Journal of Sound and Vibration (1997) **200**(5), 579–587

A MODEL FOR CALCULATING THE INSERTION LOSSES OF PIPE LAGGING

S. Kanapathipillai and K. P. Byrne

School of Mechanical and Manufacturing Engineering, The University of New South Wales, Sydney 2052, Australia

(*Received* 1 *September* 1995, *and in final form* 6 *May* 1996)

Lagging formed of jackets of porous materials such as glass wool and of impervious materials such as metal sheets are widely used to attenuate the sound radiated from pipes. A procedure is described for calculating the insertion losses produced by such lagging. The procedure yields the insertion losses corresponding to each of the low order mode types of the pipe. These are breathing, bending and ovalling mode types. Impedance and pressure transfer formulae for the various jacket types are given in terms of the fundamental properties of the jackets. The porous jacket, for example, is defined by its inner and outer diameters and the flow resistivity of the porous material. These formulae enable the frequency dependent insertion loss produced by an arbitrary pipe lagging to be determined. A comparison of the insertion losses predicted by this procedure with published results is given.

1. INTRODUCTION

The major contribution to the sound radiated from a pipe is associated with the radial vibration of the pipe wall. The usual way of attenuating the sound radiated by pipes is to lag them with porous layers or jackets such as glass wool blankets and impervious layers or jackets such as metal cladding sheets. Papers available in the readily accessible literature relating to the acoustic performance of pipe lagging generally have been concerned with presenting experimental results such as the lagging insertion losses [1–3]. It appears that few attempts have been made to predict insertion losses by the use of models which are based on the fundamental properties of the layer elements such as the thickness and flow resistivity of the porous layer. This is not surprising in view of the difficulty which has been encountered in satisfactorily predicting the transmission of sound from the inside to the outside of a pipe. It has been found, with regard to this latter problem, that theoretical predictions of the sound transmission loss from the inside to the outside of a pipe can significantly overestimate (by 30 dB) the actual sound transmission loss [4].

Pipe vibration can be classified in terms of the motion of the pipe cross-section during the vibration. The most elementary form of pipe vibration occurs in the so-called breathing type modes in which the circular cross-section of the pipe expands and contracts with the elements of the pipe wall moving only radially. The so-called bending type modes are associated with the circular cross-section of the pipe remaining circular while simply translating back and forth. In the ovalling type modes the circular cross-section of the pipe becomes oval along axes which are at right angles. More elaborate motion patterns of the pipe cross-section can also occur.

The relative contributions associated with each of these mode types to the overall level of vibration of a pipe will depend upon the particular installation. It is appropriate, in view

0022–460X/97/100579 + 09 $25.00/0/sv960626

of this, to find the insertion loss produced by a particular lagging when the lagged pipe is vibrating in each of the previously mentioned types of modes.

Thus the basic aim of this work is to devise a procedure for predicting the frequency dependent insertion losses produced by a lagging applied to an infinitely long pipe which is supporting in turn breathing, bending, ovalling and higher order structural waves travelling along the pipe.

2. PRINCIPLES OF THE ANALYSIS METHOD

The model of the lagged pipe is shown in Figure 1. The jackets which form the lagging can be classified into three types. Firstly, there is the impervious jacket type. Such jackets are usually made of metal or plastic and are defined by the nominal diameter and thickness of the jacket and the properties of the jacket material such as Young's modulus, Poisson ratio and density. Usually the outer jacket of a pipe lagging is an impervious jacket. Secondly, there is the porous jacket type. Such jackets are usually made of glass wool or mineral wool and such jackets can be defined by the inner and outer diameters of the jacket and the flow resistivity of the jacket material, and possibly other quantities which define the properties of the skeletal structure of the porous material. Thirdly, there is the air jacket. This jacket can be considered to be a porous jacket made of a zero flow resistivity porous material and so the air jacket can be considered to be a special type of porous jacket.

