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STRUCTURAL POWER FLOW ANALYSIS FOR A FLOATING RAFT ISOLATION SYSTEM CONSISTING OF CONSTRAINED DAMPED BEAMS

T. Y. Li, X. M. Zhang, Y. T. Zuo and M. B. Xu

Department of Naval Architecture and Ocean Engineering, Huazhong University of Science and Technology, 430074 *Wuhan*, *People's Republic of China*

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The power flow analysis for a floating raft isolation system consisting of constrained damped beams is completed by using the Green function, an analytical method instead of the traditional mobility approach. The transmitted power flow from a harmonic force excitation to the foundation beams via an isolator–raft beam–isolator system is calculated. Some important structural parameters that influence the transmitted power flow are discussed. The conclusions provide a theoretical basis for vibration control in engineering.

1. INTRODUCTION

The isolation of vibrating machines has been studied by many researchers. It is an important topic because many machines are mounted on flexible foundations in marine and aeronautical applications, etc., and the common objective is to minimize vibration levels. The floating raft developed recently is a new isolation system that reduces the level of noise and vibration, notably in vibrating machines [1], where Zhou obtained a mobility matrix equation after omitting the influences of near field waves of finite Euler–Bernoulli beams with flexural stiffness $EI(1 + \mathrm{i}\eta)$ and then discussed the variation of the transmitted power flow as EI and η are varied independently.

The concept of transmissibility has often been used in vibrating isolation in the past; for example, Ungar and Dietrich [2] studied the transmissibility of a simple isolation system at high frequencies. The use of power flow in a problem of this type is very valuable, because it combines both force and velocity in a single quantity. An attempt to decrease the radiation or vibration in a structure by reducing only the force or velocity amplitude and not considering the relative phase angle may not necessarily be successful, but an improvement may be ensured by decreasing the net vibrational power applied to a structure. Goyder and White [3] introduced the method of power flow and then analyzed the transmitted power flow into the foundation for both one-stage and two-stage isolating systems [4].

Structures with elastic faces and viscoelastic cores are widely used where high vibration strength and low weight structures are desired, and also where damping is required to dissipate vibrational energy. Some attempts are being made to provide effective vibration isolation by the use of an excitation system supported flexibly on a three-layer sandwich beam [5].

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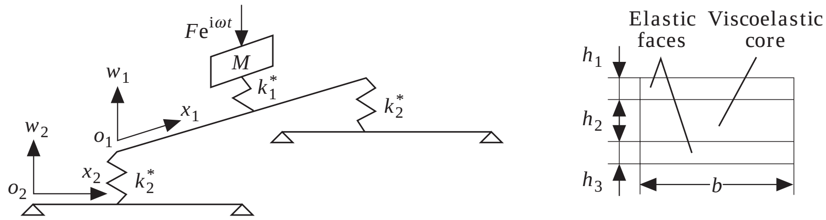

Figure 1. The mathematical model, the beam geometry and the co-ordinate systems.

In this paper, the problem of a vibration system in contact with a floating raft which consists of constrained damped beams is considered by using the Green function, where the equation of bending vibration of a sandwich beam derived by DiTaranto [6] and by Mead and Markus [7] is used. The transmitted power flow from the excitation to the flexible foundation beam is computed and some important parameters that affect the power flow are discussed. Compared with the traditional mobility approach, the method presented here avoids complex derivations in the setting up of equations for the transmitted and input mobilities of the beams at the junction and mass-attachment points. The transmitted power flow is obtained applying the expressions of displacement directly in this paper. Hence, the method given in this paper is more concise and saves computation time.

2. PROBLEM FORMULATION

The model system is shown in Figure 1. To simplify the analysis, the main mass M excited by harmonic force $F\mathrm{e}^{\mathrm{i}at}$ is attached to the center of the raft beam with an isolator, the dynamic characteristics of which are defined by the equation $k_1^* = k_1(1 + \mathrm{i}\eta_1)$, where k_1 and η_1 are the stiffness and the loss factor respectively. The three-layer sandwich raft beam, the length of which is L_1, has face layers of thickness h_1 and h_3 and a core thickness h_2. The face layers are purely elastic with Young's moduli E_1 and E_3. The core has a shear modulus $G(1 + \mathrm{i}\gamma)$, γ being the loss factor of the core material. The material and cross-sectional characteristics of the two three-layer sandwich foundation beams that have the same length L_2 are the same as those of the raft beam.

