Journal of Sound and Vibration (1997) **200**(4), 540–542

COMMENTS ON "NATURAL FREQUENCIES OF RECTANGULAR PLATES USING A SET OF STATIC BEAM FUNCTIONS IN RAYLEIGH–RITZ METHOD"

P. A. A. LAURA

Institute of Applied Mechanics (CONICET) and Department of Engineering, Universidad Nacional del Sur, 8000—Bahia Blanca, Argentina

(*Received* 3 *April* 1996)

The author is to be congratulated for developing co-ordinate functions which are combinations of sinusoidal terms and polynomials. The coefficients of the polynomial expressions are determined by the boundary conditions of the plate under analysis. Excellent accuracy is achieved [1].

It is quite surprising to the writer that the author does not refer at all to an alternative approach developed extensively by the writer and coworkers, which consists of generating approximations to the displacement function of the plate using polynomials which satisfy, at least, the essential boundary conditions.

Admittedly, the analysis has been restricted in many instances to the lower modes of vibrations, and, in a great majority of the cases, to the fundamental mode of vibration [2–18]. Among the situations treated, one may mention edges elastically restrained against rotation and in-plane stresses [3, 4], concentrated and elastically mounted masses [5, 6], non-uniform thickness [7], orthotropic materials [8], etc.

The approach has been used in the case of forced vibration problems [9, 10] and extended to circular plates [11, 12] and plates of complicated boundary shape using the conformal mapping method [13].

Furthermore, polynomial approximations have been employed in the case of plates with inner supports [14] and the technique has also been used in the case of portal frames [15], arches [16] and rings [17].

As a matter of fact, it was shown in reference [2] that using a simple one-term polynomial approximation one obtains $\lambda_1 = 9{\cdot}00$ for the fundamental eigenvalue of a clamped square plate while the author, using $p, q = 2, 2$ (Table 1 of reference [1]) obtains $\lambda_1 = 9{\cdot}012$, the "exact" result being $\lambda_1 = 8{\cdot}996$ [1]. The methodology has also been applied in the case of vibrating plates with openings having free edges [18].

ACKNOWLEDGMENTS

Research on dynamics of structural elements is sponsored by CONICET (PIA 1996–1997) and by the Secretaría General de Ciencia y Technología of Universidad Nacional del Sur (Project Director Professor R. E. Rossi).

REFERENCES

1. D. ZHOU 1996 *Journal of Sound and Vibration* **189**, 81–88. Natural frequencies of rectangular plates using a set of static beam functions in Rayleigh–Ritz method.
2. P. A. A. LAURA and B. F. SAFFELL, JR 1967 *Journal of the Acoustical Society of America* **41**, 836–838. Study of small amplitude vibrations of clamped rectangular plates using polynomial approximations.

0022–460X/97/090540 + 02 $25.00/0/sv960685

3. P. A. A. LAURA and E. ROMANELLI 1974 *Journal of Sound and Vibration* **37**, 367–377. Vibrations of rectangular plates elastically restrained against rotation and subjected to a bi-axial state of stress.
4. P. A. A. LAURA, L. E. LUISONI and C. P. FILIPICH 1977 *Journal of Sound and Vibration* **155**, 327–333. A note on the determination of the fundamental frequency of vibration of thin, rectangular plates with edges possessing defferent rotational, flexibility coefficients.
5. P. A. A. LAURA, E. A. SUSEMIHL, J. L. POMBO, L. E. LUISONI and R. GELOS 1977 *Applied Acoustics* **10**, 121–145. On the dynamic behavior of structural elements carrying elastically mounted concentrated masses.
6. P. A. A. LAURA, C. P. FILIPICH and V. H. CORTINEZ 1987 *Journal of Sound and Vibration* **117**, 459–465. Vibrations of beams and plates carrying concentrated masses.
7. P. A. A. LAURA, R. H. GUTIERREZ and R. E. ROSSI 1995 *Journal of the Acoustical Society of America* **98**, 2377–2380. Transverse vibrations of rectangular plates of bilinearly varying thickness.
8. P. A. A. LAURA and L. E. LUISONI 1978 *Computers and Structures* **9**, 527–532. Vibrations of orthotropic rectangular plates with edges possessing different rotational flexibility coefficients and subjected to in-plane forces.
9. P. A. A. LAURA and R. DURAN 1975 *Journal of Sound and Vibration* **42**, 129–135. A note on forced vibrations of a clamped rectangular plate.
10. E. A. SUSEMIHL and P. A. A. LAURA 1977 *Journal of Ship Research* **21**, 24–29. Forced vibrations of thin, elastic, rectangular plates with edges elastically restrained against rotation.
11. P. A. A. LAURA, J. C. PALOTO and R. D. SANTOS 1975 *Journal of Sound and Vibration* **41**, 170–180. A note on the vibration and stability of a circular plate elastically restrained against rotation.
12. P. A. A. LAURA, H. A. LARRONDO, D. R. AVALOS, D. V. BAMBILL, R. CARNICER and H. C. SANZI 1995 *Applied Acoustics* **44**, 125–132. Dynamic stiffening of circular plates and determination of their buckling, in-plane pressure.
13. P. A. A. LAURA and R. H. GUTIERREZ 1976 *Journal of Sound and Vibration* **48**, 327–332. Fundamental frequency of vibration of clamped plates of arbitrary shape subjected to a hydrostatic state of in-plane stress.
14. H. C. SANZI, P. A. A. LAURA and V. H. CORTINEZ 1988 *Journal of Sound and Vibration* **122**, 392–395. Fundamental frequency of vibration of a circular plate with two concentric, circular supports.
15. P. A. A. LAURA, B. H. VALERGA and C. P. FILIPICH 1987 *Journal of Sound and Vibration* **117**, 447–458. In-plane vibrations of frames carrying concentrated masses.
16. P. A. A. LAURA, C. P. FILIPICH and V. H. CORTINEZ 1987 *Journal of Sound and Vibration* **115**, 437–446. In-plane vibrations of an elastically cantilevered arc with a tip mass.
17. P. A. A. LAURA, C. P. FILIPICH, R. E. ROSSI and J. A. REYES 1988 *Applied Acoustics* **25**, 225–234. Vibrations of rings of variable cross section.
18. P. A. A. LAURA, R. H. GUTIERREZ, L. ERCOLI, J. C. UTJES and R. CARNICER 1987 *Ocean Engineering* **14**, 285–293. Free vibrations of rectangular plates elastically restrained against rotation with circular or square free openings.

