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VIBRATION ANALYSIS OF ROTATING MACHINERY USING A BISPECTRUM

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1. INTRODUCTION

The amplitude and phase of the base wave frequency component in a periodic signal can be accurately determined through the use of bispectral analysis by adding the base wave component into the original periodic signal. An example application of this method is given for the dynamic equilibrium analysis of a sewing machine.

Consider a periodic signal written as

$$x(t) = \sum_{i=1}^{n} A_i \cos(2\pi i f_0 t + \Phi_i). \tag{1}$$

Its bispectrum is determined from the following equation:

$$B_x(f_1, f_2) = \sum_{k=1}^{\infty} \sum_{l=1}^{\infty} \frac{A_l A_k A_{l+k}}{8} \mathrm{e}^{\mathrm{j}(\Phi_{l+k} - \Phi_l - \Phi_k)} \delta(f_1 - kf_0)\delta(f_2 - lf_0). \tag{2}$$

In the case in which $f_1 = kf_0$ and $f_2 = lf_0$ ($k = 1, 2, \ldots, n$; $l = 1, 2, \ldots, n$), the following equations are obtained:

$$b_x(f_1, f_2) = |B_x(f_1, f_2)| = \frac{A_l A_k A_{l+k}}{8}, \qquad \angle b_x(f_1, f_2) = \angle B_x(f_1, f_2) = \Phi_{l+k} - \Phi_l - \Phi_k. \tag{3, 4}$$

When we consider a signal of the form $z(t) = x(t) + n(t)$, where $n(t)$ is Gaussian noise, then $B_z(f_1, f_2) = B_x(f_1, f_2)$ [1]. In addition, if the Φ_i's are random variables, and Φ_i and Φ_{i+1} are not independent, the bispectrum can give the relationship of phases between various Fourier components [2].

Bispectral analysis can be adopted to separate the Fourier components of the periodic signal from the Gaussian noise to yield information about the amplitude ratio and phase difference. This information is often sufficient for problems such as brain wave analysis, sound source analysis, etc. However, if one would like to apply the bispectrum analysis to the field balancing problem, it is necessary to obtain the amplitude and phase of the base frequency component itself. The amplitude and phase of other harmonic waves can be brought about successively according to equations (3) and (4).

The method for calculating the amplitude and phase of the base wave is either by synchronizing the studied signal and some other input signal, or from the use of the

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cross-correlation function after the base frequency has been determined in some manner. However, both of these methods make the amount of calculation to be done increasingly large. Because of this, the power spectrum and the cross-correlation function are frequently used rather than bispectral analysis to separate the various harmonic components of the periodic signal from the noise. Therefore, it can be said that the lack of information about the amplitude and phase of the base wave has limited the application of the bispectrum method.

In this note a method is described for determining the amplitude and phase of the base frequency, by adding two base wave cosines with different amplitudes and by bispectral analysis. Furthermore, the amplitudes and phases of other various harmonic waves of $x(t)$ can be obtained. Because the added base wave cosine only changes the real part of the Fourier base wave component, the imaginary part and the other harmonic components are not influenced.

When calculating the bispectrum of the signal to which the base wave cosine has been added, the carrying out of the Fast Fourier Transform (FFT) is not needed again, and the amount of calculation is almost not increased. This method of determining the amplitude and phase of the base wave will extend the application of the bispectral method.

2. AMPLITUDE AND PHASE DETERMINATION OF THE BASE WAVE

A signal-flow diagram for determining the amplitude and phase of the base wave theoretically is shown in Figure 1. In the figure,

$$x + a = A_1 \cos(2\pi f_0 t + \Phi_i) + A \cos(2\pi f_0 t) + \sum_{i=2}^{n} A_i \cos(2\pi f_0 t + \Phi_i)$$

$$= A_1' \cos(2\pi f_0 t + \Phi_1') + \sum_{i=1}^{n} A_i \cos(2\pi f_0 t + \Phi_i) \qquad (5)$$

and

$$x + b = A_1 \cos(2\pi f_0 t + \Phi_i) + B_0 \cos(2\pi f_0 t) + \sum_{i=2}^{n} A_i \cos(2\pi f_0 t + \Phi_i). \qquad (6)$$

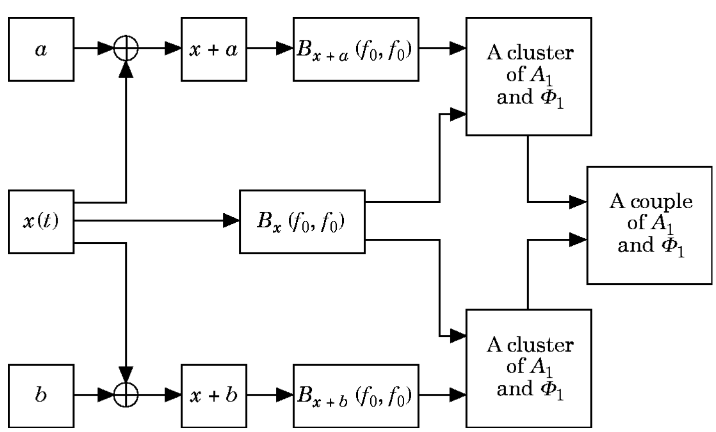

Figure 1. The principle for determining the amplitude and phase of the base wave.