The radial velocity of the outer surface of the pipe is assumed to be given by equation (1).

$$\mathbf{v}_r = V_r \cos n\phi \exp[\mathrm{j}(\omega t - k_z z)] \tag{1}$$

When $n = 0$ this equation represents a breathing type mode harmonic wave of circular frequency ω which is propagating in the positive z direction along the pipe with a phase velocity of ω/k_z. When $n = 1$, the structural wave propagating along the pipe is associated with the bending type mode while when $n = 2$, the structural wave is associated with the ovalling type mode. It should be noted that only the bending mode wave can be made to propagate at an arbitrarily small frequency. The other structural mode type waves such as the breathing, ovalling and higher order mode waves exhibit cut-offs such that they cannot propagate at frequencies below particular cut-off frequencies. Further, even if a particular structural type mode is able to propagate, it may not be able to radiate sound. Sound can only be radiated if $k_z < k = \omega/c$, the wave number in the surrounding air.

The insertion loss produced by the lagging which surrounds the pipe when the pipe is supporting a structural wave of a particular type of mode and frequency can be found if

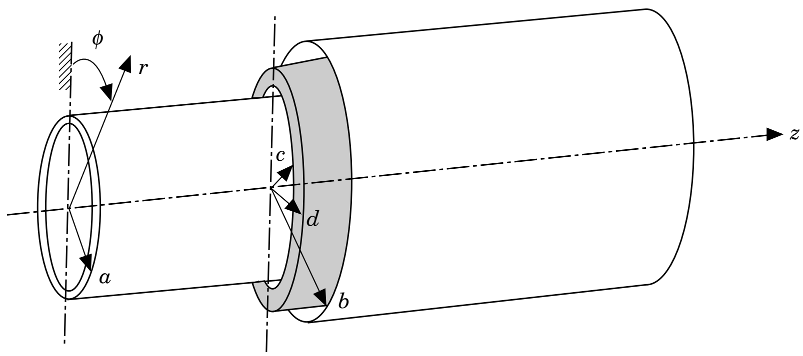

Figure 1. Model of lagged pipe.

the ratios of the sound powers radiated from a unit length of the bare and the lagged pipe respectively can be calculated when the vibration level of the pipe is unchanged. Suppose it is assumed that the acoustic pressure and the radial particle velocity on a coaxial cylindrical surface just beyond the outer surface of the pipe lagging vary according to $\cos n\phi$, that is, in the same way as the radial velocity of the pipe surface, whether or not the pipe lagging is present. The radial impedance on this cylindrical surface, being dependent only on the medium beyond this cylindrical surface, is of course unchanged by the presence or absence of the lagging. Further, the radial intensities then will vary according to $\cos^2 n\phi$ whether or not the pipe lagging is present. The required ratio of the sound powers radiated from a unit length of the bare and lagged pipe can be found from the ratio of the radial intensities at the same point on the cylindrical surface for the bare and lagged pipe. A convenient point to use is the point defined by $r = b$, $\phi = 0$ and this point is subsequently referred to as the reference point. The radial intensities can be obtained from the pressures and radial particle velocities. The radial particle velocities can be obtained from the pressures and radial impedances. Thus the basic strategy is to determine the radial impedance at the reference point and the pressures at this point with and without the lagging.

Consider first the situation when the lagging is not present. The pressure and radial impedance at the point of interest can be found by expressing the harmonic form of the wave equation, equation (2), in cylindrical co-ordinates, assuming a separable solution, equation (3) and solving the resulting differential equations (4) for $R(r)$ and $\Phi(\phi)$ to give the solutions defined by equations (5)

$$\nabla^2\mathbf{p} + k^2\mathbf{p} = 0, \qquad \mathbf{p} = \mathbf{P}R(r)\Phi(\phi)\exp[\mathrm{j}(\omega t - k_z z)], \tag{2, 3}$$

$$\mathrm{d}^2R/\mathrm{d}r^2 + (1/r)\,\mathrm{d}R/\mathrm{d}r + (k^2 - k_z^2 - n^2/r^2)R = 0, \qquad \mathrm{d}^2\Phi/\mathrm{d}\phi^2 + n^2\Phi = 0, \tag{4}$$

$$R(r) = A\mathrm{J_n}(k_r r) + B\mathrm{N_n}(k_r r), \qquad \Phi(\phi) = C\cos n\phi. \tag{5}$$