The raft beam is connected to the two foundation beams by two isolators characterized by the equation $k_2^* = k_2(1 + \mathrm{i}\eta_2)$ between the two ends of the raft beam and the same locations of the two flexible foundation beams, where k_2 and η_2 are the stiffness and the loss factor respectively. Free body diagrams of the primary system and the floating raft are shown in Figure 2. The concentrated harmonic force $\tilde{F}_0 = F_0\,\mathrm{e}^{\mathrm{i}at}$ is acting on the raft beam to represent the function of the isolator defined by $k_1(1 + \mathrm{i}\eta_1)$.

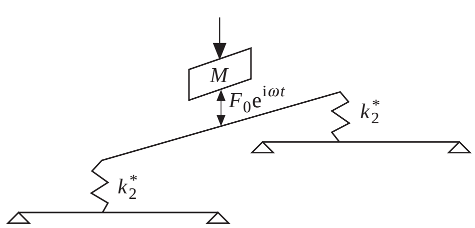

Figure 2. The free body diagram.

3. THEORY ANALYSIS

3.1. SOLUTION OF THE DIFFERENTIAL EQUATION

The differential equation of a sandwich beam is [6]:

$$\partial^6 w/\partial x^6 - g(1 + Y)\partial^4 w/\partial x^4 = (1/D_t)(\partial^2 p/\partial x^2 - gp), \tag{1}$$

where $p = -m\partial^2 w/\partial t^2 + q$, $g = [G(1 + \mathrm{i}\gamma)/h_2][(1/E_1 h_1) + (1/E_3 h_3)]$, $d = h_2 + (h_1 + h_3)/2$, $Y = (d^2 b/D_t)(E_1 h_1 E_3 h_3)/(E_1 h_1 + E_3 h_3)$, $D_t = (E_1 h_1^3 + E_3 h_3^3)b/12$, m is the mass per unit length and q is the external load.

The system can be imagined to comprise identical halves because of symmetry, each of which is acted upon by one-half of the applied force $\tilde{F}_0$, and the beams are coupled at the points $x_1 = 0$ and $x_2 = x_2'$ only. For harmonic motion, one can assume that $w_1(x_1, t) = W_1(x_1)\exp(\mathrm{i}\omega t)$, and $w_2(x_2, t) = W_2(x_2)\exp(\mathrm{i}\omega t)$. Substitution of p, $w_1(x_1, t)$ and $w_2(x_2, t)$ into equation (1) gives

$$\frac{\mathrm{d}^6 W_1}{\mathrm{d}x_1^6} - g(1 + Y)\frac{\mathrm{d}^4 W_1}{\mathrm{d}x_1^4} - \frac{m\omega^2}{D_1}\frac{\mathrm{d}^2 W_1}{\mathrm{d}x_1^2} + \frac{mg\omega^2}{D_t} W_1 = 0, \tag{2}$$

$$\frac{\mathrm{d}^6 W_2}{\mathrm{d}x_2^6} - g(1 + Y)\frac{\mathrm{d}^4 W_2}{\mathrm{d}x_2^4} - \frac{m\omega^2}{D_t}\frac{\mathrm{d}^2 W_2}{\mathrm{d}x_2^2} + \frac{mg\omega^2}{D_t} W_2 = \frac{k_2^*}{D_t}\left\{[W_1(0) - W_2(x_2')]\left[\frac{\mathrm{d}^2\delta(x_2 - x_2')}{\mathrm{d}x_2^2} - g\delta(x_2 - x_2')\right]\right\} \tag{3}$$

where $\delta(\cdot)$ is the Dirac delta function. The solution of equation (2) can be obtained easily:

$$W_1(x_1) = \sum_{i=1}^{6} A_{1i}\exp(\beta_i x_1), \tag{4}$$

where the β_i are the roots of the following simple equation of the sixth order:

$$\beta^6 - g(1 + Y)\beta^4 - (m\omega^2/D_t)\beta^2 + mg\omega^2/D_t = 0. \tag{5}$$

