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A METHOD OF ANALYZING FINITE PERIODIC STRUCTURES, PART 2: COMPARISON WITH INFINITE PERIODIC STRUCTURE THEORY

J. Wei*

8 Abberton Road, Withington, Manchester M20 1HU, England

AND

M. Petyt

Institute of Sound and Vibration Research, 1BJ, England

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The relations between the proposed method for analyzing finite periodic structures and the theory of infinite periodic structures are discussed. It is shown that the proposed method is in fact a very useful approximate approach for calculating not only the natural frequencies and modes of finite periodic structures but also the pass-bands of infinite periodic structures as a function of the phase constant.

The pass-bands of various periodic beam structures are calculated. The discussion focuses on the influence of stiffeners on the pass-bands of the structures, and provides some useful indications of the possibilities and limitations of tuning a pass-band of a periodically stiffened structure by means of tuning the properties of the stiffeners. The example calculations show that it is possible to use stiffeners to change the location and width of the lower order pass-bands.

1. INTRODUCTION

A method of analyzing finite structures with regularly spaced stiffeners and/or supports was presented by the authors in reference [1]. The method established was based on the Rayleigh–Ritz and extended Rayleigh–Ritz methods and the coupling relationships among the assumed sinusoidal displacement functions found in a finite periodic structure. In this paper, the method is further discussed in the context of periodic structure theory.

In periodic structure theory, a periodic structure is assumed to be infinitely long. It is known [2] that, when a harmonic wave is propagating along such a structure, the vibration states at one end of an element is related to those at the other end by the equation

$$\{\mathbf{w}\}_{r+1} = e^{\sigma}\{\mathbf{w}\}_r, \tag{1}$$

where $\{\mathbf{w}\}_r$ is the state vector at the rth joint, which contains all vibration variables such as displacement, slope, moment and force for a beam type structure. σ is the propagation constant. Wave propagation can occur without attenuation if σ is purely imaginary, $\sigma = j\mu$, $j = \sqrt{-1}$. μ represents the phase difference between vibration states over an element and is thus called the phase constant. The principle value of μ is between 0 and π, and the corresponding frequency bands are called pass-bands or propagation zones. The frequency regions between pass-bands are called stop-bands or attenuation zones, in which all possible values of σ are complex. Once the transfer matrix between two state vectors

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over an element is established, the eigenvalue equations can be derived by using the above equation and the propagation constant can be solved as a function of frequency, or vice versus. Various techniques have been developed to establish the eigenequation. For a beam type structure, it can be analytically derived by using characteristic receptance functions [2]. For more complicated structures, e.g., orthogonally stiffened plates and shells, approximate methods such as the finite element method [3] and the hierarchical finite element technique [4] were used. However, to use these techniques and to interpret the results requires one to have a good understanding of the wave theory and its special characteristics in a periodic structure.

The present analysis based on a finite periodic structural model differs from the periodic structure theory in that, for a free vibration problem, it directly calculates the natural frequencies and modes of the model. It is derived on the basis of classical structural dynamics and the coupling relationships found in the structure. The method is simple in concept and in application procedure. However, there are many situations, especially in the cases in which the number of period members is not small, in which it is frequency pass-bands that are more interesting. Although the natural frequencies and normal modes of a finite periodic structure can be calculated from the propagation constants [5], the reverse is not necessarily true. It will be shown to be very useful that the calculated frequencies and modes can be interpreted from the point of view of periodic structure theory. This is the main focus of the discussion in this paper. It will be shown that the present method can be used to calculate the pass-bands of a periodic structure which is infinitely long. However, the structures considered in this paper are periodic structures of symmetric element type. For a periodic structure of non-symmetric element type, additional couplings exist among the assumed displacement functions, in addition to those discussed in reference [1]. The effect of this is discussed in reference [7], where the present analysis is applied to stiffened plates and shells.