A, B and C are constants and $\mathrm{J_n}$ and $\mathrm{N_n}$ are the nth order Bessel and Neumann functions in which $k_r^2 = k^2 - k_z^2$. The radial particle velcoity, v_r can be found from the linearized Euler equation $\partial p/\partial r = -\rho\partial v_r/\partial t$. This radial particle velocity at the pipe surface can be made equal to the radial velocity of the pipe surface. The radiation condition provides a further boundary condition and so the constants AC and BC can be determined. The pressure at the reference point $r = b$, $\phi = 0$ can then be found as can the radial impedance. They are as follows:

$$\mathbf{p} = \mathrm{j}V_r\rho c\frac{k}{k_r}\left[\frac{\mathrm{J_n}(k_r b) - \mathrm{jN_n}(k_r b)}{\mathrm{J'_n}(k_r a) - \mathrm{jN'_n}(k_r a)}\right]\exp[\mathrm{j}(\omega t - k_z z)]. \tag{6a}$$

$$\frac{\mathbf{z}_\mathrm{r}}{\rho c} = -\mathrm{j}\frac{k}{k_r}\left[\frac{\mathrm{J_n}(k_r b) - \mathrm{jN_n}(k_r b)}{\mathrm{J'_n}(k_r b) - \mathrm{jN'_n}(k_r b)}\right]. \tag{6b}$$

Equation (6a) can be written as $\mathbf{p} = \mathbf{P}\exp[\mathrm{j}(\omega t - k_z z)]$. The radial intensity at the reference point $r = b$, $\phi = 0$ is then given by $0{\cdot}5\,|\mathbf{P}/\mathbf{z}_\mathrm{r}|^2\,\mathrm{Re}\,\{\mathbf{z}_\mathrm{r}\}$.

The task of computing the pressure at the reference point when the pipe is lagged is a much more complicated matter. However, the problem can be simplified by noting that the waves propagating in each of the jackets have, in the axial direction, the same wave number as the structural wave propagating along the pipe, that is, k_z. It is also useful to note that across the interfaces between the individual jackets, as well as across interfaces between the outer surface of the outer jacket and the surrounding medium and the inner surface of the inner jacket and the pipe, the acoustic pressure and the radial particle velocity are continuous. Further, these two quantities are assumed to vary according to $\cos n\phi$.

The task of computing the pressure at the reference point is undertaken in the following way. It is possible, for each of the jacket types, to devise formulae which enable the radial impedance on the inside surface of the jacket to be found when that on the outside is known. Since the radial impedance on the outside surface of the outer jacket is given by equation (6b), that on the inside surface of the outer jacket can be found. Successive application of the appropriate impedance transfer formulae enables the radial impedance at the pipe surface to be obtained. If the velocity V_r of the pipe surface is assumed to be unchanged by the presence of the lagging, the acoustic pressure on the pipe surface can be found. This pressure at $\phi = 0$, can be considered to characterise the pressure on the inner surface of the inner jacket. It is possible, just as it is for impedances, for each of the jacket types, to derive formulae which enable the acoustic pressure at $\phi = 0$ on the outside of a jacket surface to be found when that at $\phi = 0$ on the inside of a jacket surface is known. Successive application of the appropriate pressure transfer formulae enables the acoustic pressure at the reference point to be found. The radial intensity can then be found and this, along with the radial intensity associated with the unlagged pipe at the reference point, enables the insertion loss to be found. Details of the development of the impedance and pressure formulae for the two jacket types are given in Appendices A and B.

The analysis method already described can be extended to give the insertion losses in one-third octave bands. The radial intensities at the reference point, $r = b$ and $\phi = 0$, at a number of frequencies in each one-third octave band can be computed for the bare and the lagged pipe. These intensities can then be averaged to give the one-third octave band values for the bare and the lagged pipe. The one-third octave band insertion losses can then be found.