The constants A_{1i} can be determined by application of the boundary conditions of the raft beam. After cumbersome derivations, the solution of equation (3) is obtained:

$$W_2(x_2) = \sum_{i=1}^{6} A_{2i}\exp(\beta_i x_2) + \mathrm{H}(x_2 - x_2')G(x_2) = \sum_{i=1}^{6} A_{2i}\exp(\beta_i x_2) + \mathrm{H}(x_2 - x_2')[W_1(0) - W_2(x_2')]\sum_{i=1}^{6} B_i\exp(\beta_i x_2). \tag{6}$$

The constant A_{2i} can be obtained from the boundary conditions of the foundation beam. The constants B_i can be determined according to the singularity function theory [8]. $G(\cdot)$ is the Green function and $\mathrm{H}(\cdot)$ is the Heaviside function.

3.2. BOUNDARY CONDITIONS

The boundary conditions for the raft beam are as follows:

at $x = 0$: shear force $= k_2^*[W_2(x_2') - W_1(0)]$, bending moment $= 0$, $P_1 = 0$,

at $x = L_1/2$: shear force $= F_0/2$, slope $= 0$, $u_1 = 0$, where u_1 and P_1 are the longitudinal displacement of neutral axis of the first layer and the longitudinal force in the first layer respectively. P_1, u_1, the bending moment and the shear force are defined as follows [4]:

$$P_1 = (D_t/gd)[\mathrm{d}^4 W_1/\mathrm{d}x_1^4 - gY\,\mathrm{d}^2 W_1/\mathrm{d}x_1^2 - (m\omega^2/D_t)W_1], \tag{7}$$

$$u_1 = (D_t/g^2 E_1 h_1\,\mathrm{d}b)\{\mathrm{d}^5 W_1/\mathrm{d}x_1^5 - gY\,\mathrm{d}^3 W_1/\mathrm{d}x_1^3 - [(m\omega^2/D_t) + g^2 Y]\,\mathrm{d}W_1/\mathrm{d}x_1\}, \tag{8}$$

$$\text{bending moment} = (D_t/g)[-\mathrm{d}^4 W_1/\mathrm{d}x_1^4 + g(1+Y)\,\mathrm{d}^2 W_1/\mathrm{d}x_1^2 + (m\omega^2/D_t)W_1], \tag{9}$$

$$\text{shear force} = (D_t/g)[-\mathrm{d}^5 W_1/\mathrm{d}x_1^5 + g(1+Y)\,\mathrm{d}^3 W_1/\mathrm{d}x_1^3 + (m\omega^2/D_t)\,\mathrm{d}W_1/\mathrm{d}x_1]. \tag{10}$$

Applying the expressions for P_1, u_1, the bending moment and the shear force, and with the help of the above boundary conditions, a matrix equation is obtained:

$$[C]\{A_1\} = \{H\}, \tag{11}$$

where $[C]$ is a square matrix of dimension 6×6. $\{A_1\}$ and $\{H\}$ are column matrices. The elements of $[C]$ and $\{H\}$ are, for $i = 1, 2, \ldots, 6$,

$$C_{1i} = -\beta_i^5 + g(1+Y)\beta_i^3 + (m\omega^2/D_t)\beta_i, \qquad C_{2i} = \beta_i^2, \qquad C_{3i} = \beta_i^4 - m\omega^2/D_t,$$

$$C_{4i} = [-\beta_i^5 + g(1+Y)]\exp(\beta_i L_1/2), \qquad C_{5i} = \beta_i \exp(\beta_i L_1/2),$$

$$C_{6i} = (\beta_i^5 - gY\beta_i^3)\exp(\beta_i L_1/2), \qquad H_1 = gk_2^*[W_2(x_2') - W_1(0)]/D_t,$$

