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A NEW BOUNDARY INTEGRAL FORMULATION FOR THE PREDICTION OF SOUND RADIATION

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A new boundary integral formulation is presented for the evaluation of the noise radiated in a uniform medium by generic sources. To use the method one requires knowledge of the pressure, velocity, and density disturbances on a smooth closed surface surrounding the source, and to assume that the propagation is linear outside the surface itself. When applied to the prediction of transonic rotor noise the method can be used in the same manner as the Kirchhoff approach, but the new integral equations are derived without requiring the non-penetration condition in the Ffowcs Williams–Hawkings equation. The method is therefore referred as The Kirchhoff–FWH. The main advantage of the proposed formulation in respect of the Kirchhoff method is that it does not require the knowledge of the surface pressure normal derivative. Additionally, it can be applied also for bodies with permeable surfaces, while the classical FWH equation is not valid in this case. Two different formulations are presented, which differ in the way in which a time derivative is handled, and some general issues on the numerical efficiency of the two formulations are addressed. Comparisons with experiments, and with Kirchhoff and FWH methods, are presented for a hovering rotor in transonic conditions at various tip Mach numbers.

1. INTRODUCTION

In recent years, reduction of helicopter external noise has received a great deal of attention from industry, because of the more stringent certification rules and the increased sensitivity of the community and operators. The availability of fast and robust prediction codes is clearly a required step towards the development of quieter helicopters. Nowadays two different large groups of methods are available, one based on the Computational AeroAcoustics approach (CAA) and the other based on integral formulations. The first method permits one to solve at the same time both the aerodynamic and aeroacoustic problems, and is based on the solution of the fluid motion equations by classical field methods (finite volume, finite difference and finite elements) [1]. The main problem of CAA is that, in order to avoid the introduction of excessive dissipation, the required computer resources increase greatly with observer distance, and nowadays the solution can be obtained at a reasonable cost only for observers at a distance of about three times the rotor radius. The distances that are usually required in realistic calculations are, however, two or three orders of magnitude greater than the rotor radius, and, even with increase in computer speed, it is certainly not practical to apply CAA methods directly for these distances. The integral methods, instead, require knowledge of the aerodynamic flow field around the rotor, and permit one to obtain the acoustic pressure at any point of the field by executing a certain number of integrals. One of the interesting aspects of integral methods is that the required computational time is independent of the observer distance.

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Typical calculations of rotor noise are therefore executed in two steps; in the first one an aerodynamic code based on CFD/CAA or BEM methods is used to evaluate the aerodynamic field, and then an integral formulation is applied to propagate the pressure disturbance to the far field. It is important to note that the computational time required by the integral methods is usually much lower than the time required to obtain the aerodynamic solution. Several integral methods for acoustic applications are nowadays available including the classical Ffowcs Williams–Hawkings (FWH) and Kirchhoff equations, together with other methods based on the velocity potential, or on linear and non-linear versions of the FWH and Kirchhoff equations (see, e.g., references [2–4]). The FWH formulation (see, e.g., references [5–7]) permits one to compute the pressure disturbance at any point of the field once some aerodynamic quantities are known on the blade surface and in the volume around it. Sometimes FWH is referred to as a linear approach simply because in the great part of the implementations the volume quadrupole terms, that take into account the non-linearities, are neglected. However, once the volume terms are properly considered [8] good results can be obtained in transonic conditions (see, e.g., references [9–13]). On the other hand, the Kirchhoff formulation, as obtained by Farassat and Myers [14], permits one to solve linear wave propagation problems once some flow quantities are given on a closed fictitious surface surrounding the source. In order to be applied to transonic rotor noise [15–18], the surface has to be placed at a sufficient distance from the rotor in order to ensure that the propagation is governed by the linear wave equation outside the surface itself. The main advantage in respect to the FWH approach is that the acoustic calculation is generally faster, since only surface integrals have to be evaluated.

From a physical point of view it is important to realize that the Kirchhoff formula is valid for any phenomenon governed by the linear wave equation (optics, acoustics, electromagnetism, etc.), while the FWH equation is *specialized* for aeroacoustics problems. As a consequence, the Kirchhoff equation is written in terms of a single fluid quantity (the pressure disturbance $p' = p - p_0$), while FWH requires not only p' but also the fluid density ρ and the fluid perturbation velocity $\mathbf{u}$. In order to reconstruct the propagation, the Kirchhoff formulation requires some further information that is provided by knowledge of the pressure normal derivative $\partial p'/\partial n$. The necessity of specifying $\partial p'/\partial n$ can be a disadvantage for rotorcraft problems, since, if discontinuities are present, the numerical evaluation of $\partial p'/\partial n$ can introduce undesired smoothing. The other difference between the two formulations is that the surface integrals of the FWH equation are executed on a well defined physical surface (the surface of the blades), while the Kirchhoff surface is completely fictitious, being subject to the restrictions only of being smooth and of enclosing the source with all the non-linear terms. Except for the above limitations, the surface can be placed anywhere in the field, and can have a generic motion eventually different from the motion of the source itself. The degrees of freedom allowed in the definition of the Kirchhoff surface certainly represent an advantage over the FWH approach. For example, in calculation of high speed rotor noise in a delocalized condition, it is possible to use a nonrotating Kirchhoff surface in order to avoid problems with surfaces in supersonic motion.

A question now arises: Is it possible to develop an integral formulation specialized for aeroacoustics problems, but that permits the same flexibility of the Kirchhoff formulation? The answer is yes, and in this work such a new formulation is derived and applied to transonic rotor noise problems. Since the formulation combines aspects of both the FWH and Kirchhoff approaches, it is here referred as the Kirchhoff–FWH formulation (KFWH).

Two different formulations are presented which differ in the way in which time derivatives are handled, and which can be compared respectively with Farassat's formulations 1 and 1A. An analysis of the numerical efficiency of the two approaches shows how the methods that do not require any numerical evaluation of time derivatives (1A) are about twice as fast.

Finally, some comparisons with classical Kirchhoff, FWH and experimental results are shown for the UH-1H rotor in hover for tip Mach numbers up to 0·95.

2. THE FWH APPROACH

In order to obtain the new formulation, the derivations of the FWH and Kirchhoff equations are outlined here, with the aim of pointing out the differences and similarities between the two approaches.

Consider a generic body immersed in a fluid, and the surface S_b of which is described by the equation $f_b(\mathbf{x}, t) = 0$, where $f_b < 0$ for points inside the body (for simplicity, it is also assumed that the function f_b is scaled in such a way that $|\nabla(f_b)| = 1$ for $f_b = 0$). The problem can be modelled by replacing the body by fluid at rest ($p' = 0$, $\rho = \rho_0$, $\mathbf{u} = 0$), and the governing equations can be written as

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i}(\rho u_i) = 0, \qquad \frac{\partial}{\partial t}(\rho u_i) + \frac{\partial}{\partial x_j}(P_{ij} + \rho u_i u_j) = 0, \tag{1, 2}$$

where P_{ij} is the fluid compressive stress tensor, ρ is the density, and u_i is the fluid perturbation velocity. Equations (1) and (2) represent respectively mass and momentum conservation, and are valid, with the respective boundary conditions, in the two regions separated by the surface S_b. In order to obtain a single equation that is valid both for $f_b < 0$ and $f_b > 0$, the surface S_b has to be considered as a discontinuity surface, and all the fluid quantities have to be regarded as generalized functions. Exploiting the properties of generalized derivatives one can obtain a non-homogeneous version of the continuity equation that can be written as [5, 22].