3. A COMPARISON BETWEEN PREDICTED AND MEASURED RESULTS

An indication of the effectiveness of the previously described analysis method can be obtained by comparing some predicted and measured insertion losses. Experimental insertion loss data with the level of supplementary information required by the prediction procedure described here is not available in the readily accessible literature. It appears that the best available data is then given by Loney [2]. Loney has presented, for pipe lagging of the types of interest here, insertion losses derived from sound power measurements made in a reverberation chamber. Loney's measurements were made using a 12 in (304·8 mm) diameter 22 ft (6705·6 mm) long steel pipe of unspecified wall thickness. The pipe was internally acoustically excited.

It is likely that most of the sound power radiated by the pipe Loney used would have been associated with its vibration at its resonant frequencies. The associated modes of vibration of such a pipe of length L are formed by the interference of waves of the type defined by equation (1) travelling in the positive and negative z direction. This would lead to modes of vibration as defined by equation (7) where n and m are integers.

$$v_r = V_r \cos n\phi \sin (m\pi z/L) \tag{7}$$

Thus the sound power radiated by Loney's pipe in a particular one-third octave band would have been associated with the modes of vibration defined by equation (7) whose resonant frequencies fall into that one-third octave band.

A problem arises in comparing the measured and predicted insertion losses in that the measured insertion losses are based on a pipe whose vibration would have had contributions from modes with various values of n whereas the predicted insertion losses are based on a pipe whose vibration is associated with single values of n. However, it would be expected that there would be no significant effect due to the fact that the measured insertion losses are based on a finite length pipe which is supporting axial standing waves whereas the predicted insertion losses are based on an infinitely long pipe which is supporting an axial travelling wave.

Loney's paper does not contain some of the data needed for the analysis procedure described in the previous section. The wall thickness of the pipe is not given nor are the flow resistivity of the porous layer and the mechanical properties of the cladding sheet. The wall thickness of the pipe was assumed to be 10 mm. The cut-off frequency for the structural waves associated with the breathing type modes is approximately 5900 Hz for this wall thickness and this cut-off frequency is not strongly dependent upon the wall thickness. The phase velocity of the structural waves associated with the bending type modes is also not strongly dependent upon the wall thickness. The cut-off frequency for the structural waves associated wtih the ovalling type modes is approximately 490 Hz for a wall thickness of 10 mm and is strongly dependent upon the wall thickness. The porous layer was assumed to be 50 mm thick and to have a flow resistivity of 10 000 rayls/m, a typical flow resistivity of a low density glass wool. The aluminium cladding was assumed to be 0·254 mm thick and to have a Young's modulus of 71 GPa, a Poisson ratio of 0·33 and a density of 2700 kg/m^3.

The above parametric values were used to determine the predicted insertion losses for the bending and ovalling type modes and these predicted results are plotted in Figures 2 and 3 along with Loney's measured values. It can be seen that over the intermediate frequency range there is good agreement between the measured and predicted results for the bending type modes and relatively poor agreement for the ovalling type mdoes. This observation suggests that the major component of the sound power radiated from Loney's pipe was associated with the bending type modes rather than the ovalling type modes.

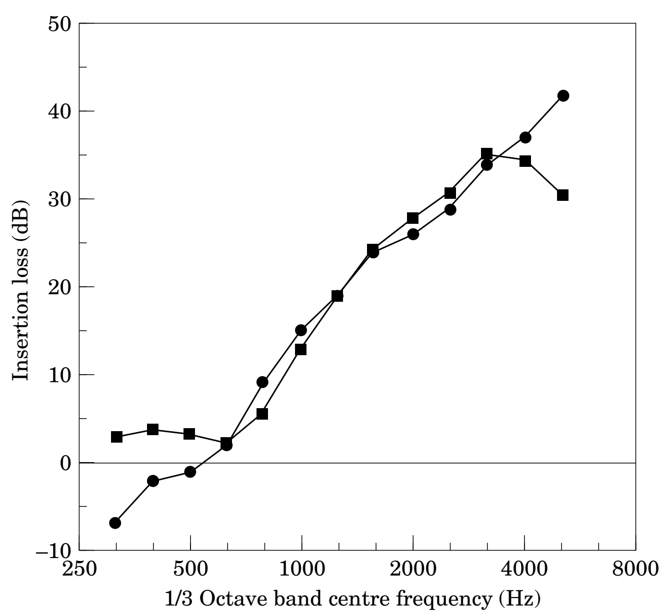

Figure 2. Comparison of predicted bending type mode insertion losses with measured insertion losses. —■—, predicted; —●—, measured.

