit area, ρ is the fluid density, D is the usual flexural rigidity of the panel, q is the dynamic pressure, c is viscous damping and $\beta = \sqrt{(M^2-1)}$, M being the flow Mach number. The deflection at time t is given by $w(\xi, t) = W(\xi)\mathrm{e}^{-\mathrm{i}\omega t}$, where $W(\xi)$ is the amplitude.

The solution to the differential equation (1) for a periodic bay with transversely rigid supports ($W(\xi = 0) = W(\xi = 1) = 0$) is given by [1, 2]

$$W(\xi) = A[\mathrm{e}^{\gamma(1-\xi)} \sin\delta \sinh\varepsilon\xi - \mathrm{e}^{-\gamma(1-\xi)} \sinh\varepsilon \sin\delta\xi] + B[\mathrm{e}^{\gamma\xi} \sin\delta \sinh\varepsilon(1-\xi) - \mathrm{e}^{-\gamma\xi} \sinh\varepsilon \sin\delta(1-\xi)] \tag{2}$$

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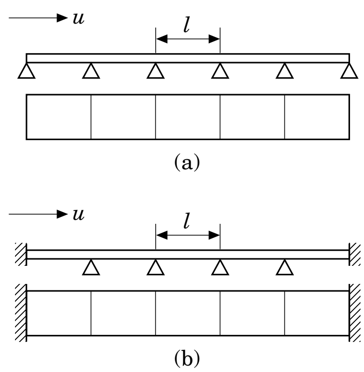

Figure 1. A multi-supported periodic panel under supersonic flow: (a) both ends simply supported; (b) both ends clamped.

where A and B are arbitrary coefficients, and the parameters γ, δ and ε are related to each other by the following equations:

$$\delta^2 = [\lambda/(4\gamma) + \gamma^2], \qquad \varepsilon^2 = [\lambda/(4\gamma) - \gamma^2], \qquad 64\gamma^6 + 16(\pi^4 Z)\gamma^2 - \lambda^2 = 0. \qquad \text{(3a–c)}$$

In reference [2], it is shown that using the boundary conditions from the Floquet principle,

$$W'(\xi = 1) = W'(\xi = 0)e^{\mu} \qquad \text{and} \qquad W''(\xi = 1) = W''(\xi = 0)e^{\mu}, \qquad \text{(4a, b)}$$

where μ is the complex propagation constant ($\mu = \mu_r + i\mu_i$), the following equation can be obtained for non-trivial values of A and B (here W' and W'' denote $dW/d\xi$ and $d^2W/d\xi^2$ respectively):

$$\det|[\alpha] - e^{\mu}[\beta]| = 0 \qquad (5)$$

Solving (5) for the eigenvalue e^{μ}, a three-dimensional graphical representation of the relationship between λ, Ω and μ_1 for undamped vibration (i.e., $g_t = 0$) is reproduced from

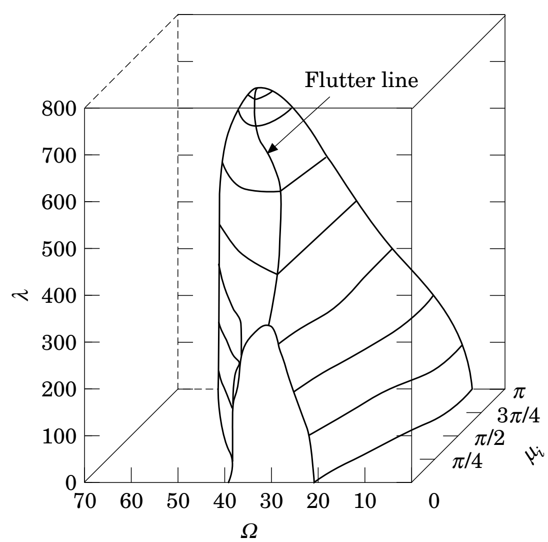

Figure 2. The variation of phase constants, pressure parameters and frequencies.

