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TRANSVERSE VIBRATIONS OF A SQUARE MEMBRANE WITH AN ECCENTRIC CIRCULAR OR QUASI-SQUARE HOLE

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1. Introduction

The present note deals with the determination of the fundamental frequency of transverse vibration of the doubly connected membrane shown in Figure 1. It is assumed that the membrane is fixed at both boundaries. Furthermore, the inner boundary P is a circle or quasi-square of very small dimensions:

$$\frac{\text{hole diameter } (2a_1)}{\text{square side } (a)} = \frac{\text{hole side† } (2a_1)}{\text{square side } (a)} = 2 \times 10^{-5}.$$

The boundary of the inner, eccentric edge is defined by

$$(x - x_c)^n + (y - y_c)^n = \alpha^n, \tag{1}$$

where n is an even number and $\alpha = a_1/a$ (see Figure 1).

For $n = 2$, one obtains a circular boundary of diameter $(2a_1)$ and, for n sufficiently large, equation (1) yields a quasi-square with rounded corners of sides $(2a_1)$. The present calculations have been performed taking $n = 50$, see Figure 2.

2. Approximate solution

In the case of normal modes of amplitude $W(x, y)$, the governing functional is

$$J(W) = \iint_{\bar{D}} (W_{\bar{x}}^2 + W_{\bar{y}}^2)\, \mathrm{d}\bar{x}\, \mathrm{d}\bar{y} = \rho\omega^2/s \iint_{\bar{D}} W^2\, \mathrm{d}\bar{x}\, \mathrm{d}\bar{y}. \tag{2}$$

Introducing the dimensionless variables

$$\bar{x} = ax, \qquad \bar{y} = ay,$$

and substituting in equation (2), one obtains

$$J(W) = \iint_D (W_x^2 + W_y^2)\, \mathrm{d}x\, \mathrm{d}y - \Omega^2 \iint_D W^2\, \mathrm{d}x\, \mathrm{d}y, \tag{3}$$

where

†The case of a quasi-square hole [1, 2].

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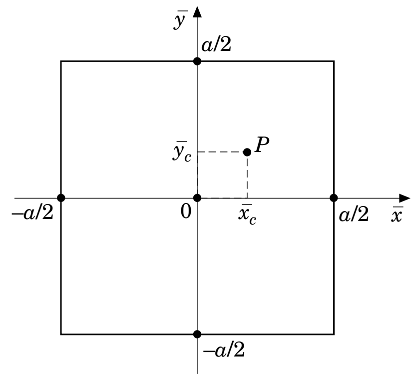

Figure 1. A vibrating square membrane with an eccentric, very small circular or quasi-square inner boundary.

$$\Omega^2 = \rho a^2 \omega^2 / S.$$

Defining

$$g(x, y) = \sqrt[n]{(x - x_c)^n + (y - y_c)^n} - \alpha, \qquad (4)$$

one can use the following approximation for the displacement amplitude:

$$W_a = C_1 \varphi_1(x, y) + C_2 \varphi_2(x, y), \qquad (5)$$

where

$$\varphi_1 = (0{\cdot}5^2 - x^2)(0{\cdot}5^2 - y^2) g(x, y), \qquad \varphi_2 = (0{\cdot}5^4 - x^4)(0{\cdot}5^4 - y^4) g(x, y). \qquad (6)$$

Applying the Ritz minimization condition, one obtains

$$\frac{1}{2}\frac{\partial J(W)_a}{\partial C_i} = \sum_{j=1}^{2} \left[\iint (\varphi_{jx}\varphi_{ix} + \varphi_{jy}\varphi_{iy})\, \mathrm{d}x\, \mathrm{d}y - \Omega^2 \iint_D \varphi_j \varphi_i\, \mathrm{d}x\, \mathrm{d}y \right] C_j = 0. \qquad (7)$$

The non-triviality condition yields a secular determinant, the lowest root of which is the fundamental frequency coefficient

$$\Omega_1 = \sqrt{\rho / S}\, \omega_1 a. \qquad (8)$$

3. NUMERICAL RESULTS

In Table 1 are depicted values of Ω_1 for the case of a circular hole: (a) as its center displaces along the x-axis and (b) as the center moves along the diagonal. It is concluded that the values determined in reference [3] are excessively high. On the other hand, when

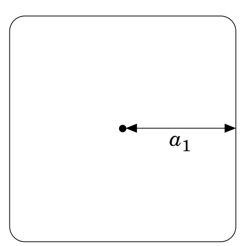

Figure 2. A quasi-square shape (n = 50).

TABLE 1

The fundamental frequency coefficients $\sqrt{\rho/S}\omega_1 a$ in the case of a circular inner boundary ($n = 2$; $\alpha = 10^{-5}$)

(a) (x_c, y_c)	Ω_1	(b) (x_c, y_c)	Ω_1
(0, 0)	5·688	(0·1, 0·1)	5·443
(0·1, 0)	5·567	(0·2, 0·2)	5·060
(0·2, 0)	5·267	(0·3, 0·3)	4·831
(0·3, 0)	5·024	(0·4, 0·4)	4·702
(0·4, 0)	4·861		

TABLE 2

The fundamental frequency coefficients $\sqrt{\rho/S}\omega_1 a$ in the case of a quasi-square inner boundary ($n = 50$; $\alpha = 10^{-5}$)

(a) (x_c, y_c)	Ω_1	(b) (x_c, y_c)	Ω_1
(0·0)	6·024	(0·1, 0·1)	5·696
(0·1, 0)	5·874	(0·2, 0·2)	5·217
(0·2, 0)	5·475	(0·3, 0·3)	4·937
(0·3, 0)	5·151	(0·4, 0·4)	4·778
(0·4, 0)	4·936		

the circular hole is located at the center of the membrane, (0, 0), the determined eigenvalue 5·688 is in good agreement with the result determined in reference [4] in the case of an acoustic soft-walled waveguide where, for a hole of infinitesimal dimension, the value 5·3346 was determined.

The case of a square membrane with a quasi-square inner edge is considered in Table 2. The data is presented in a similar fashion to that of the Table 1. The fundamental frequency coefficients turn out to be slightly larger than the corresponding ones (for the same locations) of Table 1.

It is interesting to point out that in the case of a circular membrane of diameter (2a), with a concentric circular inner boundary, the present approach yields $\Omega_1 = \sqrt{\rho/S}\omega_1(2a) = 5{\cdot}81$ for $a_1/a = 10^{-5}$, while the result determined in reference [4] is 5·78 in the case of an infinitesimal hole.

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