2. PERIODIC STRUCTURE ANALYSIS AND THE FINITE PERIODIC STRUCTURE MODEL

Mead [6] showed that the displacement of a positive-going wave in the pass-bands of a periodic beam structure may be expressed in the form

$$w_+(x) = \sum_{n=-\infty}^{\infty} A_n \, e^{j(\omega t - (\mu + 2n\pi)x/l)}, \tag{2}$$

while that for a negative-going wave is

$$w_-(x) = \sum_{n=-\infty}^{\infty} B_n \, e^{j(\omega t + (\mu + 2n\pi)x/l)}, \tag{3}$$

where l is the length of a periodic element. When a finite periodic structure is in free vibration, the positive-going wave, for example, will be reflected from the right extremity and will generate a negative-going wave. Both waves, plus some near field waves depending on the boundary at the end, will form a standing wave. When the two extreme ends of a beam are simply supported, each component in equation (2) will produce a negative-going wave which has an opposite sign but equal magnitude. So do the components of a negative-going wave at the other end. Thus, this yields

$$B_n = -A_n, \tag{4}$$

and a standing wave is formed given by

$$w(x) = w_+(x) + w_-(x) = \sum_{n=-\infty}^{\infty} A_n[\mathrm{e}^{-\mathrm{j}(\mu+2n\pi)x/l} - \mathrm{e}^{\mathrm{j}(\mu+2n\pi)x/l}]$$

$$= \sum_{n=-\infty}^{\infty} -2\mathrm{j}A_n \sin(\mu + 2n\pi)x/l. \tag{5}$$

Now let us look at the displacement expressions in which only the functions defined by the coupling relations (equation (12) in reference [1]) are retained. For a beam containing N_b elements, this gives

$$w(x) = \sum_{i=1}^{\infty} w_{m_i} \sin(m_i \pi x/N_b l), \tag{6}$$

with $m_1 = m$, $m_2 = 2N_b - m$, $m_3 = 2N_b + m$, $m_4 = 4N_b - m$, etc. If we let

$w_{m1} = -2\mathrm{j}A_0$,

$w_{m3} = -2\mathrm{j}A_1$, $\quad w_{m2} = 2\mathrm{j}A_{-1}$,

$w_{m5} = -2\mathrm{j}A_2$, $\quad w_{m4} = 2\mathrm{j}A_{-2}$,

. . . , $\quad$ etc.

and notice that, for $i = 1, 3, 5, \ldots, \infty$,

$$\sin(m_i \pi x/N_b l) = \sin(m\pi/N_b + 2n\pi)x/l, \qquad \text{for } n = 0, 1, 2, \ldots, \infty;$$

and for $i = 2, 4, 6, \ldots, \infty$,

$$\sin(m_i \pi x/N_b l) = -\sin(m\pi/N_b + 2n\pi)x/l, \qquad \text{for } n = -1, -2, -3, \ldots, -\infty;$$

then equation (6) can be rewritten as

$$w(x) = \sum_{n=-\infty}^{\infty} -2\mathrm{j}A_n \sin(m\pi/N_b + 2n\pi)x/l \tag{7}$$

Comparing equations (5) and (7), it can be seen that these two equations are the same except that μ in equation (5) is replaced by $m\pi/N_b$ in equation (7). This means that in a simply supported periodic beam, only a certain number of propagating waves can exist and form standing waves in free vibration. These waves correspond to the propagation constant μ taking the values of $m\pi/N_b$, for $m = 1, 2, \ldots, N_b$ and $2N_b$, while for a beam of the same type but infinitely long, the propagation constant may take any value between 0 and π in a frequency pass-band. This has two implications. First, it is clearly in agreement with what was proved by Sen Gupta [5] about the relations between the natural frequencies of a finite periodic system and the pass-bands of the corresponding infinite system. In each pass-band, there are, in general, N_b natural frequencies for a periodic structure containing N_b periodic elements. The values of these frequencies correspond to the phase constant μ being equally divided by N_b between 0 and π. Each of the coupling groups of $m_1 = 1, 2, \ldots, N_b - 1$ has one frequency in each of the bands, while only one frequency is contributed by either $m_1 = N_b$ or $m_1 = 2N_b$ coupling group in each band. Second, if N_b is taken to be large enough to provide an adequate resolution of μ, the proposed method can be used as an approximate method to calculate the frequency pass-bands or the propagation zones of an infinite periodic structure as the function of phase constant. In other words, the proposed method provides an approximate method that can be used to study periodic structure models of not only finite but also infinite length. The procedure of the calculation is straightforward. It involves: (1) selection of a set of appropriate displacement functions (sine or cosine functions) based on the extreme end conditions; (2) formulation of the mass and stiffness matrices of any given coupling group, following the standard procedure of Rayleigh–Ritz analysis; (3) application of the equivalent constraints of the coupling group if the structure is periodically supported; and (4) calculate the natural frequencies as $m_1 = 1$ to N_b and $2N_b$, which corresponds to $\mu = 0$ to π.