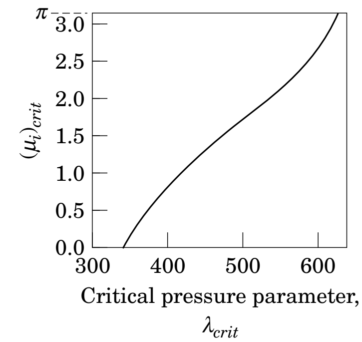

Figure 3. The relationship of the critical pressure parameters and critical phase constants.

reference [2] (see Figure 2). The "flutter line" is the locus of points characterized by the property $\partial\mu_i/\partial\Omega = 0$. These points therefore correspond to the critical conditions in which the *group velocity* $(\partial\Omega/\partial\mu_i)$ is infinite in a corresponding infinite panel of elements identical to those of the finite one. The variation of the corresponding critical phase constant and pressure parameter is presented in Figure 3.

The flutter line (see Figure 3) originates at $\lambda_{crit} = 343{\cdot}3$, $(\mu_i)_{crit} = 0$ and $\Omega_{crit} = 32{\cdot}4$, and terminates at $\lambda_{crit} = 636{\cdot}4$. $(\mu_i)_{crit} = \pi$ and $\Omega_{crit} = 52{\cdot}3$. The originating point and the terminal point correspond respectively, to the critical flutter conditions for an isolated periodic beam with both ends simply supported and both ends clamped. This is confirmed by the modal analysis of supersonic flutter of a single isolated bay.

3. PROPAGATION BANDS AND FINITE PERIODIC PANELS

The frequencies (Ω) for pressure parameters λ of vibration under supersonic flow can be obtained by the discritization process [2], similar to that prescribed in reference [4]. In reference [2], the propagation bands had been identified as the first and second bands. A close observation reveals further interesting phenomena. From $\lambda = 0$ to 292 Hz, the dispersion curve over the first propagation band exists as a continuous one from $\mu_i = 0$ to $\pm\pi$ (see Figure 4). Beyond $\lambda = 292$, however, the curve looks like that presented in Figure 5, i.e., there is a straight portion of this curve, parallel to the μ_i-axis, showing a sudden change of behaviour, and a subsidiary curved portion. Modes of frequencies corresponding to this straight portion (having equal frequencies) are all in weak coalescence [1].

At $\lambda = 343{\cdot}3$, the upper bounding frequency of the first band (for $\mu_i = 0$) just joins up with the lower bounding frequency of the second propagation band at the frequency $\Omega = 32{\cdot}4$, thus displaying the condition of $\partial\mu_i/\partial\Omega = 0$ (see Figure 6). *This is the origin of the flutter line*. With further increase of λ, this extremum point moves towards π and higher frequency values, finally just vanishing at $\lambda = 636{\cdot}4$ at $\Omega = 52{\cdot}3$ and $\mu_i = \pi$. Beyond $\lambda = 343{\cdot}3$, one can understand the existance of the first propagation band at a single frequency at which the attenuations μ_r are equal; i.e., the attenuation curves meet at the single frequencies at the specified phase constants for the finite periodic structure.

A type of standing wave is formed in finite periodic panels from the superposition of opposite going waves due to end reflections. In the propagation bands, for real Z, the eigenvalues e^{μ} always occur in complex conjugates, and so do the propagation constants $(\mu = \mu_r \pm i\mu_i)$. Observing that the real parts μ_r of the complex conjugate propagation constants are always positive in the propagation bands, one may easily conclude that a