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i}(\rho u_i) = \rho_0 u_n \, \delta(f_b) + (\rho - \rho_0)(u_n - v_n)\, \delta(f_b). \tag{3}$$

The second term on the right side disappears in the classical formulation, since the non-penetrating condition states that $(u_n - v_n) = 0$. In a similar way, the generalized version of the momentum equation can be obtained:

$$\frac{\partial}{\partial t}(\rho u_i) + \frac{\partial}{\partial x_j}(P_{ij} + \rho u_i u_j) = P'_{ij} n_j \, \delta(f_b) + (p u_i)(u_n - v_n)\, \delta(f_b). \tag{4}$$

Here $P'_{ij} = P_{ij} - p_0 \delta_{ij}$ is the perturbation stress tensor, and δ_{ij} is the Kronecker delta. Also, in this case, the second term on the right side vanishes, since flow is not allowed across S_b. It is now possible to assemble equations (3) and (4) by following a standard procedure (see, e.g., references [6, 19]). The first step is to take the generalized derivative of equation (4) with respect to x_i and to subtract the generalized time derivative of equation (3). Then the term $c^2 \partial^2 \rho / \partial x_i \partial x_i$ can be subtracted from the result of the previous operations, c being the constant speed of sound in the undisturbed medium. With some further manipulations, and considering that p_0 and ρ_0 are constant across S_b, the final form of the FWH equation can be written as

$$\Box^2 [c^2(\rho - \rho_0)] = \frac{\partial}{\partial t}[\rho_0 u_n \, \delta(f_b)] - \frac{\partial}{\partial x_i}[P'_{ij} n_j \, \delta(f_b)] + \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j}, \tag{5}$$

where $T_{ij} = P'_{ij} + \rho u_i u_j - c^2(\rho - \rho_0)\delta_{ij}$ is Lighthill's equivalent stress tensor. If the perturbations are small, the term $c^2(\rho - \rho_0)$ can be replaced by p', and therefore equation (5) can be used to evaluate the pressure disturbance. It must be pointed out that the hypothesis of small disturbances has to be valid only at the observer location, while no restriction in posed near the body. By using a standard Green function approach, equation (5) can be rewritten as an integral equation, in which the first two terms on the right side represent integrals on the surface S_b of the body (thickness and loading), and the last term generates a volume integral that describes the quadrupole contribution.

3. THE KIRCHHOFF APPROACH

In order to understand better the common aspects of the two approaches, one starts the derivation of the Kirchhoff formulation a little upstream of what is usually done. Also in this case a body B is considered, the surface S_b of which is described by the equation $f_b = 0$, and immersed in a fluid medium. The fluid motion is clearly governed by the continuity and momentum equations (1) and (2). Now consider a generic closed and smooth surface S of arbitrary shape and motion, defined by $f(\mathbf{x}, t) = 0$ ($|\nabla(f)| = 1$ for $f = 0$), and let us try to evaluate the noise radiated by the body B for observers placed outside S. If the surface S is far enough from the body B, then the fluid outside S can be considered to be inviscid, and the disturbances small. With these hypotheses, equations (1) and (2) can be rewritten as the standard wave equation

$$\frac{1}{c^2}\frac{\partial^2 p'}{\partial t^2} - \nabla^2 p' \equiv \Box^2 p' = 0. \tag{6}$$

The sound propagation outside S can therefore be modelled by replacing the volume inside S with fluid at rest ($p' = 0$), and introducing a discontinuity surface across S. At this point, by exploiting the properties of generalized derivatives, it is possible to obtain the non-homogeneous version of equation (6) [14], that is exactly the sought-for Kirchhoff equation:

$$\Box^2 p' = -\left(\frac{\partial p'}{\partial n} + \frac{M_n}{c}\frac{\partial p'}{\partial t}\right)\delta(f) - \frac{1}{c}\frac{\partial}{\partial t}[M_n p'\,\delta(f)] - \frac{\partial}{\partial x_i}[p' n_i\,\delta(f)]. \tag{7}$$

Here $\mathbf{n}$ is the unit vector normal to the surface S and pointing outwards, and $M_n = v_i n_i / c$ is the Mach number in the normal direction. The integral formulation can be easily obtained from equation (7) by using a Green function approach.

4. THE FIRST KFWH EQUATION

From the above derivations it is clear that the FWH and Kirchhoff formulations can be seen as different descriptions of the same phenomenon, since they can be obtained by starting from the same physical problem described by the same equations (1) and (2). The differences between the two formulations are due to some choices that are made in the derivation process. The first choice is that for the Kirchhoff equation some simplifying hypotheses are introduced in the early stages of derivation, while no assumption is made for the FWH equation. The second difference is that the discontinuity surface S is imposed to be coincident with the surface S_b of the body in the FWH equation, while no limitation is given for S in the Kirchhoff method. A new formulation, which combines the positive aspects of both the FWH and Kirchhoff approaches, can at this stage be obtained in a few steps, and the procedure for its derivation can be interpreted in two different ways. On the one hand, one can think of following the same approach used for the derivation of the FWH equation, using, however, a fictitious discontinuity surface S that is not necessarily coincident with S_b. On the other hand, one can think of starting from the continuity and momentum equations and following the same procedure used in the derivation of Kirchhoff formulation with the difference that the simplifying hypotheses are no longer introduced. Clearly, from a practical point of view the derivation is exactly the same. Starting from equations (1) and (2), one introduces therefore a generic discontinuity surface S, and replaces the volume inside S by fluid at rest ($p' = 0$, $\rho = 0$, $\mathbf{u} = 0$). The non-homogeneous versions of equations (1) and (2) are simply obtained from equations (3) and (4) once f_b is replaced by f. It is however, very important to note that, since the surface S is fictitious, the non-penetration condition is no longer required, and, in order to obtain correct results, one has to allow a fluid flow across S. In particular the flow can be due both to the fluid perturbation velocity ($u_n \neq 0$), and to the motion of the surface ($v_n \neq 0$), with the net flow across S being given by the difference $(u_n - v_n)$. Equations (3) and (4) can therefore be assembled by adopting the same procedure as used above except only that now the terms containing $(u_n - v_n) = 0$ can no longer be neglected. The result can be written as

$$\Box^2[c^2(\rho - \rho_0)] = \frac{\partial}{\partial t}[\rho_0 u_n\,\delta(f)] - \frac{\partial}{\partial x_i}[P'_{ij} n_j\,\delta(f)] + \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j} + \frac{\partial}{\partial t}[(\rho - \rho_0)(u_n - v_n)\,\delta(f)] - \frac{\partial}{\partial x_i}[\rho u_i (u_n - v_n)\,\delta(f)], \tag{8}$$

where $T_{ij} = P'_{ij} + \rho u_i u_j - c^2(\rho - \rho_0)\delta_{ij}$ is again Lighthill's equivalent stress tensor. Equation (8) can be interpreted as a modified version of the FWH equation extended to the case in which flow is allowed across the discontinuity surface. Clearly, if S is concident with the surface S_b of an impenetrable body, then the flow is zero and the classical FWH equation is obtained.