$$H_2 = H_3 = 0, \qquad H_4 = gF_0/2D_t, \qquad H_5 = H_6 = 0.$$

To solve equation (11), one can generally assume that

$$A_{1i} = a_{1i}F_0 + b_{1i}[W_2(x_2') - W_1(0)] \tag{12}$$

Now assuming that $F_0 = 1$, $W_2(x_2') - W_1(0) = 0$ and $F_0 = 0$, $W_2(x_2') - W_1(0) = 1$ respectively, then a_{1i} and b_{1i} can be obtained from equations (11) and (12). Through the same operation, A_{2i} can also be obtained from the boundary conditions of the foundation beam.

3.3. SOLVING THE UNKNOWN VARIABLES

Combining equations (4), (6), (11) and (12) and assuming $x_1 = 0$, $x_2 = x_2'$ yields:

$$W_2(x_2') - W_1(0) = F_0\left[-\sum_{i=1}^{6} a_{1i}\right] \Bigg/ \left[1 + \sum_{i=1}^{6}(b_{1i} + A_{2i}\exp(\beta_i x_2') + B_i\exp(\beta_i x_2'))\right]. \tag{13}$$

Consideration of the expressions of $W_1(x_1)$ and $W_2(x_2)$ gives

$$W_1(L_1/2) = \sum_{i=1}^{6} a_{1i}F_0\exp(\beta_i L_1/2) + [W_2(x_2') - W_1(0)]\sum_{i=1}^{6} b_{1i}\exp(\beta_i L_1/2), \tag{14}$$

$$W_2(x_2') = -[W_2(x_2') - W_1(0)]\sum_{i=1}^{6}(A_{2i} + B_i)\exp(\beta_i x_2'). \tag{15}$$

Both are functions of F_0. Then the dynamic analysis of the excitation system gives

$$F_0 = F - M\omega^2[k_1(1 + \mathrm{i}\eta_1)W_1(L_1/2) - F]/[k_1(1 + \mathrm{i}\eta_1) - M\omega^2]. \tag{16}$$

Substitution of $W_1(L_1/2)$ into equation (16) yields F_0, a function of F. Therefore, $[W_2(x_2') - W_1(0)]$ and $W_2(x_2')$ are also functions of F.

4. POWER FLOW ANALYSIS

When a force $F = f\exp(\mathrm{i}\omega t)$ is acting on a structure, a velocity $V = v\exp[i(\omega t + \theta)]$ is generated at the same point. The net vibrational power flow transmitted to the structure is defined by [3]

$$P_s = \frac{1}{T}\int_0^T FV\,\mathrm{d}t = \tfrac{1}{2}fv\cos\theta = \tfrac{1}{2}\mathrm{Re}(FV^{\otimes}), \tag{17}$$

where θ is the phase difference and $\otimes$ denotes the complex conjugate. From equation (17), the transmitted power flow from the excitation system to the two foundation beams via the floating raft is

$$\begin{aligned} P_s &= 2\,\mathrm{Re}\{-\mathrm{i}\omega k_2(1+\mathrm{i}\eta_2)[W_1(0) - W_2(x_2')]W_2(x_2')^{\otimes}\}/2 \\ &= \omega\,\mathrm{Re}\{\mathrm{i}k_2(1+\mathrm{i}\eta_2)[W_2(x_2') - W_1(0)]W_2(x_2')^{\otimes}\}. \end{aligned} \tag{18}$$

5. RESULTS AND DISCUSSION

The beams used here are Al–PVC–Al sandwich beams and the relative coefficient for symmetrical configuration is $h_1 = h_3$. The common parameters are $M = 5{\cdot}0$ kg, $b = 60$ mm and $k_1 = k_2 = 5{\cdot}0 \times 10^4$ N/m in this paper. The dynamic properties of PVC are taken from [5] and these are given in the Appendix. The results from equation (18) are plotted in Figures 3–6. In each case the power flow, which is redefined by $P_s' = 10\lg(P_s/F^2)$ (dB), has been plotted against the frequency f (Hz).

