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VIBRATION ISOLATION IN A THIN RECTANGULAR PLATE USING A LARGE NUMBER OF OPTIMALLY POSITIONED POINT MASSES

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A new approach is presented for dealing with the problem of vibration control in a plate over a moderately wide frequency band. The band of interest does not contain the first few eigenvalues, nor is it restricted to very high frequencies, but typically contains five or six eigenfrequencies in the 10's of eigennumber. The idea is to make minor modifications to the structure which are sufficient to change its frequency response so that the transmission of vibrations in a given frequency band is suppressed. This work is illustrated by application to a rectangular plate which carries 50 identical point masses in variable positions. A novel method for selecting optimal mass position is demonstrated and compared with the results for mass positions determined by a Genetic Algorithm (G.A.).

1. INTRODUCTION

In modern structural design there is great interest in computerized redesign in order to create structures which have certain beneficial features. The algorithms used to choose suitable modifications are of central importance to this process and form important aids to designers. Such mechanisms are now a common part of many CAD packages, often being integrated into stress analysis packages where the stresses or weights within the structure are the parameters which usually must be reduced or modified. Commercial software for optimization of vibrational characteristics is not yet so well established, but considerable research is being performed in this area. For example, Hinton *et al*. [1] considered the problem of maximization of the fundamental frequency for prismatic structures modelled numerically by using the Finite Strip Method.

Because of the large number of possible variables usually present in numerical problems, and the difficulty in finding most of their derivatives, it is rather difficult to utilize traditional calculus optimization techniques in such work. Consequently, techniques such as the Genetic Algorithm (G.A.), are becoming popular (traditional optimization methods have been discussed in some detail by Siddall [2], while Goldberg [3] has given a thorough account of the G.A. as a tool). There is, however, still some reluctance

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to use modern stochastic methods, owing to a lack of understanding of why the methods seem to work or how to manipulate them to best effect. Keane [4, 5] has undertaken a number of different optimization trials and discusses the choice of good default values for the control parameters of G.A.'s.

In this work, attention is focused on modifications to the vibratory response of a structure. Such a possibility has been investigated by a number of authors, sometimes utilizing the Genetic Algorithm to perform the modifications. For instance, Belegundu *et al*. [6] studied minimization of sound power in plates where the thickness was varied, while Keane [7] demonstrated control over the vibrational energy transmitted through a truss by variation of the lengths of the component beams. In the present work, the aim is to design a structure which has no, or reduced, resonance peaks in its vibration transmission characteristics over a given frequency band.

The need for this type of control is perhaps most keenly felt by the aerospace industry; e.g., in the design of satellites. A good example which shows the motivation for this is that of a telescope mounted on a satellite. The telescope is typically aligned by reaction wheels, which inevitably cause the whole structure to vibrate, if only slightly. It is, however, very sensitive to angular deviations, so that vibrations at its mounting points must be restricted to frequencies lower than the exposure time, frequencies high enough to have very low response amplitudes or those which are not excited by the central system. For most terrestrial applications, structural mass is not usually the limiting factor in design and vibration problems can be mitigated by siting vibration sensitive equipment on a very massive slab. This is clearly not an option for an aerospace structure. Another common form of noise control is damping. Within the atmosphere, there will always be a certain amount of damping, regardless of the materials that the structure is built from, due to the viscous drag of the air surrounding it. This damping can be augmented by applying viscoelastic coatings to many of the vibrating parts, but this is at the cost of increasing the overall mass. Neither of these approaches can readily be utilized for space systems. Clearly, structural redesign, to gain improved passive noise performance, might be a fruitful area for investigation.

2. THE CASE STUDY

The structure chosen for study here is very simple, but may be considered as a highly idealized part of a more complex design. A long thin rectangular plate with aspect ratio 3·5:1, simply supported on all four sides, is considered. This plate is forced at one end and its response calculated at the other. Because of its elongated shape these responses may be considered as indicative of the plate's ability to act as a vibration filter: i.e., if vibrational energy may be prevented from passing from one end to the other by modifications to the plate, similar approaches might work for more complex geometries over similar distances. Here the modifications consist of 50 small masses placed on the plate between the drive and response points. The total mass of these additions is some 10% of the plate mass, so the physical effect is that of a plate with slight density variation. The plate and mass parameters used for the computations are given in Table 1. The frequency band over which the vibrations are to be suppressed is chosen here as 100–110 Hz, which contains modes 37–42 of the bare plate. The method for calculating the frequency response of the loaded plate has been given by McMillan and Keane [8]. In this method the plate is modelled by Kirchhoff's thin plate theory with a simple velocity damping term and the masses are considered as frequency dependent point forces.

TABLE 1

Plate parameters

Young's modulus, E (GPa)	206·8
Poisson's ratio, ν	0·29
Density, ρ (kg/m^3)	7820
Plate thickness, h (m)	0·01
Length, a (m)	7
Breadth, b (m)	2
Forcing points (x_i, y_i) (m)	(0·859, 1·141)
	(1·193, 1·052)
	(0·948, 0·807)
Response points (x_o, y_o) (m)	(6·141, 0·859)
	(5·807, 0·948)
	(6·052, 1·193)

It was found previously [8] that in such cases there is some advantage to restricting the added masses to lie on the node lines (zeros) of the highest eigenfunction in the band of interest, so that this eigenfunction remains unaffected by the masses. The addition of a few small masses to the plate then distorts the other mode shapes of the plate, although they are still rather similar in form to the mode shapes of the plate without masses. The frequencies of these modes will drop for each successive mass added. However, since the highest mode shape in the band is not distorted, its frequency will remain fixed. This leads to a reduced modal density across the band, since the eigenvalues originally lying above the band being considered remain greater than that of the highest mode in the band. Unfortunately, as the number of added masses increases, there is a possibility that the next highest mode will be sufficiently distorted that its frequency will fall below that of the top one in the band. This reordering of the modes, or leap-frogging, could be avoided if the masses were sited very close to, but not on, the node lines of the top mode.

