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AN ELECTROMECHANICAL VIBRATION ABSORBER

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1. INTRODUCTION

A dynamic vibration absorber is a passive device which is used to reduce excessive vibrations in a mechanical system. The absorber typically consists of an inertial device which changes the dynamic response of the original vibrating system over a certain frequency range. The absorbing device may be a linear mass–spring oscillator, a pendular or rotary oscillator, a fluid or granular system, or a distributed elastic structure. The concept of a vibration absorber was first developed in 1902 by Frahm, who designed a fluid tank system for the reduction of rolling motion in the German ships *Bremen* and *Europa* [1]. The theory and design of vibration absorbers for many technical applications is discussed in reference [2].

Although a vibration absorber can produce a reduction in the vibration amplitude of the original mechanical system, this improvement is obtained at the expense of large amplitude mechanical vibrations in the absorber. This large amplitude oscillation is one of the major problems in the design of practical absorber systems. Space and weight limitations or structural failures often limit the performance of the vibration absorber.

In this letter, we present a simple experimental demonstration of an electromechanical vibration absorber. The mechanical absorber system is replaced by an electromechanical transducer and a resonant electrical circuit. It is shown that by tuning the electrical circuit properly, the vibration amplitude of the original vibrating system to which the absorber is attached may be reduced significantly. This reduction is accompanied by large electrical oscillations in the resonant circuit, instead of the undesirable large amplitude mechanical oscillations in the conventional absorber. A mathematical analysis of the electromechanical vibration absorber is given, and some initial experimental results are presented.

A vibration absorber in which piezoelectric materials are used in conjunction with a resonant electrical circuit is discussed in references [4] and [5]. These piezoelectric systems are applicable to the reduction of bending vibrations, while the approach given here, in which an electromechanical voice coil transducer is used instead of a piezoelectric material, allows a greater variation of absorber parameter values, and may be applied to a much larger class of machine and structural vibration problems. A recent magnetorestrictive device for vibration reduction is described in reference [6].

2. ANALYSIS

An initial model for the electromechanical vibration absorber is shown in Figure 1. The primary vibrating system is modelled as a single-degree-of-freedom oscillator, with mass m, spring stiffness k and damping coefficient b. The absorber consists of a coil, a fixed permanent magnet and an RLC electrical circuit connected in series with the coil. The coil is fixed to the mass m and moves in an annular gap in the permanent magnet. The purpose

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of the absorber is to reduce the vibration amplitude of the mass m when the mass is driven by the external force $f(t)$.

The dynamic equations for the displacement $x(t)$ of the mass and the current $i(t)$ in the coil are

$$m\frac{\mathrm{d}^2x}{\mathrm{d}t^2}+b\frac{\mathrm{d}x}{\mathrm{d}t}+kx+Ti=f, \qquad L\frac{\mathrm{d}^2i}{\mathrm{d}t^2}+R\frac{\mathrm{d}i}{\mathrm{d}t}+\frac{1}{C}i-T\frac{\mathrm{d}^2x}{\mathrm{d}t^2}=0. \tag{1, 2}$$

The quantity T is the transducer constant which relates the current in the coil to the magnetic force on the coil. In the simplest transducer model [3], the transducer constant is given by

$$T=2\pi nrB, \tag{3}$$

where n is the number of turns in the coil, r is the radius of the coil, and B is the uniform radial magnetic field strength in the annular gap. As discussed in reference [3], the transducer constant T also relates the electrical potential e across the terminals of the coil to the velocity of the coil with respect to the permanent magnet.

To analyze the coupled equations for the displacement $x(t)$ and the current $i(t)$, we assume that the external force $f(t)$ is sinusoidal, and write

$$f(t)=F(\omega)\,\mathrm{e}^{\mathrm{j}\omega t}, \qquad x(t)=X(\omega)\,\mathrm{e}^{\mathrm{j}\omega t}, \qquad i(t)=I(\omega)\,\mathrm{e}^{\mathrm{j}\omega t}, \tag{4–6}$$

where $X(\omega)$ and $I(\omega)$ are the steady state complex amplitudes of the displacement and current, respectively. After substitution of equations (4), (5) and (6) into equations (1) and (2), the steady state amplitudes $X(\omega)$ and $I(\omega)$ can be obtained in the form

$$X=\frac{(F/k)(1-r_e^2\omega^2/\omega_1^2+\mathrm{j}2\zeta_e r_e\omega/\omega_1)}{(1-\omega^2/\omega_1^2+\mathrm{j}2\zeta_1\omega/\omega_1)(1-r_e^2\omega^2/\omega_1^2+\mathrm{j}2\zeta_e r_e\omega/\omega_1)-r_T^2\omega^2/\omega_1^2}, \tag{7}$$

$$I=\frac{(F/T)r_T^2\omega^2/\omega_1^2}{(1-\omega^2/\omega_1^2+\mathrm{j}2\zeta_1\omega/\omega_1)(1-r_e^2\omega^2/\omega_1^2+\mathrm{j}2\zeta_e r_e\omega/\omega_1)-r_T^2\omega^2/\omega_1^2}, \tag{8}$$