The transmitted power flows are plotted in Figure 3 for different locations of the raft beam on the foundation beams and for one-stage isolation system of an excitation system supported flexibly on a viscoelastic sandwich beam at its mid-point [5]. In Reference [5], a vibration analysis of an excitation system supported flexibly on a three-layer sandwich beam is presented. Both response and transmissibility are evaluated for different geometrical and physical parameters. The solution to this problem is also obtained by approximating the sandwich beam by a lumped mass supported on a spring and dashpot. In Figure 3, the other parameters are $L_1 = L_2 = 500$ mm, $h_1 = 5$ mm, $h_2 = 10$ mm and $\eta_1 = \eta_2 = 0{\cdot}05$. It is obvious that floating raft isolation system gives an increased reduction in power flow and decreases the first resonant frequency of the system, compared with the one-stage isolation system. The major resonance is about 10 Hz, less than the natural

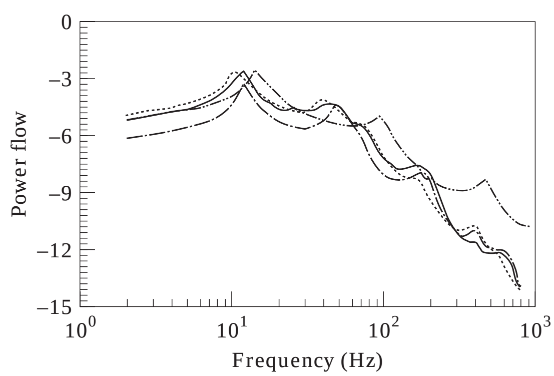

Figure 3. Power flow with variation of x_2'/L_2. —··—, One-stage system; ······, 0·1; ——, 0·3; —·—, 0.5.

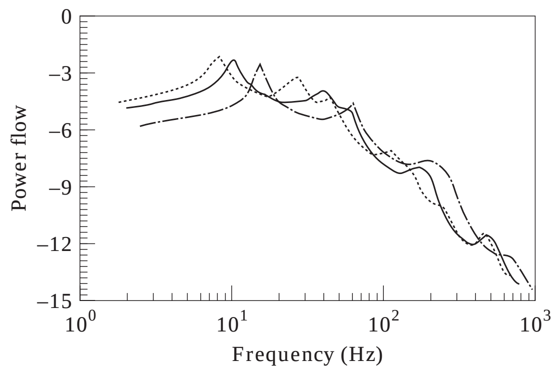

Figure 4. Power flow with variation of L_2/L_1. ······, 0·5; ——, 1·5; —·—, 2·0.

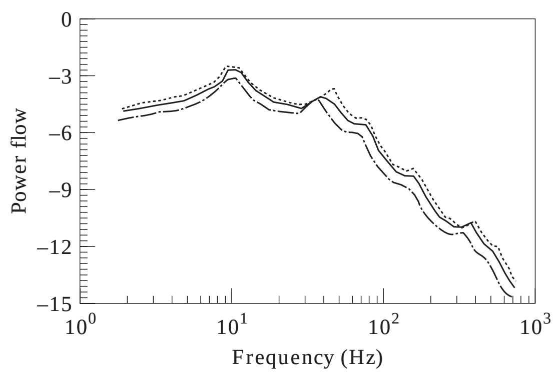

Figure 5. Power flow with variation of h_2/h_1. ······, 1·0, ——, 3·0; —·—,, 4·0.

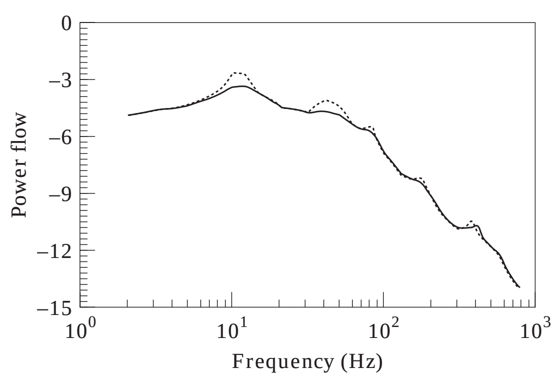

Figure 6. Power flow with variation of η_1. ······, 0·04; ——, 0·10.

frequency of the isolation system with a single degree of freedom, $(1/2\pi)\sqrt{k_1/M} = 15{\cdot}9$ Hz, because the use of three-layer sandwich beams makes the whole system somewhat flexible.