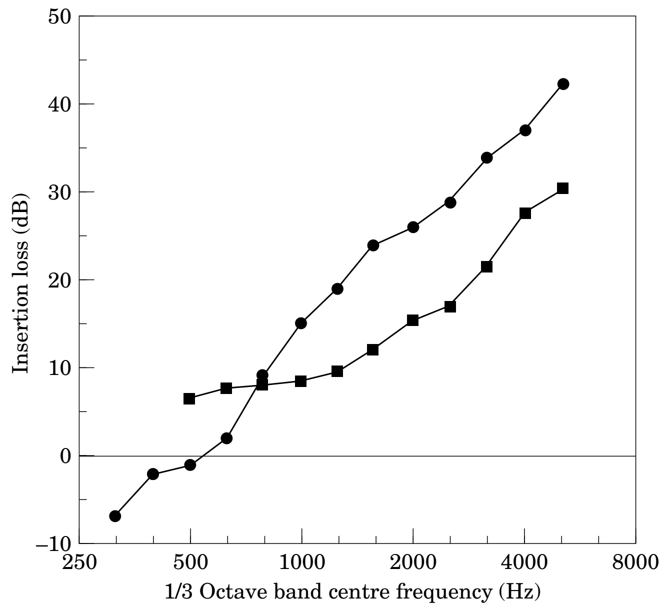

Figure 3. Comparison of predicted ovalling type mode insertion losses with measured insertion losses. key as Figure 2.

There are several points which should be noted with regard to Loney's results. Firstly, Loney's pipe was internally acoustically excited and the first higher order acoustic mode in the pipe, which would be effective in exciting the bending mode, would have a cut-off frequency of approximately 685 Hz, while the first acoustic mode which would be effective in exciting the ovalling mode would have a cut-off frequency of approximately 1135 Hz. Thus it is to be expected that the pipe would not have radiated significant sound power in the bending mode at frequencies below 685 Hz and below 1135 Hz in the ovalling mode. Secondly, there is some doubt about the validity of Loney's results at upper frequencies as the insertion losses he measured with five different types of lagging all yielded the same insertion losses in the 5 kHz one-third octave band.

4. CONCLUSIONS

A procedure for predicting the frequency dependent insertion losses produced by a pipe lagging defined in terms of the fundamental properties of the lagging elements has been described. The procedure yields the insertion losses for the individual types of pipe modes such as the breathing, bending and ovalling mode types. A comparison with some of the available measured data suggests that the procedure is capable of yielding useful predictions. However, a proper experimental program needs to be undertaken to provide a proper assessment of the procedure.

REFERENCES

1. M. E. Hale and B. A. Kugler 1975 *Petroleum Division*, *ASME Winter Annual Meeting*, *Houston*, *Texas*. The acoustic performance of pipe lagging systems.
2. W. Loney 1984 *Journal of the Acoustical Society of America* **76**, 150–157. Insertion loss tests for fibreglass pipe insulation.

3. K. P. BYRNE 1991 *4th Western Pacific Regional Acoustics Conference, Brisbane*, 112–119. The acoustic performance of plastic foam and fibrous preformed thermal pipe insulation for small diameter pipes.
4. G. L. BROWN and D. C. RENNISON 1974 *Proceedings of the Noise, Shock and Vibration Conference, Monash University, Melbourne*. Sound radiation from pipes excited by plane acoustic waves.

APPENDIX A: POROUS JACKET IMPEDANCE AND PRESSURE FORMULAE

The porous jacket has inner and outer radii of r_I and r_O as shown in Figure A.1 and is made of a material such that the propagation of acoustic waves in the gas which saturates the porous material is completely defined by the flow resistivity of the gas-porous material combination. This is the simplest model which can be used to examine wave propagation in such materials. More complex models can be devised if required. With the simplest model, the propagation of acoustic waves in the gas which saturates the porous material is governed by the following equation, which is expressed in terms of the acoustic pressure, p,

$$c^2\nabla^2 p = \partial^2 p/\partial t^2 + (R_1/\rho)\partial p/\partial t. \tag{A.1}$$

c and ρ are the velocity of sound and density of the gas which saturates the porous material and R_1 is the flow resistivity of the gas in the porous material.