In the case being considered here, the highest mode originally in the chosen band of interest is the 42nd. As masses are added close to its node lines it would be slightly affected and its frequency would gradually drop. In the limiting case of the masses being placed arbitrarily close to the node lines, after the positioning of a critical number of masses, the 42nd mode would suddenly flip to a lower frequency and different mode shape and the 43rd would become similar to the previous 42nd, which is precisely what has already been described above. Despite the possibility of leap-frogging, this positioning of masses does form a useful bottleneck against eigenfrequencies dropping down into the band of interest. Furthermore, an effective optimization method might also seek to avoid this. Note, however, that this kind of approach pays no attention to variations in the mode shapes of the modes in the band of interest. Clearly, such mode shapes affect the forced response of the plate and could therefore be used by an optimizer to further reduce vibration transmission, albeit at less significant level. This approach is not pursued further here although, as is noted later, the effect is observed in some of the results presented.

When applying an optimizer to the question of the optimum mass positions, careful consideration must be given to the objective function; i.e., the function by which each mass position set is graded. In this case, the average of the frequency responses at frequencies of 100·5, 101·5, 102·5, ... , 109·5 Hz is taken over three points on the plate when subjected to white noise forcing at three other points. As has already been noted these forcing and response points were chosen to lie at opposite ends of a rather long and narrow plate so that the objective function then measures the ease of vibration transmission along the plate. Additionally, the points were placed at the corners of equilateral triangles with the distance between them being approximately half the wavelength of a travelling wave of frequency between 100 and 110 Hz. This makes it impossible for the optimizer to choose mass positions which led to all the forcing or response points occurring at nodes. This leads to sets of mass positions which give isolation characteristics which are not overly sensitive to the exact forcing and response points chosen for study.

3. SEQUENTIAL POSITIONING

In the previous section, some general comments have been made concerning optimization of the added mass positions. Restricting the possible positions to the node lines of the 42nd mode has the effect of making the position of each mass a function of one variable (ξ) rather then two (x, y), namely

$$\xi = \left\{ \begin{array}{lll} x, & \text{for } 0 < x < 7 & \text{and} \quad y = 0{\cdot}5 \\ x + 7, & \text{for } 0 < x < 7 & \text{and} \quad y = 1{\cdot}0 \\ x + 14, & \text{for } 0 < x < 7 & \text{and} \quad y = 1{\cdot}5 \\ y + 21, & \text{for } x = 1{\cdot}4 & \text{and} \quad 0 < y < 2 \\ y + 23, & \text{for } x = 2{\cdot}8 & \text{and} \quad 0 < y < 2 \\ y + 25, & \text{for } x = 4{\cdot}2 & \text{and} \quad 0 < y < 2 \\ y + 27, & \text{for } x = 5{\cdot}6 & \text{and} \quad 0 < y < 2 \end{array} \right\},$$

which simplifies the problem considerably.

The basis of the optimization method proposed here, is that it starts with a given structure (the plate without any added masses) and improves it slightly (by adding just one mass). This new improved structure is then further improved (by the addition of another mass), and so on. Although this methodology can be followed until all available masses are placed, this solution may not be the absolute optimum, since positioning two masses at the same time may achieve a better optimum than two masses applied one after the other, for example. For this reason, it was decided also to use an alternative method, here a Genetic Algorithm, to make comparisons between the qualitative and quantitative results of each, while in both cases positioning the mases to lie on the nodal lines of the 42nd mode.

To make the problem more tractable when positioning the masses sequentially, the possible locations were discretized, so that they lay at 0·05 m intervals. The objective function was then calculated for the plate carrying just one mass, for each allowable position. The optimum position of that mass was then taken to be the position which gave the lowest objective function. A second mass was added, and its optimum position was found, and so on for all the masses. The variations of objective function with mass position for the addition of each mass are shown in Figures 1(a)–(g). It is important to note that none of the curves cross, which indicates that regardless of where the $(n+1)$th mass is placed, the result is better than it would have been if the nth mass had been put there, and no $(n+1)$th mass used.

In Figure 2 are shown the effects on the objective function of the first, eighth, 15th, 22nd, 29th, 36th, 43rd and 50th masses, and it can be seen that a law of diminishing