It is interesting to note that equation (8) can be rearranged in order to have the same formal aspect as the classical FWH equation. Upon defining the quantities U_i and L_{ij} as

$$U_i = u_i + [(\rho/\rho_0) - 1](u_i - v_i), \qquad L_{ij} = P'_{ij} + \rho u_i (u_j - v_j), \tag{9, 10}$$

equation (8) can be rewritten as

$$\Box^2[c^2(\rho - \rho_0)] = \frac{\partial}{\partial t}[\rho_0 U_n\,\delta(f)] - \frac{\partial}{\partial x_i}[L_{ij} n_j\,\delta(f)] + \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j}, \tag{11}$$

which is identical to the classical FWH equation if u_n is replaced with U_n and P'_{ij} with L_{ij}. The terms U_i and L_{ij} introduced here can be interpreted respectively as a modified velocity and a modified stress tensor, which take into account the flow across S. It is thus possible to conclude that the FWH equation is still valid for permeable surfaces if the modified velocity and stress tensor are introduced.

The Green function G of the unbounded three-dimensional space is defined as $G = \delta(g)/r$, where $r = \|\mathbf{x} - \mathbf{y}\|$, $g = t - \tau - r/c$, and where $\mathbf{x}$ and $\mathbf{y}$ represent, respectively, observer and source positions, and t and τ observer and source times. By executing a convolution of equation (8) with the Green function G, it is possible to recast the equation (11) in an integral form, that, for a non-deformable surface S, can be written as

$$4\pi c^2(\rho-\rho_0)=\frac{\partial}{\partial t}\int_S\left[\frac{\rho_0 u_n+(\rho-\rho_0)(u_n-v_n)}{r|1-M_r|}\right]_{ret}\mathrm{d}S$$
$$+\frac{1}{c}\frac{\partial}{\partial t}\int_S\left[\frac{P'_{nr}+\rho u_r(u_n-v_n)}{r|1-M_r|}\right]_{ret}\mathrm{d}S+\int_S\left[\frac{P'_{nr}+\rho u_r(u_n-v_n)}{r^2|1-M_r|}\right]_{ret}\mathrm{d}S$$
$$+\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\int_V\left[\frac{T_{rr}}{r|1-M_r|}\right]_{ret}\mathrm{d}V+\frac{1}{c}\frac{\partial}{\partial t}\int_V\left[\frac{3T_{rr}-T_{ii}}{r^2|1-M_r|}\right]_{ret}\mathrm{d}V$$
$$+\int_V\left[\frac{3T_{rr}-T_{ii}}{r^3|1-M_r|}\right]_{ret}\mathrm{d}V, \tag{12}$$

where $M_r=v_i r_i/c$ is the Mach number in the observer direction, $T_{rr}=T_{ij}r_i r_j$, and $T_{ii}=T_{11}+T_{22}+T_{33}$. Also, V is the volume external to the surface S ($f>0$), and the symbol $[]_{ret}$ means, as usual, evaluation at the retarded time $\tau^*=t-r/c$. In order to obtain equations (12), the formula [7, 20]

$$\frac{\partial}{\partial x_i}\left[\frac{\delta(g)}{r}\right]_{ret}=-\frac{1}{c}\frac{\partial}{\partial t}\left[\frac{r_i\,\delta(g)}{r^2}\right]_{ret}-\frac{r_i\,\delta(g)}{r^3} \tag{13}$$

has been used to transform the space derivative, which appears in the loading term into time derivative. A similar, but more complex expression, that was first obtained by Farassat and Brentner [8], was also used to execute the analogue transformation for the quadrupole term. Equation (12) can be seen as a boundary integral equation that, for any point external to a generically moving surface S, relates the density disturbance to the values of pressure, velocity and density on the surface itself, and to the Lighthill stress tensor in the volume external to the surface. This equation has been derived directly from the equations of conservation of mass and momentum without any further assumptions, and so can be applied to a generic surface independently of whether or not the propagation is linear outside the surface. If the surface S is placed on the body, and the body itself is assumed to be impenetrable, then the classical FWH equation is obtained and the non-linear propagation effects are taken into account by the quadrupole volume terms. By moving the surface S away from the body a sort of *mixed* formulation is obtained, in which part of the non-linearities is taken into account by the quadrupole volume terms, and part by the surface integrals. At the end, if the surface is far enough from the body, the Lighthill stress tensor outside S can be neglected and, by using the relation $c^2(\rho-\rho_0)=p'$, valid if perturbations are small, equation (12) can be rewritten as

$$4\pi p'=\frac{\partial}{\partial t}\int_S\left[\frac{\rho_0 u_n+(\rho-\rho_0)(u_n-v_n)}{r|1-M_r|}\right]_{ret}\mathrm{d}S$$
$$+\frac{1}{c}\frac{\partial}{\partial t}\int_S\left[\frac{P'_{nr}+\rho u_r(u_n-v_n)}{r|1-M_r|}\right]_{ret}\mathrm{d}S+\int_S\left[\frac{P'_{nr}+\rho u_r(u_n-v_n)}{r^2|1-M_r|}\right]_{ret}\mathrm{d}S. \tag{14}$$

This formula, together with equation (19), is the main result of this paper, and is here referred to as the first Kirchhoff–FWH equation (KFWH).

5. ANALYSIS OF THE FORMULATION

As has been seen, two hypotheses are required for the simplification of equation (12) into equation (14): the first assumption requires that the quadrupole term outside S be negligible, and the second one requires that the disturbances near the observer be small enough, in such a way that $c^2(\rho - \rho_0)$ can be replaced by p'. In order to neglect the quadrupole terms the perturbations, however, have to be small in all of the volume V, and this is certainly the more restrictive assumption Compared with those of the classical Kirchhoff approach, the hypotheses required seem to be similar, since both of the formulations can be used only in the case of small perturbations. It is important to point out that the complete KFWH formulation (equation 12), including also the volume terms, is valid also in the case of non-small perturbations; allowing, for example, the presence of a gross fluid flow across S. However, if one wants to neglect the volume terms one has to impose the small perturbation hypothesis, restricting in this way the validity of the formulation to the case of negligible fluid flow, as happened in the classical Kirchhoff formulation. It is interesting to note that the KFWH formulation can be written in a simplified form upon considering that some of the integrands are of second order in respect to the fluid perturbations, and therefore they can be neglected in the case of small perturbations. The result can be written as

$$4\pi p' = \frac{\partial}{\partial t}\int_S \left[\frac{\rho_0 u_n - p'/c^2 v_n}{r|1 - M_r|}\right]_{ret} \mathrm{d}S$$

$$+ \frac{1}{c}\frac{\partial}{\partial t}\int_S \left[\frac{P'_{nr} - \rho_0 u_r v_n}{r|1 - M_r|}\right]_{ret} \mathrm{d}S + \int_S \left[\frac{P'_{nr} - \rho_0 u_r v_n}{r^2|1 - M_r|}\right]_{ret} \mathrm{d}S. \qquad (15)$$

