Journal of Sound and Vibration (1997) **201**(2), 256–257

COMMENTS ON "AN ASSESSMENT OF TIME INTEGRATION SCHEMES FOR NON-LINEAR DYNAMIC EQUATIONS"

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(*Received* 10 *June* 1996)

A recent work by Xie [1] presents a good assessment of seven integration schemes for non-linear dynamic equations. The author applies the schemes to two types of non-linear dynamic systems with softening and hardening elastic springs, respectively. The performances of these schemes, by using different time steps, were compared and assessed in terms of each required computational time and a defined energy error. The following points were concluded in his assessment [1]: (1) The worst scheme among the seven is the Newmark method with $\beta = 0{\cdot}3025$ and $\gamma = 0{\cdot}6$.

(2) As far as the relative computational costs are concerned, the most efficient method is the central difference method. Also, the Runge-Kutta method costs slightly more than the Newmark method. (3) If the time step is too large a converged solution of the non-linear equilibrium equation may not be found by the method of central difference and the Runge-Kutta method. For other schemes that are unconditionally stable for linear systems, numerical results obtained in non-linear cases when using such a large time step are meaningless and sometimes chaotic.

A work by Low [2] supports most of the above findings. Three numerical schemes were used in reference [2] for the structural dynamics response analysis. The condition that must be met to give a stable and satisfactory solution was investigated. In his comparison for a problem with non-linear stiffness, it was shown in Figure 3 [2] that the central difference scheme is better than the Runge-Kutta and the Newmark methods in terms of computational costs. The Runge-Kutta scheme costs slightly less than the Newmark method. This is a finding that is different from Xie's [1]. It should be noted that the Newmark method with $\beta = 1/6$ and $\gamma = 1/2$ used in reference [2] is equivalent to the linear acceleration method. In fact, a positive damping is introduced if γ is greater than 1/2 [3]. This could be a reason why the Newmark method with $\beta = 0{\cdot}3025$ and $\gamma = 0{\cdot}6$ gave poor results [1].

It is also confirmed in reference [2] that the choice of the time step ($h = \Delta t$) will depend essentially on the least vibration period (T) in multi-degree-of-freedom systems. This can still be applicable to Xie's work and the same assessment conclusion would be expected.

An on-going work by Low suggests that the frequency ratio ($\Omega_r = \omega_{I+1}/\omega_i$) of multi-degree-of-freedom vibratory systems should be considered in determining the performances of integration schemes. Different trends are shown in cases with $\Omega_r \cong 1$ (close frequencies) and $\Omega_r \gg 1$.

REFERENCES

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AUTHOR'S REPLY

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(*Received* 22 *July* 1996)

Dr. Low is complimented for his interest in my paper. I am pleased to know that his previous work supports most of my findings. However, it must be noted that the three points listed in Dr. Low's letter represent only part of the conclusions in my paper and the Newmark method used in Dr. Low's work is different from any of the seven schemes used in my assessment.