4. THE PASS-BANDS OF PERIODICALLY SUPPORTED BEAMS

In order to demonstrate the above discussion, the method is first applied to two periodically supported beams, beam (a) and beam (b). Beam (a) is a periodically simply supported beam, and beam (b) is a periodically sliding supported beam. The typical elements of these two beam structures are shown in Figures 1(a) and 1(b), respectively. As discussed in the above section, the frequency pass-bands can be calculated using the

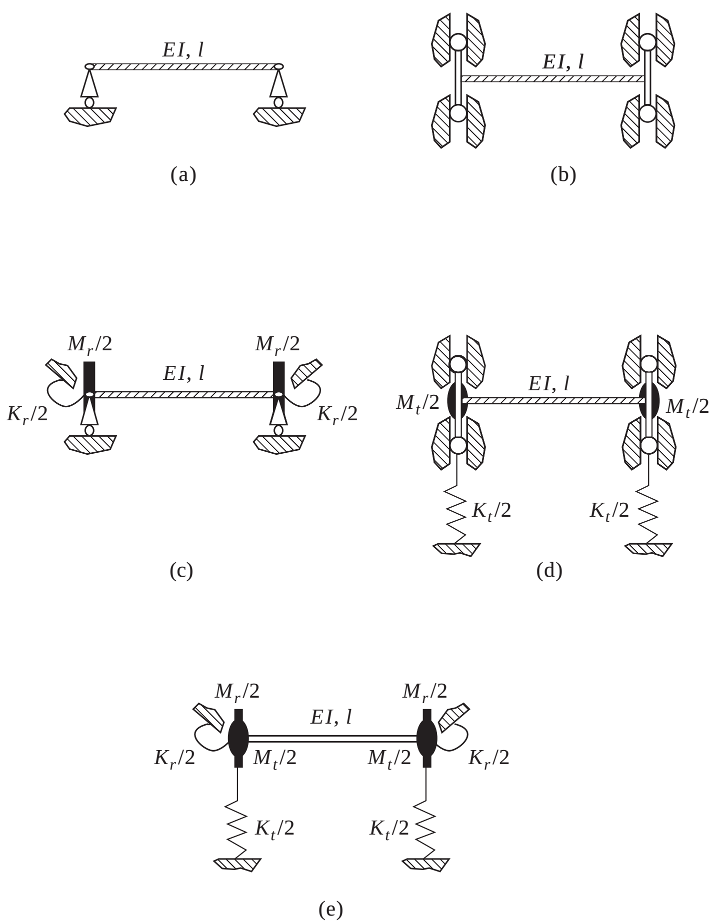

Figure 1. The elements of five periodic beams. (a) Periodically simply supported beam; (b) periodically sliding supported beam; (c) periodically simply supported beam with M_r and K_r stiffeners; (d) periodically sliding supported beam with M_t and K_t stiffeners; (e) periodically stiffened beam with M_t, M_r, K_t and K_r stiffeners.