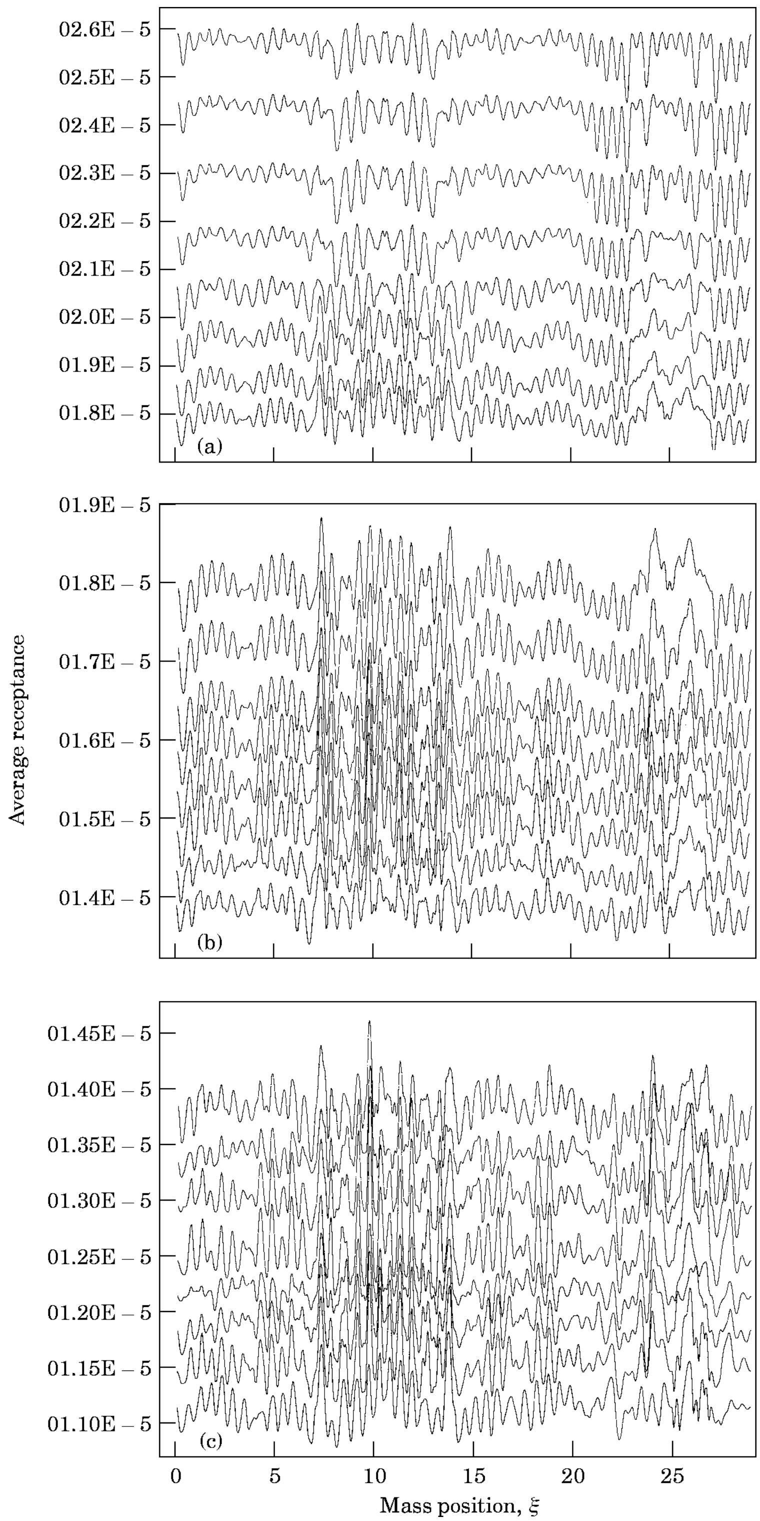

Figure 1(a to c).

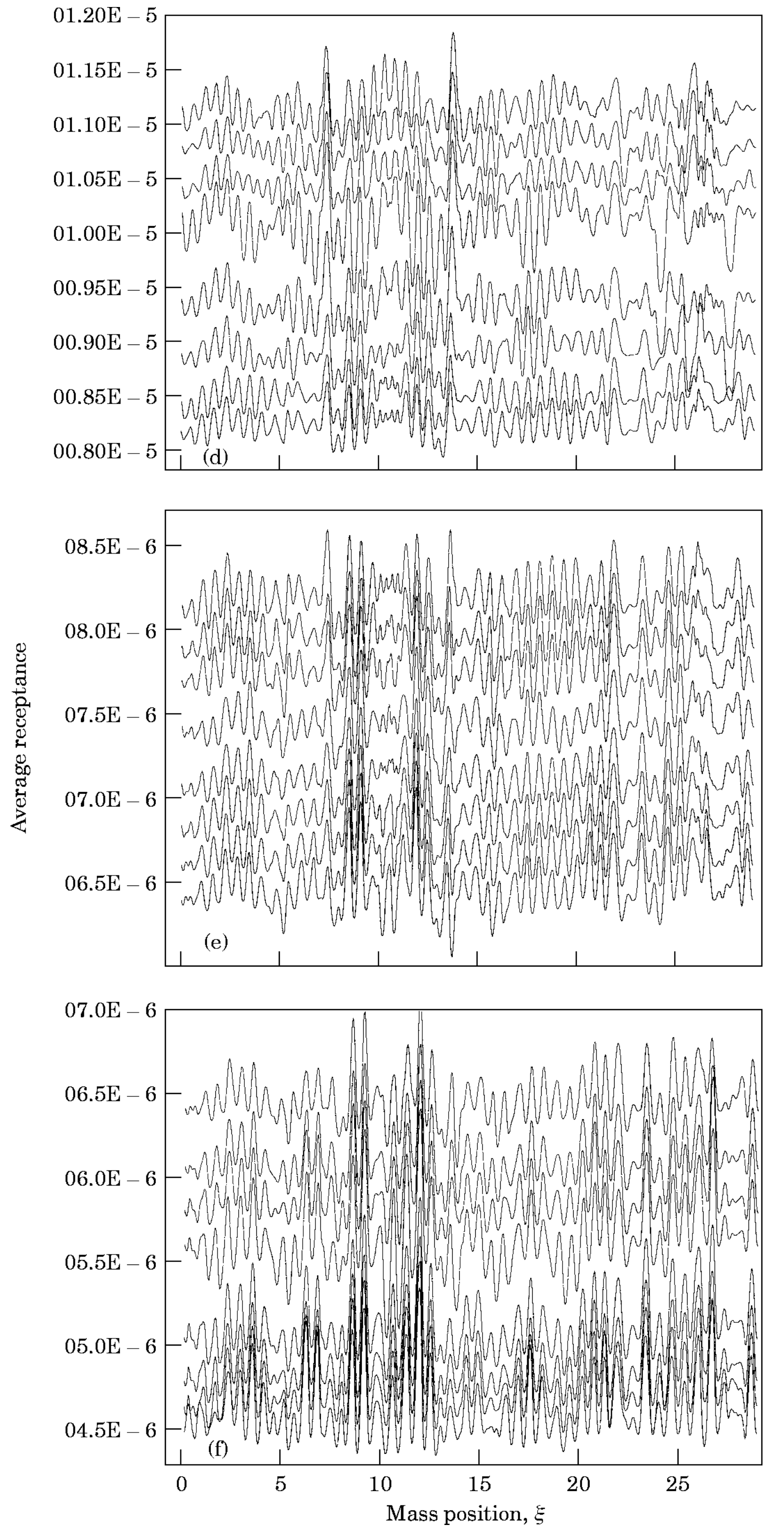

Figure 1(d to f).