The differential form of this equation, restricted to the case of an impermeable surface S coincident with S_b ($u_n = v_n$), was derived by Myers and Hausmann [21] for the study of the convergence properties of some singular integrals that appear in the Kirchhoff equation when used for scattering or aerodynamic applications. In reference [21], the authors start from the classical Kirchhoff equation and use the linearized inviscid mass and momentum equations in order to replace respectively $\partial p/\partial n$ and $\partial p/\partial t$. The equation obtained is therefore written in terms of $\mathbf{u}$ and p and does not contain $\partial p/\partial n$.

The differences between the various formulations can also be interpreted in terms of the fluid quantities involved in the integral representations. The first step is the classical Kirchhoff equation that is valid for any phenomenon governed by the linear wave equation, independently of the nature of the phenomenon itself, and is written in terms of a single physical quantity. If one wants to avoid the presence of the pressure normal derivative one has to specialize the formulation for aeroacoustic applications by making use of the linearized mass and momentum equations, obtaining in this way equation (15) which, however, is written not only in terms of p but also of $\mathbf{u}$. As a last step one can make use of the complete mass and momentum equations and derive equation (12) which, when the quadrupole terms are properly considered, is also valid in the presence of flow. On the other hand, if one neglects the volume integrals the KFWH formulation is valid for the same conditions as the classical Kirchhoff one. The main practical advantage is that KFWH contains only quantities that are directly available from CFD codes, without the need of executing derivation of CFD data. This aspect can be of a certain importance if shocks are present in the field around the surface S, as happens in delocalized conditions. In this case, in fact, the evaluation of the pressure derivative has to be executed with great care (for example, by introducing some sort of *upwinding* consistent with the formulation used in the CFD code) in order not to degrade the quality of the acoustic result. The same results could also be obtained by using the linearized continuity equation to replace $\partial p/\partial n$ in the Kirchhoff formulation (or, if one prefers, using equation (15)). As to the computational cost of the new formulation it should be almost identical to that of the Kirchhoff approach except that for the storage requirements, KFWH requires the storage of five scalar quantities (ρ, p and three components of $\mathbf{u}$) for each node, while only two quantities (p and $\partial p/\partial n$) are required by Kirchhoff.

In respect to FWH, the first clear advantage of KFWH is that, like the Kirchhoff method, it permits one to avoid the evaluation of the volume integrals, and therefore reduces the computational cost of the acoustic calculations, as a volume integration is reduced to a surface one. Another important aspect is that, for a surface S coincident with S_b, the new formulation can be applied to the prediction of the noise radiated by rotating blades when a fluid flow exists across S_b; for example, when suction or injection devices are applied for boundary layer control, or when blades with porous surfaces are used. On the other hand, the classical FWH equation is not valid in these cases, since it is derived with the assumption that no flow exists across S. A final aspect to be mentioned is that KFWH can be applied to any radiation problem, whether the source is a body in motion in the fluid, or any other mechanism. In fact, once p', $\mathbf{u}$ and ρ are known on a proper surface surrounding the source, the method can be applied independently of the source itself.

6. THE SECOND KFWH EQUATION

The presence, in the integrals of equation (14), of time derivatives of quantities depending on the retarded time is a critical aspect that can generate problems if the numerical derivation is not executed with great care. In fact, in order to execute numerically the time derivative, there is the need to evaluate the retarded times twice, and this fact, joined with the higher accuracy required in each retarded time evaluation, almost doubles the computational time in respect of other methods in which the numerical derivative does not appear (see the Appendix). The time derivatives can be moved inside the integrals by following the same procedure used by Farassat in deriving his formulation 1A [7, 22].

Taking in account that, for a generic function $Q = Q(\mathbf{y}, \tau)$,

$$\frac{\partial}{\partial t}[Q(\mathbf{y}, \tau)]_{ret} = \left[\frac{1}{1 - M_r}\frac{\partial Q}{\partial \tau}\right]_{ret} \tag{16}$$

and using the relations

$$\dot{r} = -v_r, \qquad \dot{r}_i = -v_i, \tag{17}$$

$$\dot{\hat{r}}_i = \frac{\partial}{\partial \tau}\left(\frac{r_i}{r}\right) = \frac{-v_i + \hat{r}_i v_r}{r}, \tag{18}$$

where $\hat{r}_i = r_i/r$, one can rewrite equation (14) as

$$4\pi p' = \int_S \left[\frac{\rho_0(\dot{U}_i n_i + U_i \dot{n}_i)}{r|1 - M_r|^2}\right]_{ret} \mathrm{d}S + \int_S \left[\frac{\rho_0 U_i n_i K}{r^2|1 - M_r|^3}\right]_{ret} \mathrm{d}S$$

$$+ \frac{1}{c}\int_S \left[\frac{\dot{F}_i \hat{r}_i}{r|1 - M_r|^2}\right]_{ret} \mathrm{d}S + \int_S \left[\frac{F_i \hat{r}_i - F_i M_i}{r^2|1 - M_r|^2}\right]_{ret} \mathrm{d}S + \frac{1}{c}\int_S \left[\frac{F_i \hat{r}_i K}{r^2|1 - M_r|^3}\right]_{ret} \mathrm{d}S, \tag{19}$$

where

$$K = \dot{M}_i \hat{r}_i r + M_r c - M^2 c, \qquad F_i = L_{ij} n_j \tag{20, 21}$$

Even if this formulation is more complex than equation (14), it has the great advantage, from the computational point of view, that it does not require any numerical evaluation of derivatives of quantities depending on the retarded time.

Another useful version of formula (14) can be obtained if the integration is executed on the acoustic surface Σ, leading to [20]

$$4\pi p' = \frac{\partial}{\partial t}\int_\Sigma \left[\frac{\rho_0 u_n + (\rho - \rho_0)(u_n - v_n)}{r\Lambda}\right]_{ret} \mathrm{d}\Sigma$$

$$+ \frac{1}{c}\frac{\partial}{\partial t}\int_\Sigma \left[\frac{P'_{nr} + \rho u_i(u_n - v_n)}{r\Lambda}\right]_{ret} \mathrm{d}\Sigma + \int_\Sigma \left[\frac{P'_{nr} + \rho u_i(u_n - v_n)}{r^2\Lambda}\right]_{ret} \mathrm{d}\Sigma, \tag{22}$$

where $\Lambda = \sqrt{1 - M_n^2 - 2M_n \cos\theta}$. It is possible to show that, with an appropriate numerical approach [16, 23–26], this formula has the great advantage that can be applied when the surface S is moving supersonically, while equations (14) and (19) present a singularity in this case.