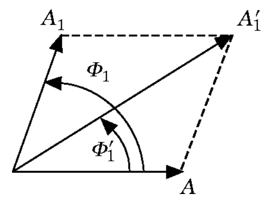

Figure 2. The amplitude and phase of the base wave after having added $A\cos(2\pi f_0 t + \Phi_1)$.

Assuming that b_x, $\angle b_x$, b_{x+a}, $\angle b_{x+a}$, b_{x+b} and $\angle b_{x+b}$ represent the amplitudes and phases of the bispecrums B_x, B_{x+a} and B_{x+b} of x, $x+a$ and $x+b$, respectively, we have

$$b_x(f_0, f_0) = \frac{A_1^2 A^2}{8}, \qquad \angle b_x(f_0, f_0) = \Phi_2 - 2\Phi_1, \tag{7, 8}$$

$$b_{x+a}(f_0, f_0) = \frac{A_1'^2 A^2}{8}, \qquad \angle b_{x+a}(f_0, f_0) = \Phi_2 - 2\Phi_1'. \tag{9, 10}$$

As shown in Figure 2, the relation of A_1' to A and A_1 is

$$A_1'^2 = A_1^2 + A^2 + 2A_1 A\cos\Phi_1, \tag{11}$$

while Φ_1 and Φ_1' possess the relation

$$\tan\Phi_1 = \frac{A_1\sin\Phi_1}{A_1\cos\Phi_1 + A}. \tag{12}$$

Reorganizing equations (7)–(12), the following equations are obtained:

$$b_{x+a}(f_0, f_0) = b_x(f_0, f_0)\left[1 + \left(\frac{A}{A_1}\right)^2 + \frac{2A}{A_1}\cos\Phi_1\right], \tag{13}$$

$$\angle b_{x+a}(f_0, f_0) = \angle b_x(f_0, f_0) + 2\Phi_1 - 2\tan^{-1}\frac{\sin\Phi_1}{\cos\Phi_1 + A/A_1}. \tag{14}$$

It is difficult to formulate the expressions containing A_1 and Φ_1 beyond those given in equations (13) and (14). Allowing $\tau_1 = A/A_1$ and rearranging the above equations yields

$$\tau_1 = -\cos\Phi_1 \pm \left[\frac{b_{x+a}(f_0, f_0)}{b_x(f_0, f_0)} - \sin^2\Phi_1\right]^{-1/2}, \tag{15}$$

$$\tau_1 = \sin\Phi_1\left[\tan\frac{\angle b_x(f_0, f_0) + 2\Phi_1 - \angle b_{x+a}(f_0, f_0)}{2}\right]^{-1} - \cos\Phi_1. \tag{16}$$

Equations (15) and (16) separately represent a curve of τ_1 and Φ_1, in which τ_1 is greater than 0. A_1 and Φ_1 are determined by the intersecting points of the two curves, as shown in Figure 3.

It is easy to understand that there are more than one intersecting points. Two conditions must be supplied for calculating the two unknowns—the amplitude and phase of the base wave. This is the reason why the base wave cosine $B \cos(2\pi f_0 t)$ must also be added to $x(t)$.

Therefore, substituting b_{x+b} and $\angle b_{x+a}$ and $\angle b_{x+a}$, respectively, in equations (15) and (16), a cluster of intersecting points can be obtained. That is, putting $\tau_2 = B/A_1$ we obtain

$$\tau_2 = -\cos \Phi_1 \pm \left[\frac{b_{x+b}(f_0, f_0)}{b_x(f_0, f_0)} - \sin^2 \Phi_1 \right]^{-1/2}, \tag{17}$$

$$\tau_2 = \sin \Phi_1 \left[\tan \frac{\angle b_x(f_0, f_0) + 2\Phi_1 - \angle b_{x+b}(f_0, f_0)}{2} \right]^{-1} - \cos \Phi_1. \tag{18}$$

Among the intersecting points of curves (15) and (16) and other intersecting points of curves (17) and (18), there must be two points which have the same Φ_1. Φ_1 is the phase of the base wave which we wish to obtain. The amplitude of the base wave can be computed from τ_1 or τ_2.