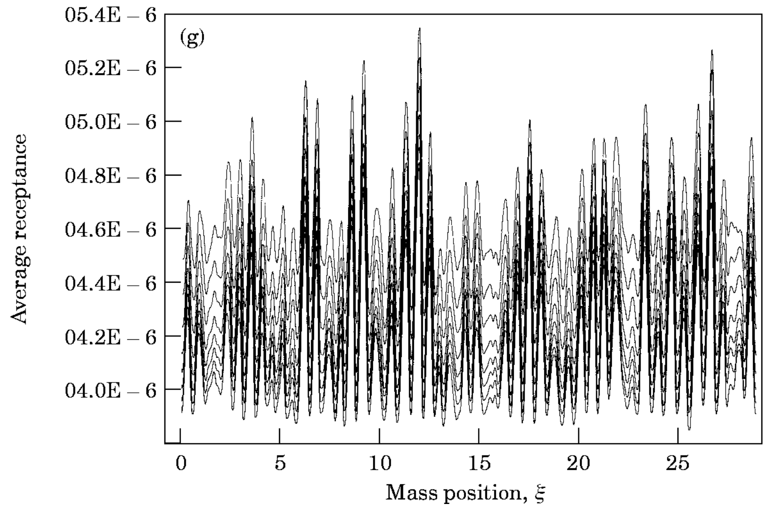

Figure 1. Variation of spatially and frequency averaged response of plate with the mass position (ξ): sequential positioning; (a) 1st–8th masses; (b) 8th–15th masses; (c) 15th–22nd masses; (d) 22nd–29th masses; (e) 29th–36th masses; (f) 36th–43rd masses; (g) 43rd–50th masses.

returns is operating. In Figure 3 is shown the objective function for the addition of each individual mass, which demonstrates much the same thing. However, a closer examination reveals some interesting details: there are two kinks in the otherwise smooth curve, which correspond to the addition of the 25th and 39th masses. Study of the individual traces for the 24th and 25th masses reveals minima occurring at the same points but that the minima for the 25th mass are significantly deeper. It would seem that, on the addition of the 25th mass, one or more of the resonances are shifted down out of the 100–110 Hz band, thereby suddenly making the structure more readily optimized. A similar explanation applies for the addition of the 39th mass.

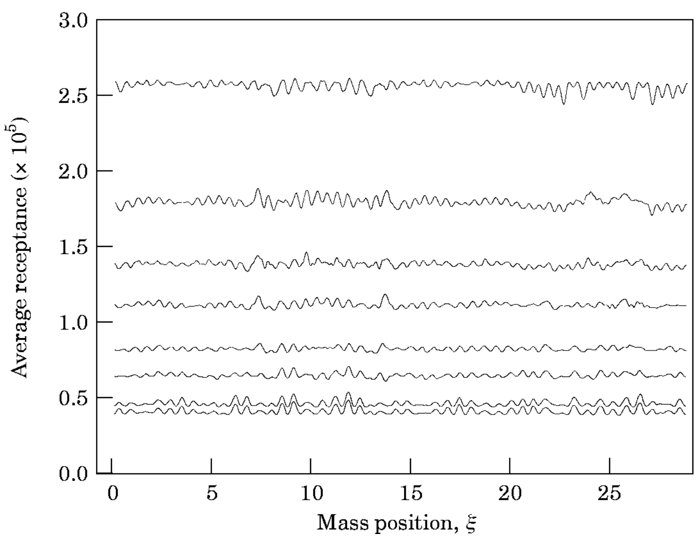

Figure 2. Variation of the spatially and frequency averaged response of plate with the mass position (ξ) for the 1st, 8th, 15th, 22nd, 29th, 36th, 43rd and 50th masses: sequential positioning.

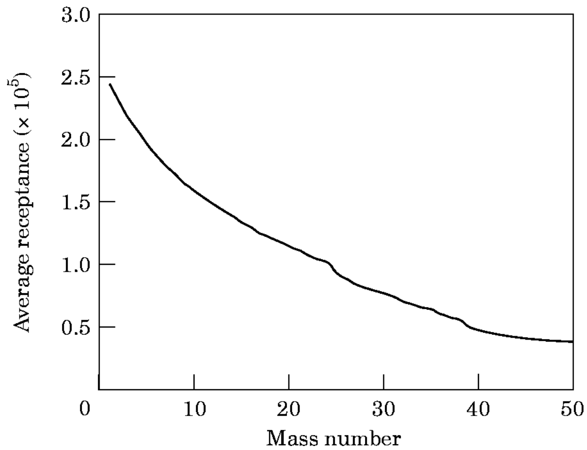

Figure 3. Variation of the spatially and frequency averaged response of plate with the number of optimally positioned masses: sequential positioning.