7. RESULTS

In order to test and validate the KFWH approach, an existing code for the evaluation of Kirchhoff and FWH integrals has been improved by adding some new modules. The code consists of a series of common modules (input–output, kinematics, retarded time evaluation, etc.) that execute the tasks that are independent of the specific acoustic formulation, and of some specific modules devoted to the evaluation of the different integral kernels. In particular, the code is able to evaluate the FWH thickness, loading and quadrupole terms in the subsonic case, and the Kirchhoff integrals for both stationary and subsonic moving surfaces. The integrals in each formulation can be evaluated with three different methods: integration on the physical surface, with numerical evaluation of the time derivatives; integration on the physical surface, with analytical evaluation of the time derivative (Farassat's formulation 1A and extensions); and integration on the acoustic surface, with numerical evaluation of the time derivatives. Recently, the possibility of evaluation of the Kirchhoff integrals for supersonically moving surfaces, with integration on the acoustic surface and evaluation of multiple emission times, also has been added [24]. On each surface or volume element, a first order isoparametric description of both geometry and aerodynamic data is used, and the integrals are evaluated by using Gaussian quadrature.

The KFWH formulation has been introduced in the code by adding some modules that permit the evaluation of the integral kernels of equations (14) and (19), and the input of the relative aerodynamic data. The code has been used to execute some acoustic calculations for the well known two-bladed untwisted UH-1H model rotor in non-lifting hovering conditions, for which experimental results are available for various tip Mach numbers. The purpose of the tests was mainly to compare the capabilities of the KFWH approach in respect to the classical Kirchhoff and FWH formulations. The emphasis was therefore placed not only on the direct comparison with experimental results, which clearly depends also on the quality of the CFD solution used as input for acoustic calculations, but also on the relative results achievable by the different acoustic methods when the same aerodynamic solution is used to provide the input data. Three different conditions were therefore chosen, corresponding to three different tip Mach numbers, namely 0·85, 0·90 and 0·95, and for each condition the same CFD results have been used to provide the different input data required by the different acoustic formulations. The three CFD solutions were provided by DLR, and have been obtained with an Euler code for hovering rotors developed at DLR [27]. The code is an explicit finite volume Euler solver that adopts a scheme that reduces to a second order central difference on a Cartesian grid with constant grid sizes, and is second order accurate on smoothly stretched meshes. In order to avoid spurious oscillations, a blend of first and third order dissipative terms is introduced. An explicit Runge–Kutta time-stepping scheme is used, and a multi-grid algorithm is implemented in order to accelerate convergence. The results for the three Mach numbers have been obtained with the same grid of $128 \times 41 \times 97$ elements (chord $\times$ vertical $\times$ span). Due to the symmetry of the problem, only one blade is modelled with the imposition of proper periodicity conditions. A section in the x–y plane of the aerodynamic mesh is shown in Figure 1 together with one of the fixed Kirchhoff surfaces used in the calculations. It is possible to see that the outer portion of the CFD mesh has a cylindrical

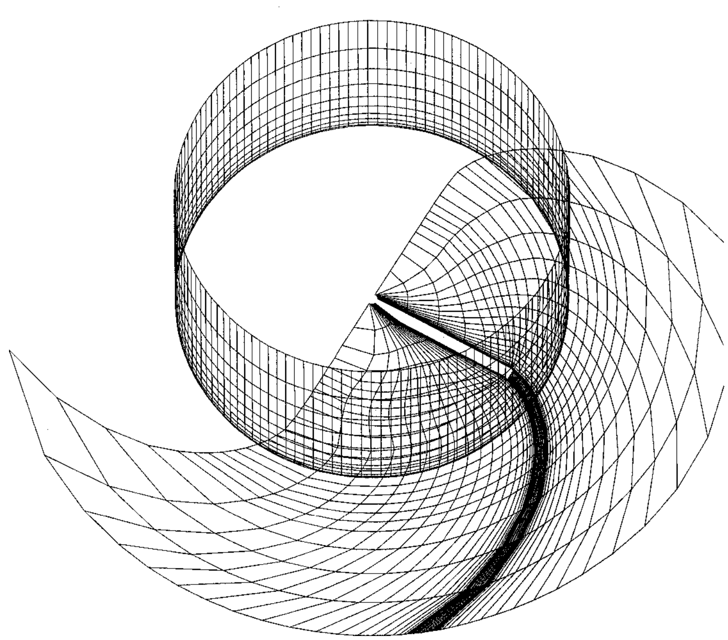

Figure 1. CFD and Kirchhoff meshes: a section of the aerodynamic mesh along the plane $z = 0$, together with the fixed cylindrical Kirchhoff surface.

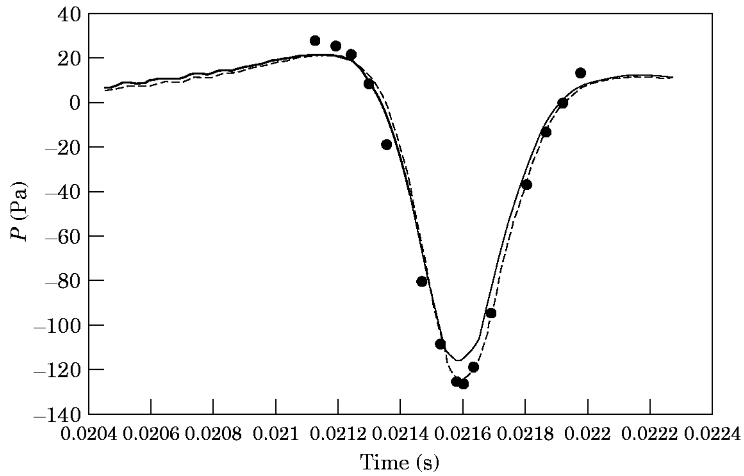

Figure 2. UH-1H: a comparison of results from the rotating surface KFWH method (——), the rotating surface Kirchhoff method (- - - -) and experiments (●), for $M = 0{\cdot}85$.

shape, and this facilitates the interpolation on the fixed Kirchhoff surface. The first result reported in Figure 2 refers to the case $M = 0{\cdot}85$, and shows the pressure disturbance for an observer placed in the rotor plane at a distance equal to 3·09 times the rotor radius R. A Kirchhoff surface surrounding the blade and rotating together with it has been extracted from the nodes of the aerodynamic mesh to obtain the surface sketched in Figure 4. Due to the symmetry of the problem (non-lifting condition), only the upper portion of the surface was considered which gave a total of 1023 nodes (31 vertical $\times$ 33 span). The outer boundary of the mesh was placed at a distance of $1{\cdot}15R$. The mesh obtained was then used to evaluate the pressure disturbance with both the KFWH and the Kirchhoff approach, by using the CFD data available at each node to compute the input data required by the two formulations. In the case of the Kirchhoff approach, another surface adjacent to the original one has been used to extract the pressure normal derivative. The pressure disturbance obtained with the KFWH approach is represented in the figure by a solid line, while the dashed line represents classical Kirchhoff, and the large