3. AMPLITUDE AND PHASE DETERMINATION OF THE BASE WAVE

According to Figure 1, the amount of calculation needed to find B_{x+a} and B_{x+b} through bispectrum analysis is rather large. The more practical method for determining the amplitude and phase of the base wave is introduced as follows.

In Figure 1, comparing the Fourier components of $x + a$ with the Fourier components of x, the only difference between the two is the real part of the base wave component. According to this, B_{x+a} and B_{x+b} can be calculated from B_x directly and, in addition, the FFT is not needed for $x + a$ and $x + b$. The method for calculating the bispectrum $B_x(f_1, f_2)$ of an actually measured signal $x(t)$ is well known [3].

When $x(t)$ is sampled with a sampling period of Δt, the total number of data points is $N = 2^n$ and K groups of data are obtained. After pre-processing, the FFT for each group of data is calculated. Then,

$$^kB_x(f_0, f_0) = \frac{1}{T} {^kX_{f0}^2} {^kX_{2f0}^*}, \tag{19}$$

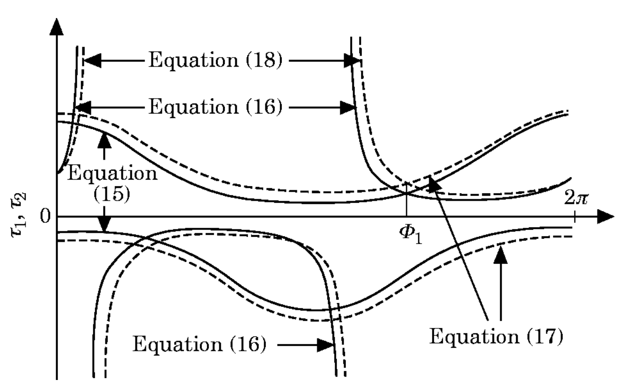

Figure 3. Determining A_1 and Φ_1 from two clusters of intersecting points.

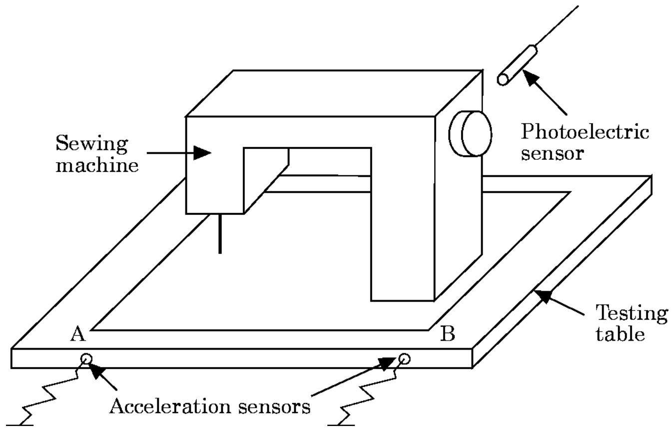

Figure 4. A schematic diagram of the testing table for field balancing.

where $T = N\Delta t$ and ${}^kX_{f0}$ is the FFT component of $x(t)$ when the frequency is f_0.

Therefore,

$$
\begin{aligned}
{}^kB_{x+a}(f_0, f_0) &= \frac{1}{T}\,{}^kX^*_{2f0}({}^kX_{f0} + A)^2 \\
&= \frac{1}{T}\,{}^kX^2_{f0}\,{}^kX^*_{2f0} + \frac{A}{T}\,{}^kX^*_{2f0}(A + 2({}^kX_{f0})),
\end{aligned}
\tag{20}
$$

$$
\begin{aligned}
B_{x+a}(f_0, f_0) &= \frac{1}{K}\sum_{k=1}^{K} {}^kB_{x+a}(f_0, f_0) \\
&= B_x(f_0, f_0) + \frac{A}{TK}\left[A\sum_{k=1}^{K} {}^kX^*_{2f0} + 2\sum_{k=1}^{K} {}^kX^k_{f0}X^*_{2f0}\right].
\end{aligned}
\tag{21}
$$

In calculating $B_x(f_0, f_0)$, according to equation (19), the quantity ${}^kX_{f0}{}^kX^*_{2f0}$ must be computed first; while, according to equation (21), the additional calculations for getting $B_{x+a}(f_0, f_0)$ are only the average calculation to ${}^kX^*_{2f0}$ and the average calculation to ${}^kX_{f0}{}^kX^*_{2f0}$.