The harmonic form of equation (A.1) is

$$\nabla^2 \mathbf{p} + \mathbf{k}^2\mathbf{p} = 0, \tag{A.2}$$

where the complex wave number $\mathbf{k}$ is given by $\mathbf{k} = k[1 - \mathrm{j}R_1/\rho\omega]^{1/2}$. The characteristic impedance is given by $\mathbf{Z}_O = \rho c[1 - \mathrm{j}R_1/\rho\omega]^{1/2}$.

Suppose now that a separable solution to equation (A.2) of the form of equation (A.3) is proposed, then

$$\mathbf{p} = \mathbf{P}R(r)\Phi(\phi)\exp[\mathrm{j}(\omega t - k_z z)]. \tag{A.3}$$

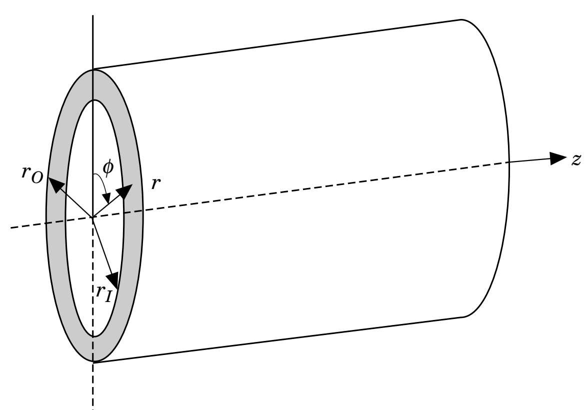

Figure A.1. Porous jacket model.

Substitution of this equation into equation (A.2) leads to the differential equations (A.4) whose solutions are given by equations (A.5).

$$\mathrm{d}^2R/\mathrm{d}r^2 + \mathrm{d}R/\mathrm{d}r + (\mathbf{k}^2 - k_z^2 - n^2/r^2)R = 0, \qquad \mathrm{d}^2\Phi/\mathrm{d}\phi^2 + n^2\Phi = 0, \qquad \text{(A.4a, 4b)}$$

$$R(r) = A^*\mathrm{J_n}(\mathbf{k_r}r) + B^*\mathrm{N_n}(\mathbf{k_r}r), \qquad \Phi(\phi) = C^*\cos n\phi \qquad \text{(A.5a, 5b)}$$

A^*, B^* and C^* are constant and $\mathrm{J_n}$ and $\mathrm{N_n}$ are nth Bessel and Neumann functions in which $\mathbf{k_r}$ is given by $\mathbf{k_r}^2 = \mathbf{k}^2 - k_z^2$. The similarity of equations (A.2) to (A.5) with equations (2) to (5) is evident.

The proceeding equations can be used to write equations (A.6) and (A.7) for the acoustic pressure and the radial particle velocity.

$$\mathbf{p} = \mathbf{P}C^*[A^*\mathrm{J_n}(\mathbf{k_r}r) + B^*\mathrm{N_n}(\mathbf{k_r}r)]\cos n\phi\,\exp[\mathrm{j}(\omega t - k_z z)], \qquad \text{(A.6)}$$

$$v_r = -\frac{\mathbf{P}C^*\mathbf{k_r}}{\mathrm{j}\mathbf{Z}_O\mathbf{k}}[\mathrm{A}^*\mathrm{J'_n}(\mathbf{k_r}r) + B^*\mathrm{N'_n}(\mathbf{k_r}r)]\cos n\phi\,\exp[\mathrm{j}(\omega t - k_z z)]. \qquad \text{(A.7)}$$