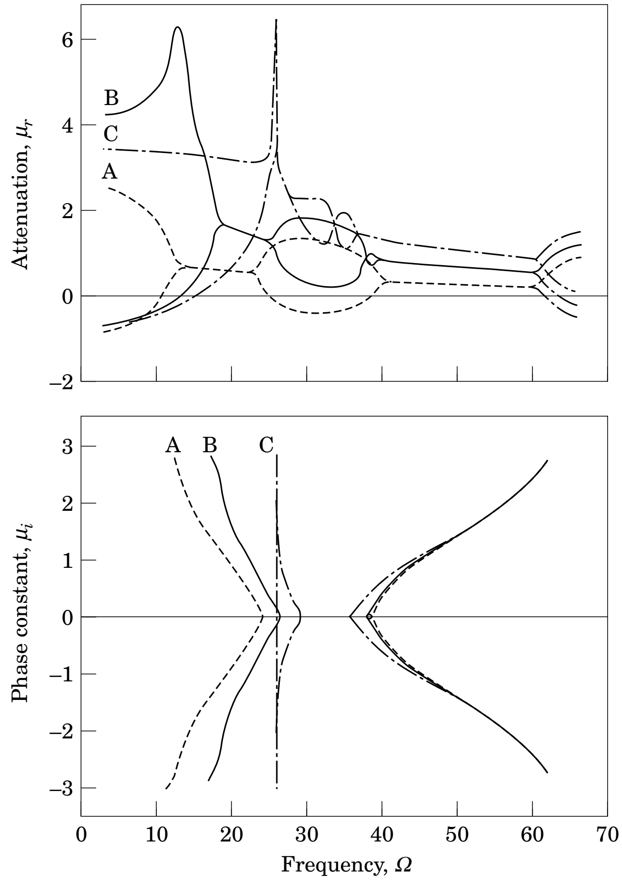

Figure 4. Attentuation and phase constants versus frequency for: A, $\lambda = 100$; B, $\lambda = 200$; C, $\lambda = 300$.

wave moving along the flow direction (with phase constant $-\mu_i$) increases in amplitude along its direction of propagation by the factor $\exp(\mu_r)$, while the wave moving opposite to the flow (with phase constant $+\mu_i$) attenuates along its direction of propagation, by the same factor.

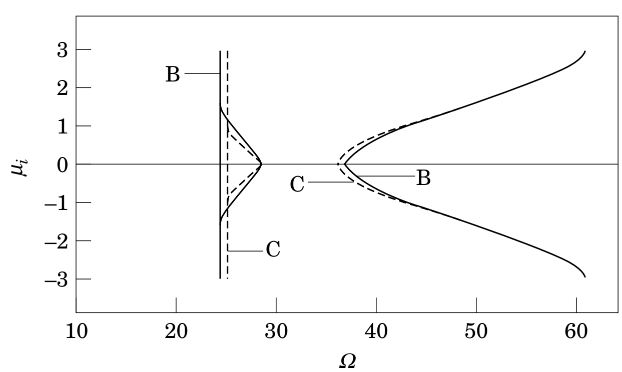

Figure 5. Phase constants versus frequency for: B, $\lambda = 292{\cdot}2$; C, $\lambda = 300$.