The amount of calculation is very small. When a windowing process is applied, the difference only amounts to multiplying the average value of ${}^kX^*_{2f0}$ and ${}^kX^*_{f0}{}^kX^*_{2f0}$ by some coefficient. As for calculating A_1 and Φ_1 from equations (15), (16), (17) and (18), the rough range of Φ_1 can be carried out by a smaller step size, depending on the accuracy that is desired.

4. BISPECTRUM APPLICATION TO THE FIELD BALANCING OF A SEWING MACHINE

In the case of rotary machinery, such as a sewing machine, the main motor has rotary motion even if its revolving axle parts have already been balanced themselves. This is due to the action of the other moving parts connected with the axle. Some periodic loads will be produced that possess the same frequency as the rotary axle. These loads are equivalent to the dynamic unbalance of a rotary axle. The dynamic balancing method for the entire rotary machinery is the field balancing method [4]. In reference [4], the author fixed the head of the sewing machine on a self-made testing table and let the axle revolve (Figure 4). Two acceleration sensors were mounted on to the testing table and were used to pick up the signal of the vibration at points A and B.

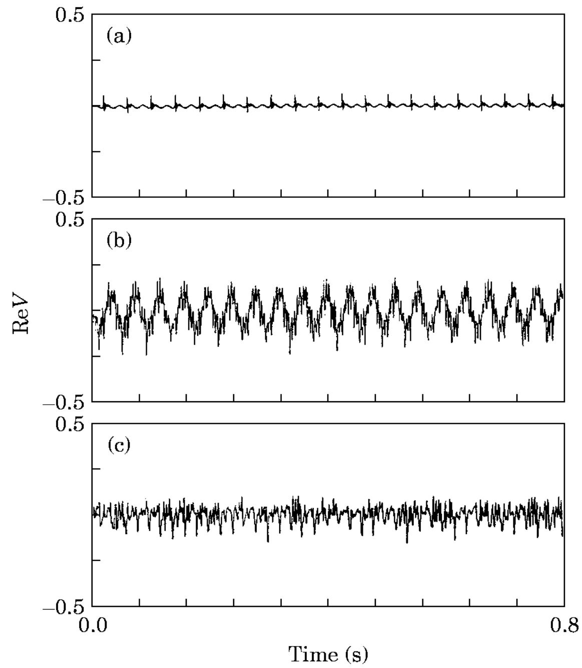

Figure 5. (a) A standard signal. (b) The vibrating acceleration at point B before balancing. (c) The vibrating acceleration at point B after balancing.

Because the rotary speed of the lower axle is twice as large as that of the upper axle in the sewing machine, the vibrational signals obtained mainly contain the base wave and the second harmonic component. The standard signal can be obtained from the belt pulley which is connected with the upper axle. As long as the amplitudes and phases relative to the standard signal of the base wave and the second harmonic of vibrating acceleration in points A and B can be measured, field balancing for the sewing machine can be undertaken.

Typically, the power spectrum and cross-spectrum analysis are used to extract the amplitudes and phases of the base wave and the second harmonic from noise. However, in this case, the amount of computation would be large. By using the method introduced in this paper, the amplitude and phase of the base wave can be calculated by using the bispectrum. Because the FFT is not needed for the standard signal, as long as the FFT for the signal of vibrating acceleration is calculated, then $B_x(f_0, f_0)$, $B_{x+a}(f_0, f_0)$ and $B_{x+b}(f_0, f_0)$ can be obtained according to equations (19), (20) and (21).

The amplitudes and phases of the base wave and the second harmonic obtained from this method are entirely consistent with that from the power spectrum and cross-spectrum analysis. The waveform of the standard input signal is shown in Figure 5(a). The vibrating acceleration measured at point B before balancing is shown in Figure 5(b). The amplitude and phases of the base wave and the second harmonic, which are obtained from the bispectrum, are shown in Table 1. After balancing, the waveform of the vibrating acceleration is shown in Figure 5(c).

5. CONCLUSIONS

The method for determining the base wave amplitude and phase of a periodic signal from the bispectrum of the noise-containing periodic signal, which is introduced in this note, does not increase the amount of calculation.

TABLE 1

The amplitudes and phases of the base wave and the second harmonic obtained from the bispectrum

	Before balancing		After balancing	
	Point A	Point B	Point A	Point B
Base wave amplitude (m/s^2)	0·00998	0·0966	0·00176	0·00849
Base wave phase (degrees)	15·2	−155	−26·2	−72·5
Second wave amplitude (m/s^2)	0·0012	0·0031	0·0009	0·0023
Second wave phase (degrees)	86	88·1	89	92

The relative ratios of amplitudes and phases of various harmonic components of a periodic signal involving the bispectrum can be turned into absolute quantities. The application of the bispectrum is extended.

The correctness of the method is supported by the results of the computer simulation and the field balancing of a sewing machine.

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