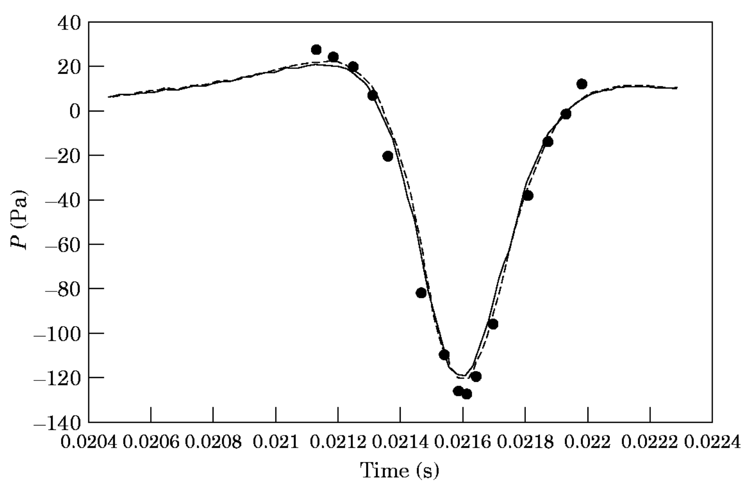

Figure 3. UH-1H: a comparison of results from the fixed surface KFWH method (——), the fixed surface Kirchhoff method (- - - -), and experiments (●), for $M = 0{\cdot}85$.

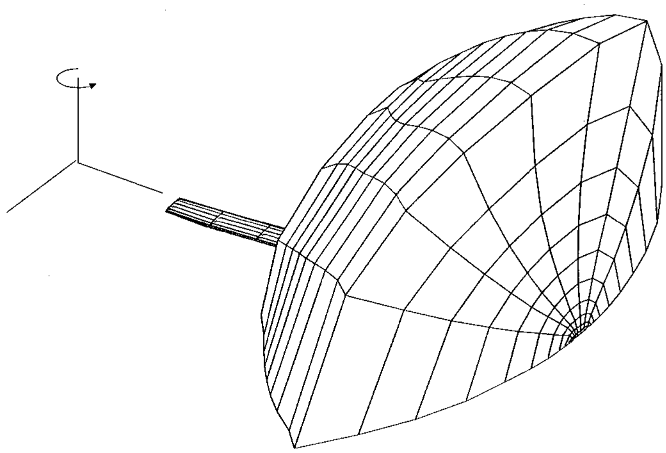

Figure 4. Rotating Kirchhoff: a view of the rotating Kirchhoff surface extracted from the aerodynamic mesh, together with a portion of the blade.

dots represents the experimental results. The same case is considered in Figure 3, but the calculation has been executed by using a cylindrical Kirchhoff surface fixed in respect to the air and surrounding the entire rotor (as the one sketched in Figure 1). The height of the cylinder was about one rotor radius and this permitted one to neglect the contributions of the upper and lower bases of the cylinder itself. The cylindrical surface was discretized with 512 equally spaced points along the azimuth and 31 points in the vertical direction clustered near the rotor plane (also, in this case, only the upper portion of the surface was discretized due to the symmetry along the rotor plane). Since the CFD data were given in the rotating frame, a transformation was required in order to obtain them in the fixed frame. The operation is made easier by the cylindrical shape of the outer part of the aerodynamic mesh. In particular, a bilinear interpolation among the nodes of the aerodynamic mesh that form the lateral surface of the cylinder was used. As a result, for each node the unsteady aerodynamic data were obtained in 512 equally spaced time steps along the rotation period. In the case of the Kirchhoff approach, another cylindrical surface close to the original one was used to evaluate $\partial p/\partial n$. Finally, the same case is considered also in Figure 5, where the results obtained with the KFWH approach, applied to the rotating Kirchhoff surface are compared with the results of the FWH formulation, including thickness, loading and quadrupole terms. In particular the quadrupole terms are evaluated by using a mesh of $33 \times 31 \times 18$ nodes (chord $\times$ vertical $\times$ span). It is important to note that the outer boundary of the mesh for the quadrupole calculation was coincident with the surface used for the KFWH calculation. This means that the two formulations should provide identical results independently of whether or not the integration domain encloses all of the non-linear region, since the two approaches neglect exactly the same terms. It is possible to see from Figures 2, 3 and 5 that all the results obtained compare well with experimental results; but, more importantly, also compare very well with the results of the Kirchhoff and FWH approaches (only a slight difference exists between the Kirchhoff and KFWH results for the moving surface). It must be pointed out that in the above calculations the formulation (19) was used, but almost indistinguishable results can be obtained with formulation (14) once the proper dt are chosen for derivation. In this case, however, the computational cost is greater, as shown in the Appendix.

Similar results can also be obtained for the other two tip Mach numbers. In

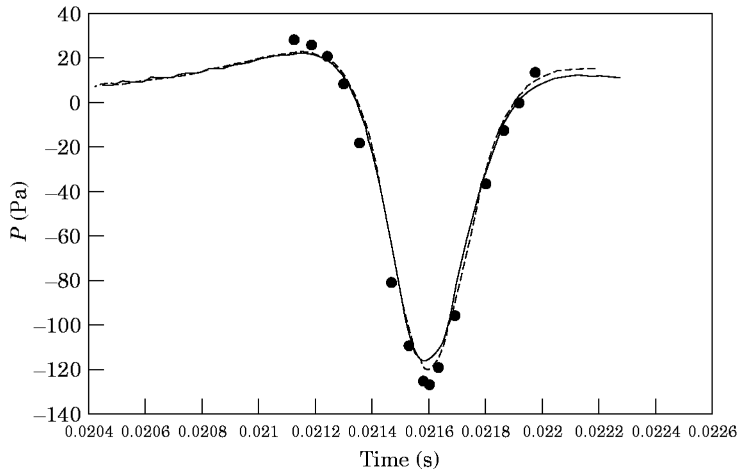

Figure 5. UH-1H: a comparison of results from the rotating surface KFWH method (——), the FWH thickness + loading + quadrupole method (- - - -) and experiments (●), for $M = 0{\cdot}85$.

Figures 6 and 7 the results for $M = 0{\cdot}90$ and $M = 0{\cdot}95$ are given for a fixed cylindrical surface of radius equal to $1{\cdot}3R$. Also, in these cases, the agreement with experiment is satisfactory, and the differences in the slopes of the pressure disturbances are probably due to an excess of dissipation introduced in the aerodynamic solutions. What is important, however, is that the KFWH and Kirchhoff methods produce almost the same results.