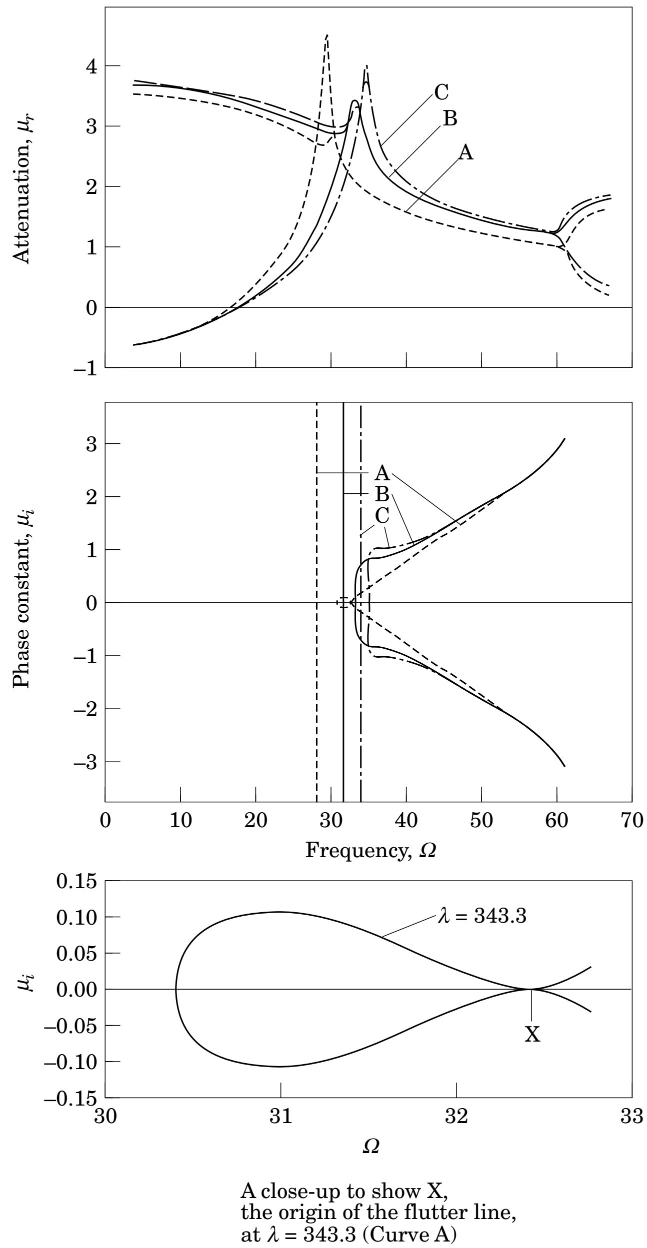

Figure 6. Attenuation and phase constants versus frequency for: A, $\lambda = 343{\cdot}3$; B; $\lambda = 399$; c; $\lambda = 420$.

At a support r of the finite structure of N spans ($r = 0$ for the first support over which flow occurs), the state vector v_r (slope or bending moment) can be expressed as a superposition of opposite going waves, one increasing in amplitude and the other decreasing in amplitude along their respective direction of propagation:

$$v_r = v_{along\,flow} + v_{against\,flow}$$

$$v_r = A^+ \exp[(\mu_r - \mathrm{i}\mu_i)r] + A^- \exp[(\mu_r + \mathrm{i}\mu_i)r + \mathrm{i}\zeta]$$

where ζ is the phase factor incorporated for reflection ($\zeta = 0$ or π). For a bounded edge, $A^+ = A^-$. Thus the net superposed state vector is

$$v_r = A^+ \exp(\mu_r r)[\exp(-\mathrm{i}\mu_i r) \pm \exp(\mathrm{i}\mu_i r)].$$

Thus either

$$v_r = K \exp(\mu_r r)\sin(\mu_i r) \qquad (6a)$$

or

$$v_r = K \exp(\mu_r r)\cos(\mu_i r), \qquad (6b)$$

where K is some normalized constant. If v_r represents support slope, then equations (6a) and (6b) apply, respectively, to the cases with extreme edges both clamped and both simply supported. If v_r represents support bending moment, the correlation is just reversed.

For an N-bay structure, the permissible phase constants are given by $\mu_i = j\pi/N$, where the values of j are given by the following rules.

For both ends simply supported [2]:

$j = 1, 2, \ldots, N$ for the first (and subsidiary) propagation band,

$j = 0, 1, 2, \ldots, N - 1$ for the second propagation band;

and for both ends clamped

$j = 0, 1, 2, \ldots, N - 1$ for the first (and subsidiary) propagation band,

$j = 1, 2, \ldots, N$. for the second propagation band.

The frequency–pressure relationship of a single bay and that of a five-bay structure (with simply supported ends or clamped ends) are presented in Figures 7–9. From the discritization rule, it indeed follows that the five-bay case also includes the one-bay case [2]. This *is* equally true for both ends clamped and both ends simply supported [4]. Figure 7, or Figure 8, is included for any N-bay periodic panel with simply supported or clamped ends respectively. In Figure 9, the results presented for the five-bay case (common to both simply supported and clamped ends) correspond only to the cases for $\mu_i = \pi/5$, $2\pi/5$, $3\pi/5$ and $4\pi5$, while those for $\mu_i = 0$ or π are not shown, although it is understood that the one-bay case (Figures 7 or 8) applies also to the five-bay case (of either simply supported or clamped ends). This procedure of representation is adopted here following the earlier literature [1] (see Table 1).