These two equations can be used to give an expression for the radial impedance at radius r. An expression for the radial impedance $\mathbf{Z}_I$ at the inner surface where $r = r_I$ can then be found in terms of the radial impedance $\mathbf{Z}_O$ at the outer surface where $r = r_O$. The relevant expressions are:

$$\frac{\mathbf{Z}_I}{\mathbf{Z}_O} = -\mathrm{j}\frac{\mathbf{k}}{\mathbf{k_r}}\frac{\mathrm{J_n}(\mathbf{k_r}r_I) + \boldsymbol{\alpha}\mathrm{N_n}(\mathbf{k_r}r_I)}{\mathrm{J'_n}(\mathbf{k_r}r_I) + \boldsymbol{\alpha}\mathrm{N'_n}(\mathbf{k_r}r_I)}, \qquad \text{(A.8)}$$

where $\boldsymbol{\alpha}$ is given by:

$$\boldsymbol{\alpha} = \frac{(\mathbf{Z}_O/\mathbf{Z}_O)(\mathbf{k_r}/\mathbf{k})\mathrm{J'_n}(\mathbf{k_r}r_O) + \mathrm{j}\mathrm{J_n}(\mathbf{k_r}r_O)}{(\mathbf{Z}_O/\mathbf{Z}_O)(\mathbf{k_r}/\mathbf{k})\mathrm{N'_n}(\mathbf{k_r}r_O) + \mathrm{j}\mathrm{N_n}(\mathbf{k_r}r_O)}. \qquad \text{(A.9)}$$

Equation (A.8) is the impedance transfer formula for a porous jacket.

Equations (A.6) and (A.7) can be manipulated to give the pressure transfer formula which gives the pressure, $\mathbf{P}_O$ at $\phi = 0$ on the outer surface where $r = r_O$ when the pressure $\mathbf{P}_I$ at $\phi = 0$ on the inner surface where $r = r_I$ is known. The pressure transfer formula is given by equation (A.10) in which $\boldsymbol{\alpha}$ is again as given by equation (A.9).

$$\mathbf{P}_O = \mathbf{P}_I\frac{\mathrm{J_n}(\mathbf{k_r}r_O) + \boldsymbol{\alpha}\mathrm{N_n}(\mathbf{k_r}r_O)}{\mathrm{J_n}(\mathbf{k_r}r_I) + \boldsymbol{\alpha}\mathrm{N_n}(\mathbf{k_r}r_I)}. \qquad \text{(A.10)}$$

APPENDIX B: IMPERVIOUS JACKET IMPEDANCE AND PRESSURE FORMULAE

The impervious jacket has a nominal radius of a and a thickness of h as shown in Figure B.1. The jacket is made of a material with a Young's modulus of E, a Poisson ratio of v, a loss factor of η and a density of ρ. The so-called Donnell equations (B.1) are commonly used to define the vibrations of thin walled shells of which the jacket of interest here is a specific example. The directions of the pipe wall displacement components, u, v and w, which are used in these equations, are also shown in Figure B.1.

$$\frac{\partial^2 u}{\partial z^2} + \frac{1-v}{2a^2}\frac{\partial^2 u}{\partial\phi^2} + \frac{1+v}{2a}\frac{\partial^2 v}{\partial z\partial\phi} + \frac{v}{a}\frac{\partial w}{\partial z} - \frac{\ddot{u}}{c_p^2} = 0, \qquad \text{(B.1a)}$$

$$\frac{1+v}{2a}\frac{\partial^2 u}{\partial z\partial\phi} + \frac{1-v}{2}\frac{\partial^2 v}{\partial z^2} + \frac{1}{a^2}\frac{\partial^2 v}{\partial\phi^2} + \frac{1}{a^2}\frac{\partial w}{\partial\phi} - \frac{\ddot{v}}{c_p^2} = 0, \qquad \text{(B.1b)}$$

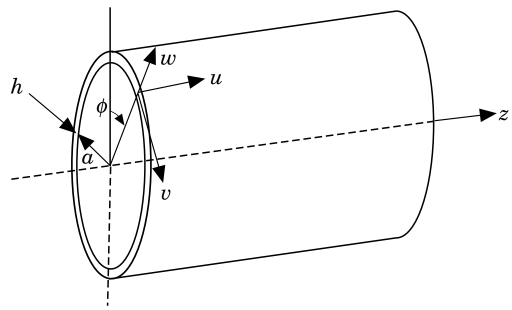

Figure B.1. Impervious jacket model.