For the floating raft isolation system, the frequencies of peaks are almost the same for different x_2' values. At low frequencies, the transmitted power flow reaches the minimum value as $x_2'/L_2 = 0{\cdot}5$, but in the medium and high frequencies domains there is a steady decrease in power flow of about 84 dB per decade, together with some small superimposed fluctuations. Detailed study shows this is true for frequency-averaged values and light damping in the medium and high frequencies band. That is to say, there is little scope for reducing the transmitted power flow at medium and high frequencies by changing the location of the raft beam attachment points.

The effects of changing length ratio L_2/L_1 of the beams on the transmitted power flow is plotted in Figure 4, where $L_1 = 500$ mm, $h_1 = 5$ mm, $h_2 = 10$ mm, $x_2' = L_2/2$ and $\eta_1 = \eta_2 = 0{\cdot}05$. The peak amplitudes and the frequencies of peaks change with the beam length ratio. The major resonance decreases and the transmitted power flow decreases at low frequencies as the ratio increases. However, in the medium and high frequency domains there is no consistent effect by changing the ratio. At very high frequencies, the ratio has little or no effect because resonances in the foundation beams are effectively damped out. Where maritime vessels are concerned, the lengths of raft and foundation beams have to adapt according to the scope of excitation frequency.

The power flow curves are plotted in Figure 5 for a core thickness ratio h_2/h_1, where $L_1 = L_2 = 500$ mm, $h_1 = 5$ mm, $\eta_1 = \eta_2 = 0{\cdot}05$ and $x_2' = L_2/2$. The three curves are very similar and the transmitted energy decreases at an approximate uniform rate as the thickness ratio increases because of the increase of the overall loss factor with the increase of h_2/h_1. With a continued increase in h_2/h_1, the overall loss factor attains a maximum and then decreases at a uniform rate [5]. This decrease in the loss factor tends to cause an increase in transmitted power flow, but this tendency is offset by the decrease in stiffness associated with the increase of h_2/h_1.

The calculation also shows that increasing the loss factor of the isolator reduces the resonant values of transmitted power flow (seen in Figure 6). However, at medium and high frequencies, it has little effect on vibration isolation. This is to say, increasing the hysteretic loss factor of the isolating system does not have the same disadvantageous effect on high frequency transmissibility as does viscous damping. At very high frequencies it is positively advantageous when the isolator has its own resonances, as in the case being studied in this paper.

6. CONCLUSIONS

In this paper, the power flow via a floating raft consisting of three-layer sandwich raft beams to the foundation beams is investigated. The Green function offers a concise method of analyzing the dynamic mechanism of vibration isolation. It can be concluded that the floating raft isolation system gives more effective isolation. Furthermore, the influences on the transmitted power flow of some important parameters, such as x_2'/L_2, L_2/L_1, h_2/h_1 and η_1, are discussed. The conclusions provide theoretical guidance on reducing noise and vibration in practical engineering applications.

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APPENDIX: DYNAMIC PROPERTIES OF PVC MATERIALS

The shear modulus and loss factor of PVC at 30°C are as follows:

$$G = 420{\cdot}0 + 2{\cdot}5f \text{ N/cm}^2 \quad \text{and} \quad \gamma = 0{\cdot}24 + 0{\cdot}00125f, \qquad f \leqslant 80 \text{ Hz};$$

$$G = 570{\cdot}0 + 0{\cdot}667f \text{ N/cm}^2 \quad \text{and} \quad \gamma = 0{\cdot}28 + 0{\cdot}00075f, \qquad f > 80 \text{ Hz}.$$