As a last result, KFWH (Figure 8) and Kirchhoff (Figure 9) approaches are compared in respect to the convergence properties in terms of the radius of the cylindrical surface S. As can be seen, the behaviour of the results seems to be very similar for the two formulations. In particular, for $r/R = 1{\cdot}1$ the surface is too near to the blade and some non-linear terms are neglected, while for $r/R = 1{\cdot}3$ convergence is practically achieved and the results are almost identical to those obtained for $r/R = 1{\cdot}4$.

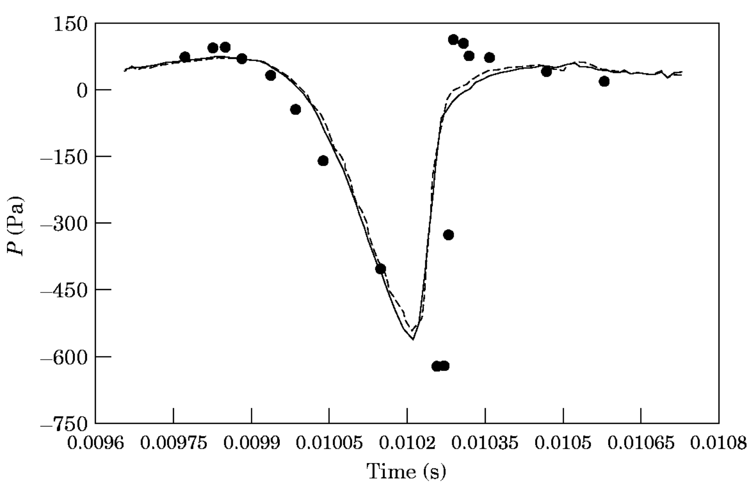

Figure 6. UH-1H: a comparison of results from the fixed surface KFWH method (——), the fixed surface Kirchhoff method (- - - -) and experiments (●), for $M = 0{\cdot}90$.

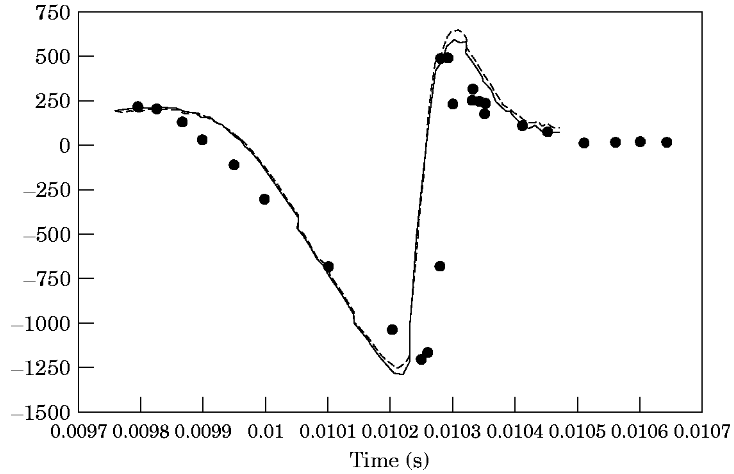

Figure 7. As Figure 6, but for $M = 0{\cdot}95$.

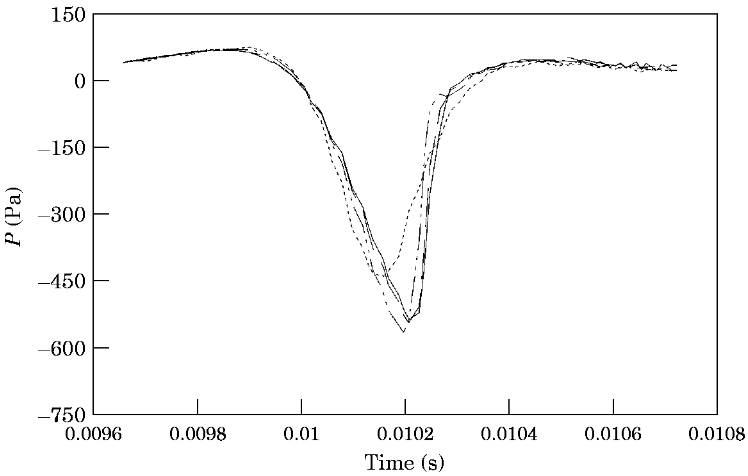

Figure 8. UH-1H: a convergence test for the KFWH method as a function of the outer radial position of the surface for $M = 0{\cdot}90$. r/R values: · · · · ·, 1·1; — - - —, 1·2; — —, 1·3; ——, 1·4.

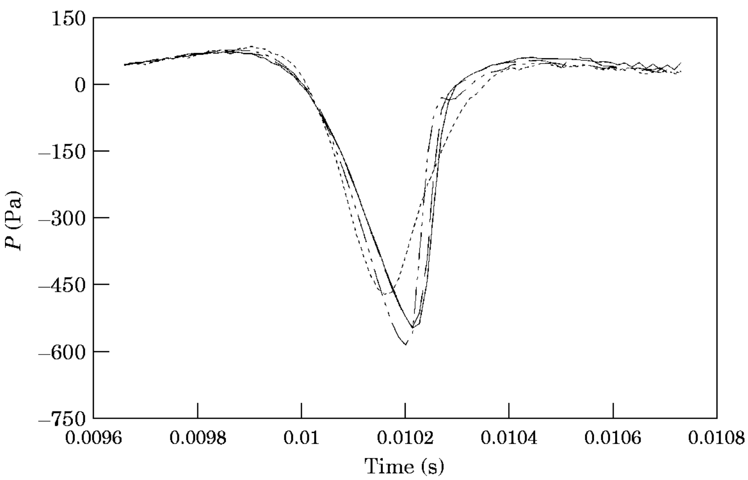

Figure 9. As Figure 8, but for the Kirchhoff method.

8. CONCLUSIONS

A new boundary integral equation has been presented that permits the evaluation of the noise radiated by arbitrary sources once pressure, velocity and density disturbances are known on a smooth closed surface surrounding the source.

The main advantages of the proposed approach in comparison to the Kirchhoff formulation is that it does not require the knowledge of $\partial p/\partial n$ and therefore it can be more easily interfaced with CFD codes. The new formulation can also be applied when a fluid flow exists across the surface of the radiating body, making possible, therefore, evaluation of the noise radiated by blades with porous surfaces or with suction or injection devices; the classical FWH equation is not valid under these circumstances.

Two different formulations have been presented. In the first one a time derivative appears outside some of the integrals and has to be evaluated numerically, while in the second one the derivative is taken inside the integrals and is evaluated analytically. Some numerical considerations concerning the comparison of these two approaches, but valid in general for any aeroacoustic formulation, show that the second formulation is about twice as fast as the first.

Numerical tests reveal that the KFWH method produces almost the same results as the Kirchhoff method, and also the convergence properties in respect to surface distance from the source seem to be similar.

Further work has to be performed to assess the accuracy required by the two approaches in terms of grid definition during the aerodynamic calculation, in order to understand whether the use of the KFWH formulation could permit one to use a less refined aerodynamic grid without affecting the accuracy in the acoustic solution.