4. GENETIC ALGORITHM

As has already been noted, a Genetic Algorithm optimization of the mass positions was performed to provide a comparison for the proposed sequential positioning strategy. This utilized the same objective function, again restricting the masses to lie on the node lines of the 42nd mode, but now without discretizing the mass positions. The G.A. used is fairly typical of those discussed by Goldberg [3] but encompasses a number of new ideas that are particularly suited to engineering design problems [4, 9]. Such methods work by maintaining a pool or population of competing designs which are combined to find improved solutions. In their basic form, each member of the population is represented by a binary string that encodes the variables characterizing the design. The search progresses by manipulating the strings in the pool to provide new generations of designs, hopefully with better properties on average than their predecessors. The processes that are used to seek these improved designs are set up to mimic those of natural selection: hence the method's name. The most commonly used operations are currently as follows: (i) selection according to fitness, i.e., the most promising designs are given a bigger share of the next generation; (ii) crossover, where portions of two good designs, chosen at random, are used to form a new design, i.e., two parents "breed" an "offspring"; (iii) inversion, whereby the genetic encoding of a design is modified so that subsequent crossover operations affect different aspects of the design; (iv) mutation, where small but random changes are arbitrarily introduced into a design. In addition, the number of generations and their sizes must be chosen, as must a method for dealing with constraints (usually by application of a penalty function).

The implementation of the G.A. used here works with up to 16 bit binary encoding (default 12). The settings adopted for the control parameters are those previously found to give robust performance when tested over a number of different optimization problems [5]. The code uses an elitist survival strategy which ensures that the best of each generation always enters the next generation and has optional niche forming to prevent dominance by a few moderately successful designs preventing wide ranging searches. Two penalty functions are available to deal with constraints. The main parameters used to control the method may be summarized as follows: N_{gen}, the number of generations allowed (here 20); N_{pop}, the population size or number of trials used per generation which is therefore inversely related to the number of generations given a fixed number of trials in total (here 500); P[best], the proportion of the population that survive to the next generation (here 0·8); P[cross], the proportion of the surviving population that are allowed to breed (here 0·8); P[invert], the proportion of this population that have their genetic material re-ordered (here 0·5); P[mutation], the proportion of the new generation's genetic material that is randomly changed (here 0·005); a Proportionality Flag which selects whether the new generation is biased in favour of the most successful members of the previous generation or alternatively if all P[best] survivors are propagated equally (here TRUE); the Penalty Function choice.

When using the G.A. to explore large design spaces with many variables, it has also been found that the method must be prevented from being dominated by a few moderately good designs which prevent further innovation. A number of methods have been proposed to deal with this problem; that used here is based on MacQueen's Adaptive KMEAN algorithm [10] which has been applied with some success to multi-peak problems [11]. This algorithm subdivides the population into clusters that have similar properties. The members of each cluster are then penalized according to how many members the cluster has and how far it lies from the cluster centre. It also, optionally, restricts the crossover process that forms the heart of the G.A., so that large successful clusters mix solely with themselves. This aids convergence of the method, since radical new ideas are prevented from contaminating such sub-pools. The version of the algorithm used here is controlled by the following: D_{min}, the minimum non-dimensional Euclidean distance between cluster centres, with clusters closer than this being collapsed (here 0·05); D_{max}, the maximum non-dimensional Euclidean radius of a cluster, beyond which clusters subdivide (here 0·1); N_{clust}, the initial number of clusters into which a generation is divided (here 25); N_{breed}, the minimum number of members in a cluster before exclusive inbreeding within the cluster takes place (here 5); α, the penalizing index for cluster members, which determines how severely members sharing an over-crowded niche will suffer, with large numbers giving greater penalty (here 0·5), i.e., the objective functions of members of a cluster of m solutions are scaled by $m^{min(\alpha,1)}\,[1-(E/D_{max})^{\alpha}]+(E/D_{max})^{\alpha}$, where E is the Euclidean distance of the member from its cluster centre (which is always less than D_{max}; moreover, when $E=D_{max}$ no penalty is applied). In addition, the implementation of the G.A. used allows the analysis of individual members of the population to be run in parallel if a multiple processor computer or cluster of computers is available.

Unlike the method described in the previous section, the G.A. controls the full set of 50 mass positions simultaneously. Since there are 50 variables, it was necessary to use a large generation size (500) to explore this problem adequately. The number of generations used by the method was restricted to 20, so that the total computational expense of the two different approaches would be comparable. The initial choice of positions used to begin the optimization process for the masses was rather regular in that the masses were placed at 0·5 m intervals in position space, which give a very poor value for the objective function. (Indeed, the results described by Keane [7] for vibration suppression in truss structures showed that regular structures often exhibited rather poor vibrational behaviour and that small random deviation from such spacing often gave an improvement.)

Figure 4 shows the objective function for each successive generation in this case. This graph is typical of a G.A. result, showing an initial very steep decline followed by a slow descent. At the end of the 20th generation, it seems unlikely that any significant further improvement would be found, although such changes cannot be ruled out.

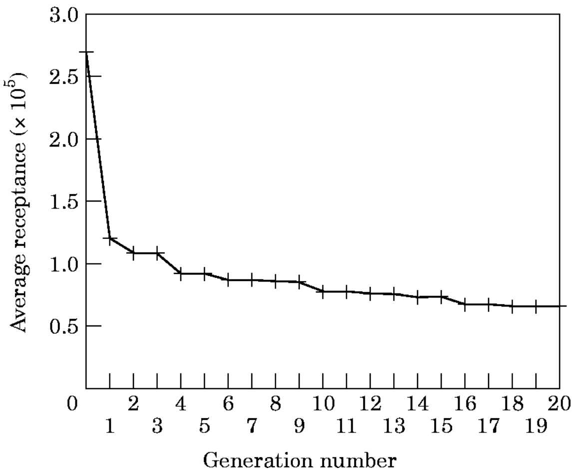

Figure 4. Variation of the spatially and frequency averaged response of plate with the generation number: Genetic Algorithm.