$$\frac{\nu}{a}\frac{\partial u}{\partial z}+\frac{1}{a^2}\frac{\partial v}{\partial \phi}+\frac{w}{a^2}+\beta_0^2\left(a^2\frac{\partial^4 w}{\partial z^4}+2\frac{\partial^4 w}{\partial z^2\partial \phi^2}+\frac{1}{a^2}\frac{\partial^4 w}{\partial \phi^4}\right)+\frac{\ddot{w}}{c_p^2}=\frac{p_a(1-\nu^2)}{Eh}. \qquad \text{(B.1c)}$$

p_a is the resultant outward pressure on the shell, $\beta_0^2 = h^2/12a^2$ and $c_p^2 = E/\rho(1-\nu^2)$.

Consider now the situation of interest here in which the travelling pressure wave on the inside of the jacket has a complex representation $\mathbf{P}_I \cos n\phi \exp[\mathrm{j}(\omega t - k_z z)]$ and that on the outside has a complex representation $\mathbf{P}_O \cos n\phi \exp[\mathrm{j}(\omega t - k_z z)]$. The complex representations of the three components of the jacket motion have similar forms. After the appropriate complex representations are substituted into equations (B.1) the following equations are obtained:

$$A\mathbf{U}+B\mathbf{V}+C\mathbf{W}=0, \qquad P\mathbf{U}+Q\mathbf{V}+R\mathbf{W}=0,$$

$$L\mathbf{U}+M\mathbf{V}+N\mathbf{W}=(\mathbf{P}_I-\mathbf{P}_O)(1-\nu^2)/\mathbf{E}h. \qquad \text{(B.2a, 2b, 2c)}$$

$\mathbf{E}$ is the complex Young's modulus by $\mathbf{E} = E(1+\mathrm{j}\eta)$. A, B, *etc*., are terms of the form $A = (\omega/c_P)^2 - k_z^2 - n^2(1-\nu^2)/2a^2$, $B = -\mathrm{j}nk_z(1+\nu)/2a$, $C = -\mathrm{j}k_z\nu/a$, $\mathbf{P} = \mathrm{j}nk_z(1+\nu)/2a$, $Q = (\omega/c_p)^2 - (1-\nu)k_z^2/2 - (n/a)^2$, $R = -n/a^2$, $L = -\mathrm{j}k_z\nu/a$, $M = n/a^2$ and $N = 1/a^2 + \beta_O^2(a^2k_z^4 + 2n^2k_z^2 + n^4/a^2) - \omega^2/c_p^2$. The impedances on the inside and outside of the jacket can be written as $\mathbf{Z}_I = \mathbf{P}_I/\mathrm{j}\omega\mathbf{W}$ and $\mathbf{Z}_O = \mathbf{P}_O/\mathrm{j}\omega\mathbf{W}$ respectively. Algebraic manipulations of equations (B.2) enable the following equation to be obtained which gives the impedance on the inside of the impervious jacket, $\mathbf{Z}_I$ when that on the outside, $\mathbf{Z}_O$ is known

$$\mathbf{Z}_I = \mathbf{Z}_O + \mathrm{j}\boldsymbol{\gamma}/\omega, \qquad \text{(B.3)}$$

where $\boldsymbol{\gamma}$ is given by:

$$\boldsymbol{\gamma} = \frac{\mathbf{E}h}{(1-\nu^2)}\left[\frac{L(QC-BR)C+M(AR-PC)C}{A(QC-BR)+B(AR-PC)}-N\right] \qquad \text{(B.4)}$$

Equation (B.3) is the impedance transfer formula for the impervious jacket. Manipulation of the previous equations allow the pressure transfer formula, equation (B.5) to be found;

$$\mathbf{P}_O = \mathbf{P}_I(1-\mathrm{j}\boldsymbol{\gamma}\omega/\mathbf{Z}_I) \qquad \text{(B.5)}$$

The term $\boldsymbol{\gamma}$ which appears in this equation is again given by equation (B.4).