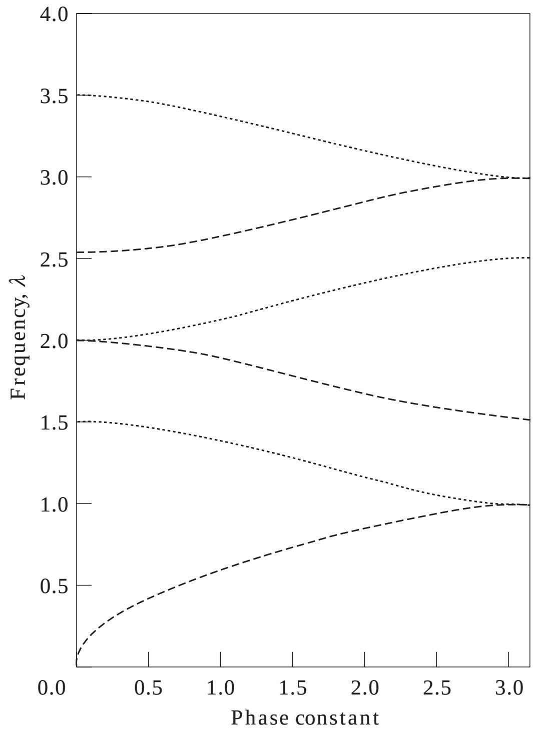

Figure 2. The pass-bands of periodically supported beams. ····, Periodically simply supported beam, beam (a); -----, periodically sliding supported beam, beam (b).

present method, with N_b being set to a large value. The first three pass-bands of these two beams are thus calculated with $N_b = 200$ and stepping m_1 from 1 to 200. The results are shown in Figure 2. A non-dimensional freqeuncy, $\lambda = (\omega^2 \rho A/(EI))^{0\cdot 25} l/\pi$, is used, which is also used throughout the discussions in this paper. Some of the natural frequencies of five-bay periodic beams are listed in Table 1. The periodically simply supported beam was the one discussed in section 4 of reference [1], where a comparison was given with the results calculated by Sen Gupta [5]. For beam (b), a series of cosine functions is used as the assumed displacement functions. The expressions of the mass and stiffness matrices and the equivalent constraints are given in Appendix A.

TABLE 1

The natural frequencies of finite periodically supported beams: $N_b = 5$

	First pass-band		Second pass-band		Third pass-band	
m_1	Beam (b)	Beam (a)	Beam (b)	Beam (a)	Beam (b)	Beam (a)
10	0·0		2·0	2·0		
1	0·4872	1·4489	1·9577	2·0565	2·6825	3·4502
2	0·6899	1·3222	1·8561	2·1807	2·7749	3·3242
3	0·8428	1·1780	1·7353	2·3215	2·8803	3·1822
4	0·9546	1·0533	1·6307	2·4460	2·9657	3·0570
5	1·0	1·0			3·0	3·0

Beam (a): periodically simple supported beam.
Beam (b): periodically sliding supported beam.

TABLE 2

The bounding frequencies of periodically supported beams corresponding to the frequencies of a locked element

M^*	Beam (a) Band 1, $m_1 = 1$	Beam (a) Band 2, $m_1 = 1999$	Beam (a) Band 3, $m_1 = 1$	Beam (b) Band 2, $m_1 = 1999$	Beam (b) Band 3, $m_1 = 1$
9	1·506215	2·502797	3·507794	1·586236	2·643222
21	1·505666	2·499983	3·500633	1·536725	2·552995
31	1·505633	2·499824	3·500204	1·526214	2·534664
41	1·505625	2·499783	3·500942	1·521015	2·525720
51	1·505622	2·499768	3·500054	1·517912	2·520423
Exact	1·5	2·5	3·5	1·5	2·5
δ	0·4%	<0·01%	<0·01%	1·7%	1·4%

Beam (a): periodically simple supported beam.
Beam (b): periodically sliding supported beam.
M^* vs the number of coupled terms used in the calculation.