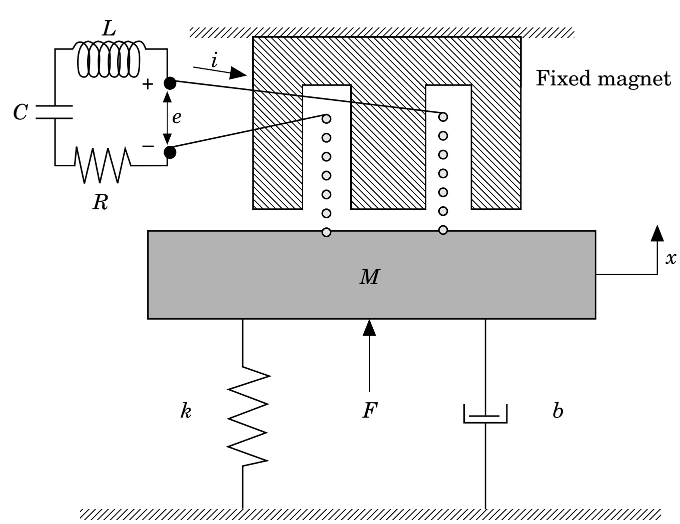

Figure 1. A model of the electromechanical vibration absorber.

where

$$\omega_1^2 = k/m, \qquad \omega_e^2 = 1/LC, \qquad \omega_T^2 = k/T^2C, \tag{9–11}$$

$$\zeta_1 = \omega_1 b/2k, \qquad \zeta_e = \omega_e RC/2, \qquad r_e = \omega_1/\omega_e, \qquad r_T = \omega_1/\omega_T. \tag{12–15}$$

By choosing the parameters of the absorber appropriately, it is possible to make the vibration amplitude of the mass m smaller, in some frequency bands, than it would be if the absorber were not present. In Figure 2 is shown the dimensionless amplitude $|X/(F/k)|$ as a function of the dimensionless forcing frequency ω/ω_1 for the numerical values $r_e = 1$, $r_T = 1$, $\zeta_1 = 0{\cdot}1$ and $\zeta_e = 0{\cdot}1$. The dashed line shows the amplitude $|X/(F/k)|$ when the absorber is not present (or, equivalently, when the transducer constant T is zero), and the solid line shows the amplitude $|X/(F/k)|$ with the absorber. By choosing $r_e = 1$, the resonant electrical circuit is tuned to the natural frequency of the original vibrating system, and the resonant peak at $\omega/\omega_1 = 1$ is reduced drastically. Figure 2 closely resembles the corresponding result for a conventional inertial vibration absorber; the resonant electrical circuit here plays the role of the tuned mechanical oscillator in an inertial absorber. In Figure 3 is shown the dimensionless current amplitude $|I/(F/T)|$ for the same numerical parameter values used in Figure 2. The current amplitude is large in the region where the vibration amplitude of the mass m is reduced, but it may be easier in may situations to accommodate the large current amplitude in the coil than to accommodate the corresponding large amplitude mechanical oscillation of the secondary mass in an inertial absorber.

The two new resonant peaks which the absorber introduces in Figure 2 may be reduced by increasing the damping in the resonant electrical circuit. In Figure 4 is shown the dimensionless amplitude $|X/(F/k)|$ as a function of the dimensionless forcing frequency ω/ω_1 for the parameter values $r_e = 1$, $r_T = 1$, $\zeta_1 = 0{\cdot}1$ and $\zeta_e = 0{\cdot}5$. The amplitude reduction

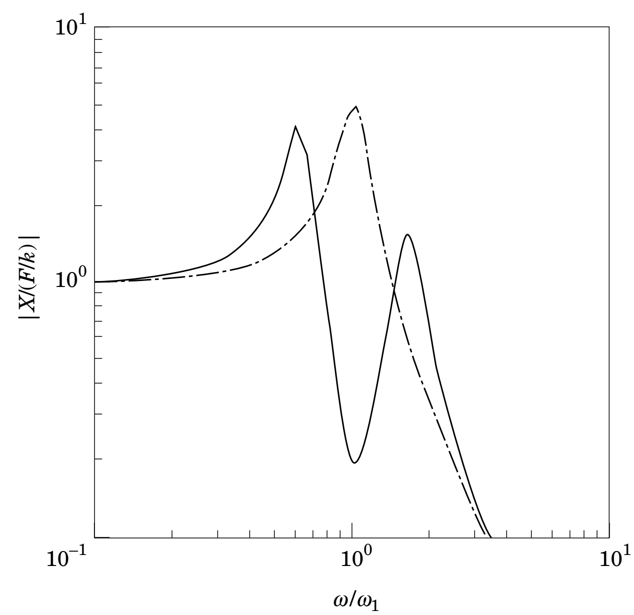

Figure 2. The dimensionless displacement amplitude $|X/(F/k)|$ as a function of the dimensionless forcing frequency ω/ω_1 for $r_e = 1$, $r_T = 1$, $\zeta_e = 0{\cdot}1$ and $\zeta_1 = 0{\cdot}1$ (solid line). The dashed line shows the response of the system for $T = 0$ (no absorber).