ACKNOWLEDGMENTS

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APPENDIX: TIME DERIVATIVES

In this Appendix some problems concerned with numerical evaluation of the time derivatives that appear in equation (14) are addressed. It will be shown that the numerical evaluation can be handled correctly without affecting the accuracy of the solution, but this requires a computational cost that is about twice the cost required by formulation (19). The comparison is made between the first and second KFWH equations, but the analysis is general, and the results obtained are valid for the comparision of any acoustic formulation in which the time derivative is evaluated numerically (for example, Farassat's formulation 1), with the respective approach in which the derivative is evaluated analytically (for example, Farassat's formulation 1A).

Two main aspects will be addressed: the first one is the accuracy required in the evaluation of the retarded time in order not to cause problems in numerical evaluation of the derivative, and the second one refers to the problems that may arise when the observer is in motion.

A.1. RETARDED TIME

One of the main problems that makes the numerical evaluation of the time derivatives in equation (14) difficult is the fact that the quantities inside the integrals depend on the retarded time. In order to focus on the problem consider, as an example, the numerical evaluation of the time derivative $D(t)$ of a function Q depending on the retarded time: i.e.,

$$D(t)=\frac{\partial}{\partial t}[Q(t)]_{ret}=\frac{\partial Q(\tau^*)}{\partial t}, \tag{A1.1}$$

τ^* being the solution of

$$\tau - t + r(\tau)/c = 0. \tag{A1.2}$$

For a two-point scheme, the numerically evaluated derivative $D_N(t)$ can be written as

$$D_N(t)=\frac{[Q(t_2)]_{ret}-[Q(t_1)]_{ret}}{t_2-t_1}, \tag{A1.3}$$

where $D(t)=D_N(t)+O(\Delta t^2)$. The problems arise since equation (A1.2) has no analytical solution for arbitrary motion, and so it has to be solved numerically, iterating until the desired accuracy is reached. Let τ_1^* and τ_2^* be the exact solutions of equation (A1.2), respectively for instants t_1 and t_2, and let ε_1 and ε_2 be the errors achieved in the iterative solution. The estimation D_{Ne} of the numerical derivative will be

$$D_{Ne}(t)=\frac{Q(\tau_2^*+\varepsilon_2)-Q(\tau_1^*+\varepsilon_1)}{t_2-t_1}. \tag{A1.4}$$

Upon expressing $Q(\tau+\varepsilon)$ as $Q(\tau)+\varepsilon\partial Q/\partial\tau+O(\varepsilon^2)$ and taking into account equation (16), equation (A1.4) can be written, with $O(\varepsilon^2)$ terms neglected, as

$$D_{Ne}(t)=\frac{Q(\tau_2^*)-Q(\tau_1^*)}{\Delta t}+D(t)k\left(\frac{\varepsilon_2-\varepsilon_1}{\Delta t}\right), \tag{A1.5}$$

where $\Delta t=t_2-t_1$ and $k=1-M_r$. Upon neglecting now $O(\Delta t^2)$ terms, it is possible to replace $(Q(\tau_2^*)-Q(\tau_1^*))/\Delta t$ by $D(t)$ and write

$$D_{Ne}(t)=D(t)\left(1+k\,\frac{\varepsilon_2-\varepsilon_1}{\Delta t}\right), \tag{A1.6}$$

and therefore

$$\left|\frac{D_{Ne}(t)-D(t)}{D(t)}\right|\leqslant\frac{2\varepsilon k}{\Delta t}. \tag{A1.7}$$

It is thus possible to see that the relative error in the evaluation of the numerical derivatives due to the non-exact knowledge of the retarded time is of the order of $\varepsilon/\Delta t$, where ε is the maximum error in retarded time evaluation and Δt is the time step used in the derivation. Therefore, in order to avoid problems, the maximum error ε allowed in the evaluation of the retarded time has to be much smaller then the time step Δt used in the derivation.

A.2. OBSERVER IN MOTION

Now suppose that one wants to evaluate p' for n time instants, and compare the number of operations required by equation (14) and (19). The surface integrals of equation (14) can be considered as functions $I(\mathbf{x}, t)$ that depend on observer position $\mathbf{x}$ and time t, with $\mathbf{x}$ expressed as a function of t. Each term of equation (14) that contains the time derivative can be written in the form

$$4\pi p' = (\partial/\partial t)\{I(\mathbf{x}, t)\}. \tag{A2.1}$$

The derivative that appears outside the surface integrals has to be taken strictly with respect to time with the position kept constant, and so one has to compute the quantity

$$(\partial/\partial t)\{I(\mathbf{x}, t)\}_{\mathbf{x}=constant}. \tag{A2.2}$$

To evaluate this derivative numerically by using a two point scheme, one has to compute two values of the function I for two different time instants but for the same observer position. If at the time t_0 the observer position is $\mathbf{x}_0$, one can write the correct value of the numerical derivative as

$$\frac{I(\mathbf{x}_0, t_1) - I(\mathbf{x}_0, t_0)}{t_1 - t_0}. \tag{A2.3}$$

Therefore, if one wants to evaluate p' at n time instants, one needs to evaluate the function I for $2n$ pairs $(\mathbf{x}, t)$ and to solve equation (A1.2) $2n$ times. On the other hand, the formulation (19) requires only n evaluations of the function I, and therefore only n solutions of equation (A1.2) are required.

It is of interest to note that the simple evaluation of the function $I(\mathbf{x}(t), t)$ for the n instants, followed by the derivation of the time history obtained, will lead to incorrect results. In this case, in fact, one obtains a numerical evaluation of

$$(\partial/\partial t)\{I(\mathbf{x}(t), t)\} \tag{A2.4}$$

that is different from that of equation (A2.2). The difference between these two results is given by

$$\sum \frac{\partial I}{\partial x_i}\frac{\partial x_i}{\partial t} = \sum \frac{\partial I}{\partial x_i} Vobs_i, \tag{A2.5}$$

and so the error induced is proportional to the observer speed, and is zero only when the observer is at rest.

A.3. CONCLUSIONS

The main advantage of equation (19) in respect of equation (14) is that it requires half the retarded time evaluations. In addition the accuracy required in each evaluation can be lower, since there are no numerical derivatives of quantities depending on the retarded time, and so each retarded time evaluation can be obtained at a lower cost. The only disadvantage is that the integrals are slightly more complex and there is the need to evaluate and store some quantities (such as $\dot{M}$, $\dot{w}$ and $\dot{n}$) that are not required in formulation (14). For sources in generic motion the solution of equation (A1.2) is usually the most computational expensive task of an acoustic code, and therefore the disadvantages of equation (19) are usually compensated for by the advantages, and in practical applications equation (19) is about twice as fast as equation (14). The situation changes if for some specific case the solution of equation (A1.2) can be obtained in a closed form, and in this case the advantages of equation (19) can be less evident.

The above result can clearly be extended to a generic acoustic formulation, and therefore it can be concluded that formulations of Farassat's type 1A (without numerical time derivative) are about half as expensive than formulations of type 1 (with the numerical time derivative).