These two systems are of the simplest mono-coupled periodical beam type, since there is only one co-ordinate, slope or deflection at each end of an element. Mead [4] has shown that the two bounding frequencies of a pass-band of a mono-coupled system consisting of symmetric elements are the natural frequencies of a single element with its both ends either free or locked. The elements of these two beams have the same natural frequencies when their ends are free or locked. However, the propagation waves in the two systems must have opposite natures at the supported points. Therefore one system's pass-band is the other system's stop-band, as shown in Figure 2. The bounding frequencies at which the frequency curves of the two systems join together are the natural frequencies of a single element when it is free. These frequencies can be calculated accurately with the coupling groups of $m_1 = N_b$ and $2N_b$. The bounding frequencies at the other ends correspond to the natural frequencies of a locked element. These frequencies can only be estimated approximately by letting $\mu = m\pi/N_b$ very close to 0 and π, say, $m = 1$ or 1999 with $N_b = 2000$. Some of the bounding frequencies thus calculated are listed in Table 2. The last row shows the percentage errors of the results calculated with M^* (the number of the coupled functions used) equal 31. This value of M^* is used in Figure 2 and in the rest of the discussions in this paper.

The pass-bands of the above two systems with stiffeners added at the locations of the supported points are also calculated. The elements of the stiffened beam systems are shown in Figures 1(c) and 1(d), respectively. Beam (c) is beam (a) with a rotational inertia $I_r/2$ and a rotational spring $K_r/2$ being added at both ends of its elements. Beam (d) is beam (b) with an added mass $M_t/2$ and a transverse stiffness $K_t/2$ at both ends. The actual values of the stiffeners used in the calculation are as follows: $M_t = 0{\cdot}25\rho Al$, $I_r = 0{\cdot}25\rho Al^3$, $K_t = 4EI/l^3$ and $K_r = 4EI/l$, with $\rho = 8000$ kg/m^3, $E = 2{\cdot}13$ e11 N/m^2, $I = 1{\cdot}08$ e $-$ 6 m^4, $A = 3{\cdot}6$e $-$ 3 m^2 and $l = 0{\cdot}9$ m. These values are also used in the rest of the calculations.

The first four pass-bands of beam (c) are shown in Figure 3 alongside with those of beam (a). It is clear that by adding stiffeners the natural frequencies of a free element of beam (c) differ from those of beam (a). However, the natural frequencies of locked elements are unchanged. Thus the pass-bands of beam (c) are such that they have one end coinciding with the pass-bands of beam (a) and the other end moving to a new position. This is because that there is no rotational motion at the supported points in the wave motion pattern in one end of a pass-band; the importance of the rotational motion gradually increases towards the other end of the pass-band and therefore the effect of stiffeners are felt. However, the first band is exceptional, and it is additional to those of beam (a). It is bounded by two natural frequencies of a free element, which are dominated by the stiffener's resonant vibration. The motion in this band is dominated by the rotational motion of the stiffeners, with the phase difference between two adjacent stiffeners varying from $\mu = 0$ to $\mu = \pi$. Similar behaviour can be observed from Figure 4 for beam (d), compared with that of beam (b). However, in this case there is no additional band in the low frequency range. This is because the first band of beam (b) is already bounded by two natural frequencies of a free element, and adding stiffeners only moves them to different locations. The motion in the first band towards $\mu = 0$ is dominated by the stiffeners' vertical motions. The bounding frequency at that end is determined by the ratio of the transverse stiffness K_t to the transverse mass M_t and the mass of a beam element. Towards the other end ($\mu = \pi$), the beam bending stiffness becomes important, since the two ends of an element are moving out of phase. In general, the effect of adding stiffeners is such that the pass-bands become narrower than those of unstiffened ones in the cases studied.

5. THE PASS-BANDS OF A PERIODICALLY STIFFENED BEAM

Having studied the pass-bands of periodically supported beams, we are now ready to investigate the pass-bands of a periodically stiffened beam. The beam, beam (e), is a combination of beams (c) and (d), but without constraints. An element of the system is shown in Figure 1(e). It has both slope and deflection degrees of freedom at each end and therefore is a multi-coupled periodic system.