5. DISCUSSION

Comparison of the two design methods studied here (see Table 2) shows that the sequential mass positioning method eventually achieved the greatest vibration suppression, although the G.A. result was not far behind. In Figures 5 and 6 is shown the frequency response of the plates with forcing and response points as used by the optimizers and mass positions generated by the sequential method and the Genetic Algorithm respectively, and in both cases at various stages of the optimization procedures.

It may be seen that the G.A. achieved the bulk of its improvement within the first few generations. However, it is worth noting that the computational effort of calculating the objective function increases with the square of the number of masses in use. This means that the mass positioning method described earlier is very efficient to begin with but becomes rapidly more expensive as each additional mass is added. Thus, the degree of noise isolation achieved by either method per unit of computer effort would be rather similar. The G.A. has the combined advantage and disadvantage that all 50 masses are being used at each stage and so the optimization process may be terminated at any stage, whereas the sequential method showed continual improvement up to the point where the 50th mass was added (and further improvement may be obtained if yet more masses were added). In this case, perhaps a faster solution may be obtained if a smaller number of larger masses were used, although it would be unlikely that such a low objective function would be obtained.

Perhaps the greatest difference between these methods is in the type of results that are obtained. Recall that the same objective function was used for both methods, but that the G.A. was allowed more flexibility in choosing precise positions. The positions of the masses at various stages throughout both optimization procedures were recorded and these were then used to generate frequency response curves over the band 100–110 Hz with just one, different, forcing point and one, different, response point ((0·1, 0·1) and (6·9, 1·9), respectively). This procedure measures the robustness of the designs for vibration control. In Figure 7 are shown the results for 0, 10, 20, 30, 40 and 50 masses added sequentially in their optimal positions. It may be seen that the resonance near 108 Hz has been steadily pushed back to around 105 Hz, and that the response to the

TABLE 2

Optimized positions of the masses

Sequencial positioning: Co-ordinates	Sequencial positioning: No. of coincident masses	Genetic Algorithm: co-ordinates	
(3·20, 1·00)	2	(5·889, 1·000)	(3·198, 1·000)
(1·80, 1·00)	3	(2·800, 1·326)	(3·038, 0·500)
(4·20, 0·60)	1	(5·733, 1·000)	(5·600, 1·341)
(1·40, 1·75)	4	(5·085, 1·500)	(4·661, 1·500)
(5·60, 0·25)	3	(5·600, 0·669)	(5·984, 0·500)
(1·05, 1·00)	1	(2·726, 0·500)	(5·600, 0·640)
(1·10, 1·00)	2	(4·200, 0·926)	(6·685, 0·500)
(1·40, 1·30)	1	(4·702, 0·500)	(2·800, 1·191)
(0·30, 0·50)	1	(4·200, 0·856)	(3·258, 0·500)
(2·80, 025·)	1	(1·400, 0·160)	(5·212, 0·500)
(6·00, 1·00)	1	(1·176, 0·500)	(4·250, 0·500)
(0·25, 1·50)	1	(4·200, 0·608)	(2·430, 1·500)
(6·75, 0·50)	1	(3·927, 1·000)	(3·974, 1·500)
(2·80, 0·70)	2	(1·488, 1·500)	(2·433, 1·000)
(1·40, 1·35)	3	(1·155, 1·500)	(1·909, 1·000)
(6·05, 1·00)	1	(2·504, 1·000)	(3·209, 1·500)
(1·75, 1·00)	1	(2·800, 1·928)	(4·200, 1·358)
(5·25, 1·00)	2	(5·351, 1·000)	(3·269, 1·000)
(2·80, 1·25)	1	(1·400, 1·280)	–
(5·60, 0·75)	1	(5·600, 0·286)	–
(5·20, 1·00)	1	(1·198, 1·500)	–
(6·25, 1·00)	1	(4·200, 0·700)	–
(1·15, 1·00)	1	(5·167, 1·000)	–
(1·80, 1·50)	1	(5·496, 1·500)	–
(6·15, 1·00)	4	(1·827, 0·500)	–
(5·80, 1·00)	1	(4·356, 1·500)	–
(6·75, 1·00)	1	(3·209, 1·500)	–
(3·85, 1·00)	2	(5·600, 0·669)	–
(3·25, 1·00)	1	(5·280, 1·000)	–
(2·80, 1·30)	1	(4·448, 1·500)	–
(2·80, 0·75)	1	(4·427, 1·500)	–
(5·75, 1·00)	2	(5·273, 1·000)	–

lower resonances is very low from 30 masses onwards. This is rather surprising, since the forcing and response points were deliberately chosen to be in quite different positions to those used for the optimization, although still at opposite ends of the plate. In spite of this success, it may be seen that some leap-frogging of higher modes has occurred, with the effect that a spike is formed at around 109 Hz.

The G.A. results shown in Figure 8 are quite different in nature. After just eight generations, there seem to be no resonances between about 104·5 and 110 Hz, and there seems to be no evidence of leap-frogging. However, the frequency responses for all generations beyond the 8th seem to peak at around 104 Hz, rather than leaving a gently undulating low.