AUTHOR'S REPLY

D. ZHOU

School of Mechanical Engineering, Nanjing University of Science and Technology, Nanjing 210094, People's Republic of China

(*Received* 20 *May* 1996)

I would like to thank Professor Laura for his calling attention to additional literature [1] pertinent to the title problem. I am sorry that I did not pay much attention to Professor Laura's papers, partly owing to the lack of opportunities to obtain the total literature in the field when preparing my paper [2]. Besides, readers may find that in most cases, Laura's papers were mainly aimed at the calculation of the fundamental mode of vibration of plates, beams, etc., while my paper focuses on higher modes in addition to the fundamental modes of vibration of rectangular plates. My method has been extended to vibration of elastically restrained rectangular plates with eight different rotational and translational flexural coefficients [3] and line supported rectangular plates in one and two ways [4]. Good accuracy was also achieved.

In reply to his last comment, the point to note is that $\lambda_1 = 9{\cdot}012$ is actually obtained by using $p, q = 1, 1$ (Table 1 of reference [2]) in my paper. Apparently the structures are symmetric about the central axes for the clamped rectangular plates and the basis functions $Y_i(\zeta)$ in my paper are symmetric for $i = 1, 3, 5, \ldots$ and antisymmetric for $i = 2, 4, 6, \ldots$. In such cases, the accuracy of taking $p, q = 1, 1$ is the same as that of $p, q = 2, 2$ for λ_1; the antisymmetric function $Y_2(\zeta)$ does not affect the symmetric modes of vibration. If one wants, one may take advantage of the symmetry about the central axes of the plates by taking i and j to be $1, 3, 5, \ldots$ for full-symmetric, i, j to be $2, 4, 6, \ldots$ for full-antisymmetric, i to be $1, 3, 5, \ldots$ and j to be $2, 4, 6, \ldots$ or i to be $2, 4, 6, \ldots$ and j to be $1, 3, 5, \ldots$ for symmetric–antisymmetric or antisymmetric–symmetric modes of vibration in the eigenvalue equation of clamped rectangular plates; furthermore, the computational cost will be reduced.

REFERENCES

1. P. A. A. Laura 1996 *Journal of Sound and Vibration* **200**, 540–541. Comments on "Natural frequencies of rectangular plates using a set of static beam functions in Rayleigh–Ritz method".
2. D. Zhou 1996 *Journal of Sound and Vibration* **189**, 81–88. Natural frequencies of rectangular plates using a set of static beam functions in Rayleigh–Ritz method.
3. D. Zhou 1995 *Computers and Structures* **57**, 731–735. Natural frequencies of elastically restrained rectangular plates using a set of static beam functions in the Rayleigh–Ritz method.
4. D. Zhou 1994 *International Journal of Solids and Structures* **31**, 347–358. Eigenfrequencies of line supported rectangular plates.