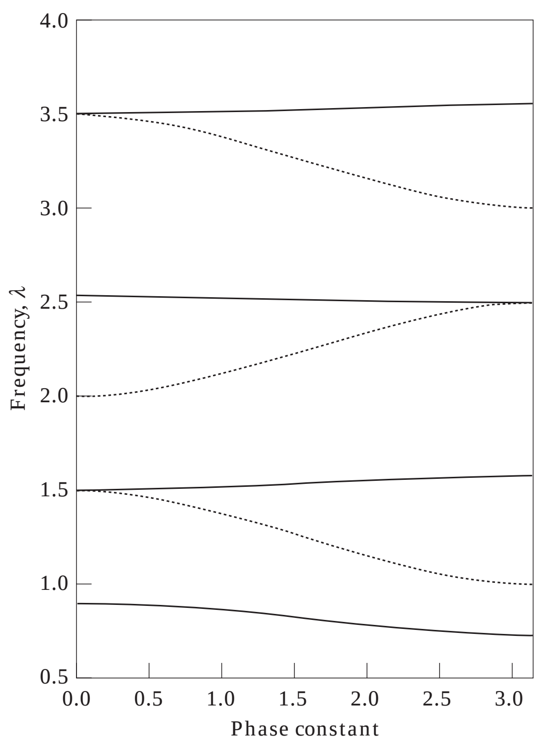

Figure 3. The pass-bands of a periodically simply supported beam with stiffeners. ——, Simply supported beam with I_r and K_r, beam (c); ····, simply supported beam without stiffener, beam (a).

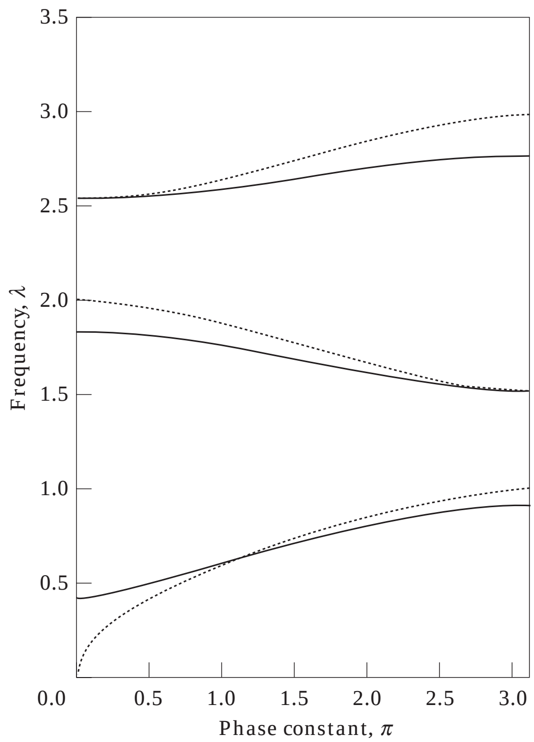

Figure 4. The pass-bands of a periodically sliding supported beam with stiffeners. ——, sliding supported beam with M_t and K_t, beam (d); ····, sliding supported beam without stiffener, beam (b).

If the two extreme ends of the beam are assumed to be simply supported, the natural frequencies of the structure may be calculated using sine functions as the assumed displacement functions. The expressions for the mass and stiffness matrices were given in reference [1]. The frequencies are plotted in Figure 5 as a function of the phase constant $\mu = m\pi/N_b$, together with the pass-bands of beams (c) and (d). The first pass-band of beam (e) is bounded by the lower bounding frequencies of the first bands of beams (c) and (d). This indicates that the dominant motion in the band changes from transverse vibration into rotational vibration of the stiffeners as the propagation constant changes from 0 to π. The two ends (at $\mu = 0$ and $\mu = \pi$) of the second pass-band coincide with the upper bounding frequencies of the first band of beams (c) and (d), but it is not bounded by them. The maximum value of the frequency curve is evidently not at either end, but in between. This indicates that for a multi-coupled system the bounding frequency of a pass-band may not necessarily occur at $\mu = 0$ or $\mu = \pi$. In other words, the frequency curve in a pass-band may not monotonically increase or decrease as μ increases. The bounding frequencies of other bands occur at $\mu = 0$ or $\mu = \pi$ and therefore correspond to the natural frequencies of a single element with the two different types of co-ordinate, slope or deflection, appropriately locked or free; i.e., the natural frequencies of a free element of beam (c) or beam (d). This can be seen from the third and fourth pass-bands, which are bounded by both upper bounds of the second or third pass-bands of beams (c) and (d), respectively. It is important to notice that these two bands, and indeed the higher bands as well, are close to those of beam (d) instead of beam (c). This indicates that the motion in these bands is very much dominated by inter-stiffener motion, while the stiffeners are moving predominantly up and down close to the motion in beam (d).