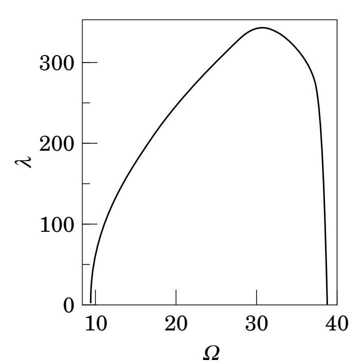

Figure 7. The λ-Ω relationship for the one-span case (both ends simply supported).

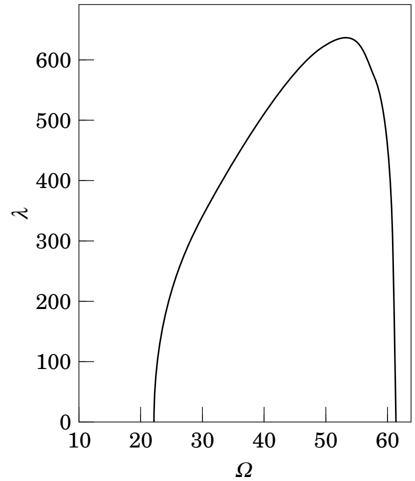

Figure 8. The λ-Ω relationship for the one-span case (both ends clamped).

In general, for free vibration ($\lambda = 0$) of a periodic panel of N spans with clamped ends, except for the highest (higher bounding) frequency in a propagation band, all the lower $N - 1$ frequencies are identical to the natural frequencies of a periodic panel with simply supported ends, except for the lowest (lower bounding) one. This fact can be confirmed by a modal/exact analysis. This implies that the work of reference [2] actually has a wider range of application than has been originally reflected.

For predicting the critical values of the pressure parameters, the curve of Figure 3 is to be discretized at the critical phase constants $(\mu_i)_{crit} = (j\pi/N)$, $j = 0, 1, 2, \ldots, N - 1$ for simply supported ends [2] and $j = 1, 2, \ldots, N$ for clamped ends.

4. CONCLUSIONS

The present paper is meant to discuss some of the significant observations made on flutter in the earlier literature [2] and the doctoral dissertation of the first author [5]. The behaviour of the propagation bands, the characteristics of propagating waves and the

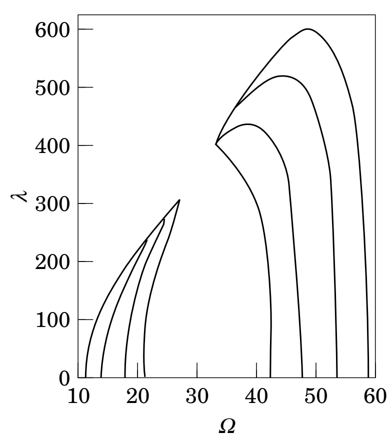

Figure 9. The λ-Ω relationship for the five-span case (both ends simply supported/clamped). The one-span case is also present (Figure 7 or Figure 8).

TABLE 1

*Critical flutter parameters for one-dimensional multi-bay panels: for simply-supported edges, ignore rows marked by +; for clamped edges, ignor rows marked by **

Number of spans, N		Λ_{crit} Dowell [1]	Λ_{crit} Present	Ω_{crit}	$(\mu_i)_{crit}$
1	*	343·0	343·3	32·4	0
	+	—	636·4	52·3	π
2	*	343·0	343·3	32·4	0
		485·0	481·0	42·0	$\pi/2$
	+	—	636·4	52·3	π
5	*	343·0	343·0	32·4	0
		395·0	395·0	33·1	$\pi/5$
		450·0	447·0	39·0	$2\pi/5$
		525·0	520·0	44·0	$3\pi/5$
		605·0	600·0	49·0	$4\pi/5$
	+	—	636·4	52.3	π

occurrence of the standing waves in finite periodic structures with clamped or simply supported ends are discussed briefly.

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