An I-DEAS [12] finite element vibration analysis was also performed for the two optimized plate systems. The main part of the grid was formed with some 2800 three

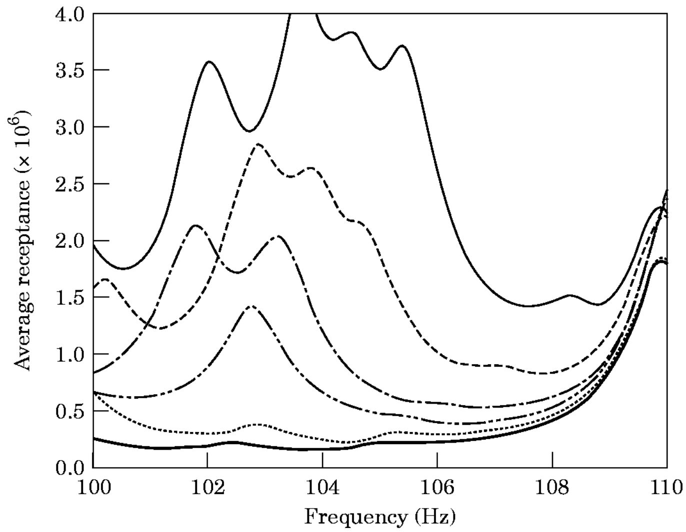

Figure 5. The spatially averaged frequency response of the plate with an optimally positioned masses: sequential positioning: ———, No masses; - - - - -, 10 masses; -·-·-·-·, 20 masses; -··-··-··-··, 30 masses; - - - - - - - -, 40 masses; ———, 50 masses.

noded-triangular elements, and was distorted locally to enable the masses to be modelled as lumped masses at the plate element nodes. Triangular plate elements were chosen, as these are based on Kirchhoff plate theory rather than Mindlin thick plate theory, which is the default option for quadrilateral elements. In Figures 9(a)–(c) are shown the mode shapes with frequencies lying in the range 100–110 Hz for the plate with masses positioned by the sequential method. The eigennumbers for these run from 40 to 42, confirming that only one mode has been able to leapfrog the original 42nd. The modes of this plate system are characterized by having localized areas of low and high deflection

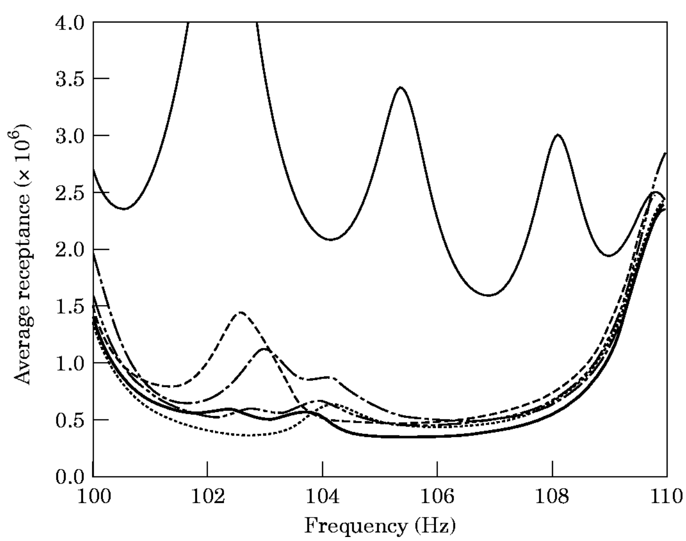

Figure 6. The spatially averaged frequency response of the plate with 50 masses: Genetic Algorithm. ———, Initial positions; - - - - -, after four generations; -·-·-·-·, after eight generations; -··-··-··-··, after 12 generations; - - - - - -, after 16 generations; ———, after 20 generations.

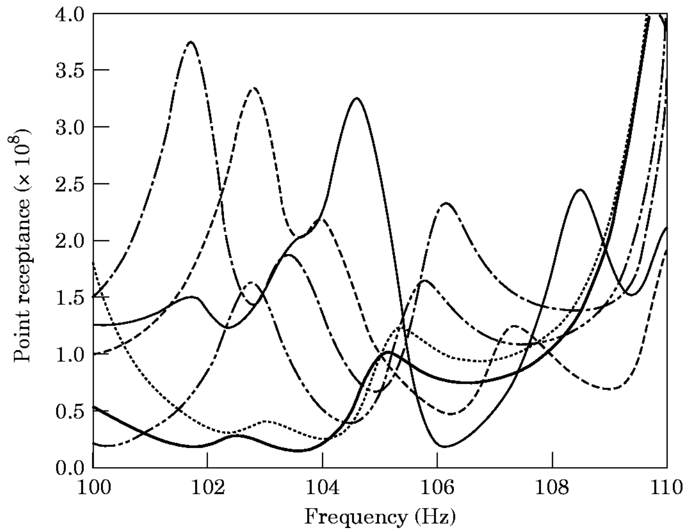

Figure 7. The frequency response of the plate at (6·9, 1·9) due to forcing at (0·1, 0·1) with optimally positioned masses; sequential positioning. Key as Figure 5.

maxima, so it seems that in this case the noise isolation is achieved, at least in some small part, by sculpting of the mode shapes. The corresponding mode shapes of the plate system where the masses were positioned by the G.A. are shown in Figures 10(a)–(c). In this case the eigennumbers run from 39 to 41. The maxima of these mode-shapes are generally much more uniform, exhibiting many peaks of a similar height. It would seem that here the noise isolations is achieved only by reducing the number of modes in the frequency band of interest.

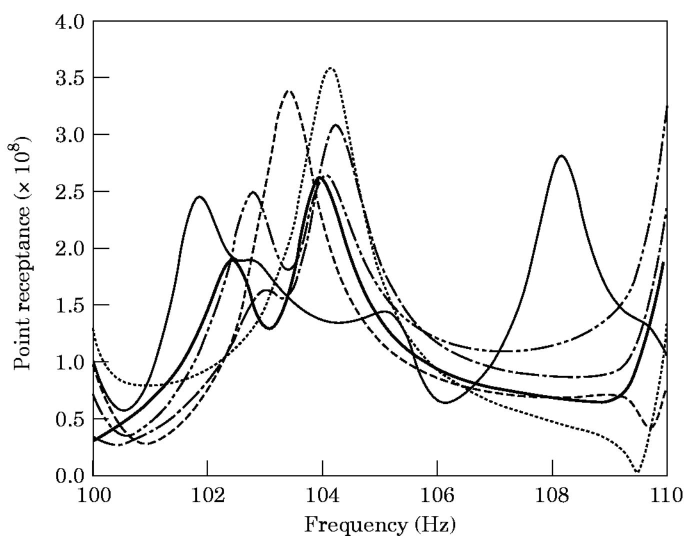

Figure 8. The frequency response of the plate at (6·9, 1·9) due to forcing at (0·1, 0·1) with 50 masses; Genetic Algorithm. Key as Figure 6.