The pass-bands of beam (e) shown in Figure 5 are calculated using sine functions; i.e., the two extreme ends are assumed to be simply supported. Since a pass-band of a periodic structure should be independent of the extreme end conditions (indeed, there should not be an end), there is no reason why the cosine functions should not be used and still give the same results. In other words, the same pass-band should also be calculated by assuming the two extreme ends to be sliding supported. This is of course true, and its is to be proved in the following.

For a simply supported beam using sine functions and a sliding supported beam using cosine functions, the beam mass and stiffness matrices are diagonal and are the same for both beams except for $m = 0$, which only exists for the sliding supported beam. For the coupling groups of $m_1 = 1$ to $N_b - 1$, the mass and stiffness matrices of the stiffeners for the simply supported beam can be expressed as (combining equations (8) and (12a) of reference [1]),

$$[\mathbf{M}]_S = [M_{ij}], \qquad \text{with } M_{ij} = 0{\cdot}5N_b\{M_t(-1)^{i-j} + I_r(\pi/L)^2 m_i m_j\}, \tag{8a}$$

$$[\mathbf{K}]_S = [K_{ij}], \qquad \text{with } K_{ij} = 0{\cdot}5N_b\{K_t(-1)^{i-j} + K_r(\pi/L)^2 m_i m_j\}. \tag{8b}$$

Those for the sliding supported beam are

$$[\mathbf{M}]_C = [M_{ij}], \qquad \text{with } M_{ij} = 0{\cdot}5N_b\{M_t + I_r(\pi/L)^2 m_i m_j(-1)^{i-j}\}, \tag{9a}$$

$$[\mathbf{K}]_C = [K_{ij}], \qquad \text{with } K_{ij} = 0{\cdot}5N_b\{K_t + K_r(\pi/L)^2 m_i m_j(-1)^{i-j}\}, \tag{9b}$$

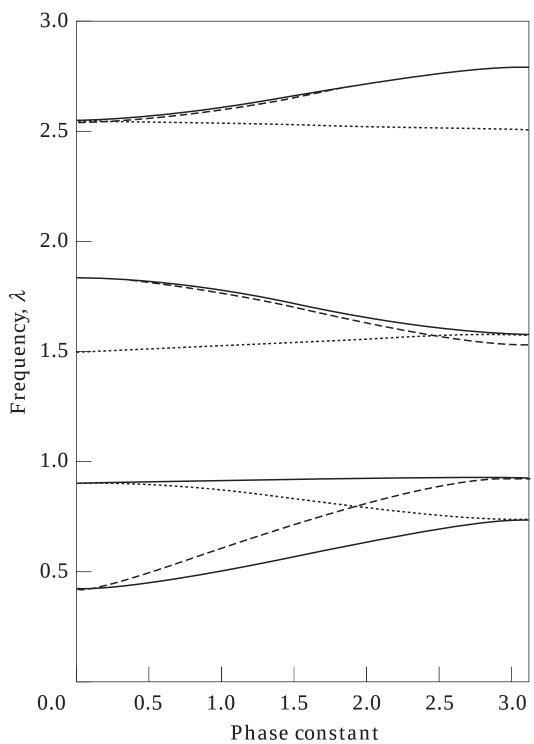

Figure 5. The pass-bands of a periodically stiffened beam. ——, Stiffened beam with M_t, I_r, K_t and K_r, beam (e); ····, simply supported beam with I_r and K_r, beam (c); -----, sliding supported beam with M_t and K_t, beam (d).