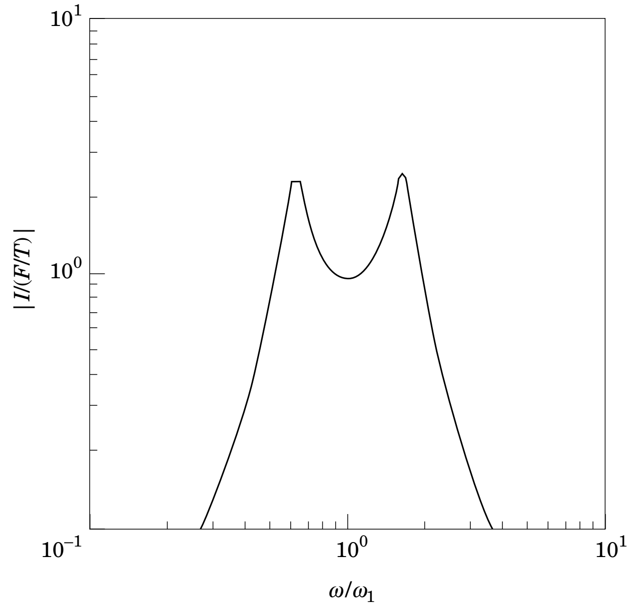

Figure 3. The dimensionless current amplitude $|I/(F/T)|$ as a function of the dimensionless forcing frequency ω/ω_1 for $r_e = 1$, $r_T = 1$, $\zeta_e = 0{\cdot}1$ and $\zeta_1 = 0{\cdot}1$.

at the original resonant peak $\omega/\omega_1 = 1$ is not as great as in Figure 2, but the two new resonant peaks in Figure 2 are substantially reduced. In Figure 5 is shown the dimensionless current amplitude $|I/(F/T)|$ for the same parameter values as used in Figure 4.

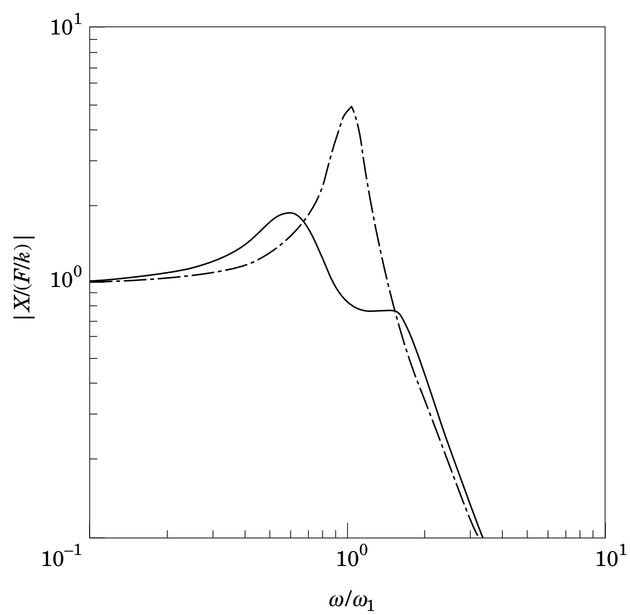

Figure 4. The dimensionless displacement amplitude $|X/(F/k)|$ as a function of the dimensionless forcing frequency ω/ω_1 for $r_e = 1$, $r_T = 1$, $\zeta_e = 0{\cdot}5$ and $\zeta_1 = 0{\cdot}1$ (soild line). The dashed line shows the response of the system for $T = 0$ (no absorber).