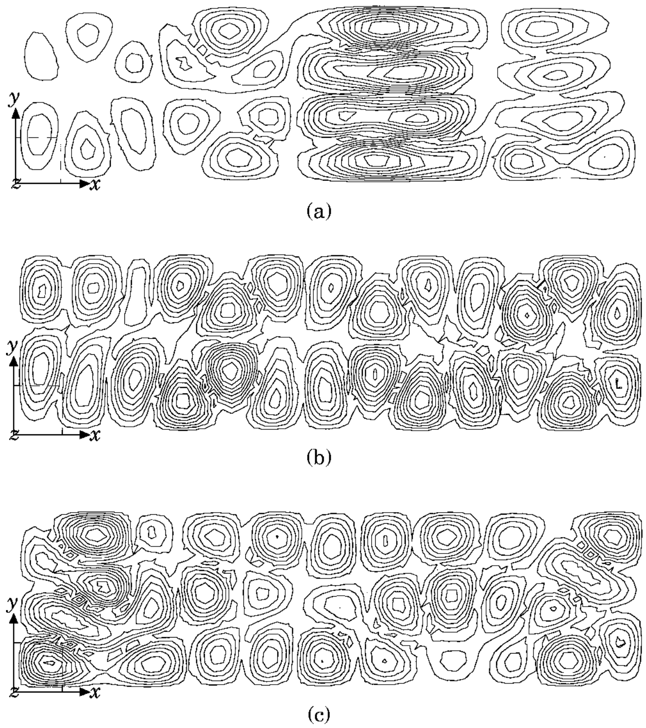

Figure 9. (a) Mode shape 40 of the plate with 50 masses, 102·68 Hz, sequential positioning; (b) mode shape 41 of the plate with 50 masses, 104·81 Hz, sequential positioning; (c) mode shape 42 of the plate with 50 masses, 109·45 Hz, sequential positioning.

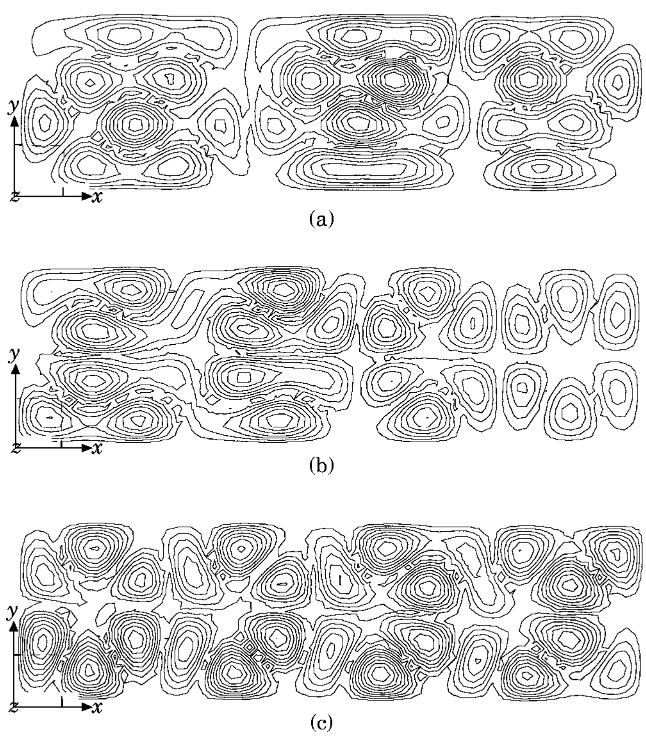

Figure 10. (a) Mode shape 39 of the plate with 50 masses, 100·40 Hz, Genetic Algorithm; (b) mode shape 40 of the plate with 50 masses, 102·94 Hz, Genetic Algorithm; (c) mode shape 41 of the plate with 50 masses, 104·20 Hz, Genetic Algorithm.

6. CONCLUSIONS

In this paper, it has been shown that it is possible to achieve a certain amount of vibration isolation in a long narrow plate by loading it with a number of small point masses. A novel method for choosing the mass positions has been described and shown to be at least as good as using a Genetic Algorithm for this task when the masses are constrained to lie on the nodal lines of the highest mode studied. For the example chosen, a factor of five reduction in the frequency and spatially averaged response amplitude has been achieved using 50 masses, each of around 0·2% of the total mass of the plate. Moreover, the isolation has been shown to be tolerant of variations in the forcing and response points used to measure the vibration transmission characteristics of the plate. It is also worth noting that it is quite likely that 50 individual masses are not sufficient to move the original six eigenfrequencies from out of the band studied.

Finally, it would seem from the studies reported here that if the main objective of the search were to widen a gap between modes, the G.A. method would be the more successful. On the other hand, for a wide frequency band, over which a low response is required, the individual positioning of masses would seem to be the more fruitful method. However, the band considered should be somewhat wider than that actually desired to make room for the leap-frogging phenomena. Some modifications to the objective function, to penalize localized humps, etc., could also be added. In both cases it would also be possible to exploit mode shapes modifications directly by allowing the added masses to be positioned over the whole area of the plate rather than simply on or near the modal lines of the highest mode of interest. This approach would lead to significantly greater computational costs when using the sequential positioning scheme, however, and in such cases the genetic algorithm would probably be the more useful method.

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