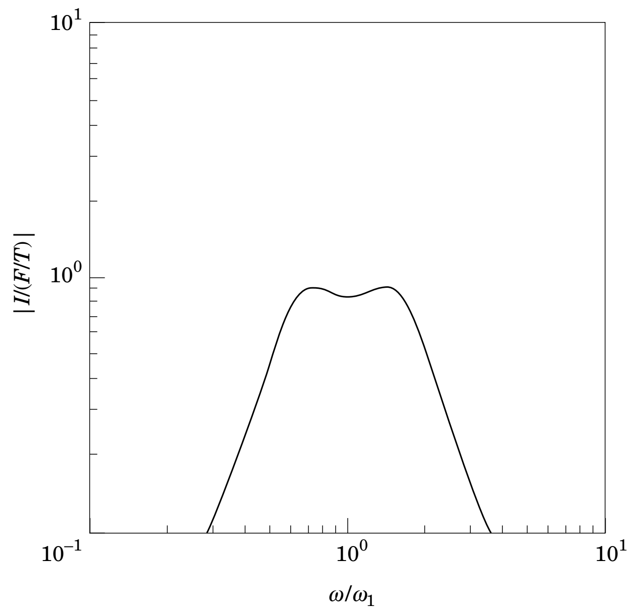

Figure 5. The dimensionless current amplitude $|I/(F/T)|$ as a function of the dimensionless forcing frequency ω/ω_1 for $r_e = 1$, $r_T = 1$, $\zeta_e = 0{\cdot}5$ and $\zeta_1 = 0{\cdot}1$.

3. EXPERIMENTS

As can be observed in Figure 6, the apparatus consisted of a mild steel strip of width 0·05 m and thickness 0·0015 m. The strip was clamped at one end, thereby rendering it a cantilever. A sinusoidal driving force was applied to the cantilever by attaching a small dc motor with a lead rod attached eccentrically to its rotating shaft. The damping ratio of the unforced cantilever–motor system was measured to be $\zeta_1 \approx 0{\cdot}1$. A piezoelectric strip (AMP Inc., Model #ST1-028k) was placed on the surface of the cantilever in order to monitor and measure the vibration amplitude caused by the rotating eccentric mass.

A coil of diameter 0·0048 m and inductance 37·5 mH was attached to the bottom of the cantilever so that the coil oscillated between two permanent magnets ($B \approx 4000$ Gauss),

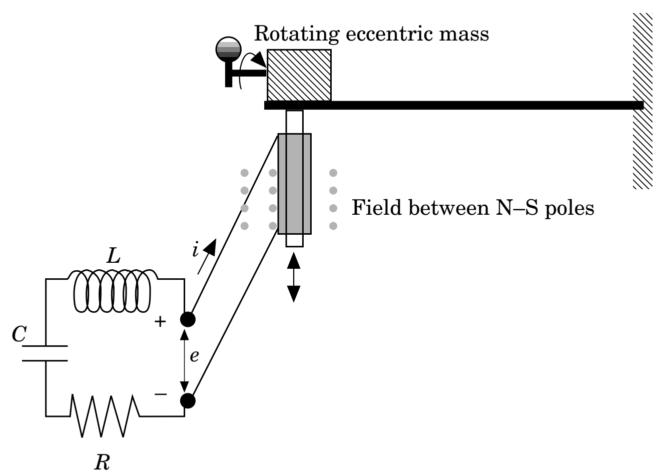

Figure 6. A schematic of the experimental set-up of the vibrating cantilever.

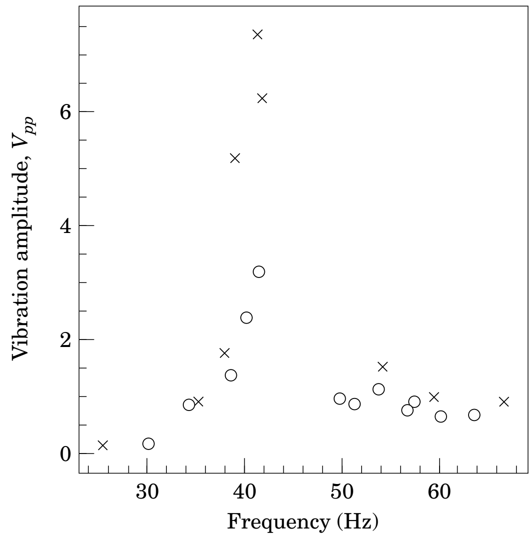

Figure 7. The experimentally obtained values of the cantilever displacement amplitude as a function of forcing frequency with and without the absorber. ×, Value for system without absorber; ○, value for system with absorber.

approximating the voice coil arrangement assumed in the mathematical model. The coil was connected in series with an *RLC* circuit. A set of tests were performed in which the resonance of the electrical circuit was tuned to the natural frequency of the cantilever, and the driving frequency of the motor was swept through a range which included the first natural frequency of the unforced cantilever–motor system. The experimentally obtained vibration amplitude of the cantilever with and without the electromechanical absorber is shown in Figure 7. The data show a dramatic decrease in vibration amplitude near the beam resonance, as predicted by the theory. The antiresonance of the absorber system predicted by the analysis could not be observed with the present experimental system. The load reduction effect of the absorber was so strong that the speed of the motor increased suddenly when the antiresonance was approached. Further experiments with a more precisely controlled force input are expected to give an even larger vibration reduction and better agreement with the theoretical results.

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