TABLE 3

The bounding frequencies of a periodically stiffened beam: the use of sine and cosine functions

Band		$\mu = 0$	$m_1 = 1$		$\mu = \pi$	$m_1 = 1999$
1	c	0·425355	0·425355	s	0·732863	0·732037
2	s	0·899136	0·902442	c	0·921364	0·921345
3	c	1·836975	1·836978	s	1·564733	1·576018
4	s	2·527358	2·546095	c	2·784051	2·784070

s, Sine functions; c, cosine functions.

where $i, j = 1, 2, \ldots, M^*$, and M^* is the number of the coupled functions used in the calculation. Let the unknown coefficients, $w_{mi} = u_{mi}$ for $i = 1, 3, 5, \ldots,$ and $w_{mi} = -u_{mi}$ for $i = 2, 4, 6, \ldots,$ and transfer the system matrices from w_{mi} into u_{mi} co-ordinates. This means pre- and post-multiplying the system matrices by a diagonal matrix $\mathbf{S} = \text{diag}\,\{(-1)^{i-1}\}$, $i = 1,2, \ldots, M^*$. The mass and stiffness matrices of the beam itself are unchanged. The element at the ith row and jth column in equation (8) or equation (9) is multiplied by $(-1)^{i-j}$. Thus equation (8) becomes equation (9), or vice versa, and therefore the mass or stiffness matrices of the two beam systems are the same for a given coupling group of $0 < m_1 < N_b - 1$, which means the same frequency pass bands for $0 < \mu < \pi$.

The frequencies of these two beams for the coupling groups of $m_1 = 2N_b$ and $m_1 = N_b$ are different since at $\mu = 0$ and $\mu = \pi$ two types of function represent opposite motions at the locations of the stiffeners as those of beams (c) and (d), respectively. In fact the two models are complementary to each other, and together they are able to provide both the bounding frequencies of a pass-band at $\mu = 0$ and $\mu = \pi$. The bounding frequencies thus calculated, together with the frequencies calculated by letting $m_1 = 1$ or 1999 with $N_b = 2000$, are listed in Table 3. It can be seen that the results of using a large value of N_b agree well with those of using sine and cosine functions at $\mu = 0$ and $\mu = \pi$.

The discussion so far has demonstrated the use of the proposed method in the analysis of periodic structures. Simplicity is the most important attraction of the method. The analysis procedure is not limited by the number of structure elements. When N_b is small, the results can be interpreted using classical modal analysis theory. When N_b is large, the results can be handled in groups or pass-bands and interpreted using wave theory.

6. THE EFFECTS OF STIFFENER PROPERTIES ON THE PASS-BANDS

The periodically stiffened beam, beam (e), is studied further by increasing in turn one of four properties of the stiffeners by factors of 10 and 100 times, while the others remain constant. The aim of this calculation is to investigate the effect of stiffener properties on the pass-bands of the beam and to highlight the potential and limits of using the properties of stiffeners to influence the locations of the pass-bands. All of the pass-bands below $\lambda = 3$ are shown in Figure 6. When mass M_t is increased, it is shown in Figure 6(a) that the first band gradually moves downwards. The second band approaches the first band of beam (c). The large masses force the beam to behave as if it was periodically simply supported. A similar phenomenon may be seen when the rotational inertia I_r is increased (Figure 6(b)). However, in this case the second band moves close to the first band of beam (d), as I_r resists the rotation at the joint and forces the beam to behave like a periodically sliding supported beam. In both cases, the frequency curves of the first bands become very flat, which means, for a beam of N_b elements, that there are N_b natural frequencies in these narrow frequency bands. Vibrations in the bands are dominated by the resonance of stiffeners. When the translation or rotational stiffness, K_t or K_r, is increased (see Figures 3(c) and 3(d)), one

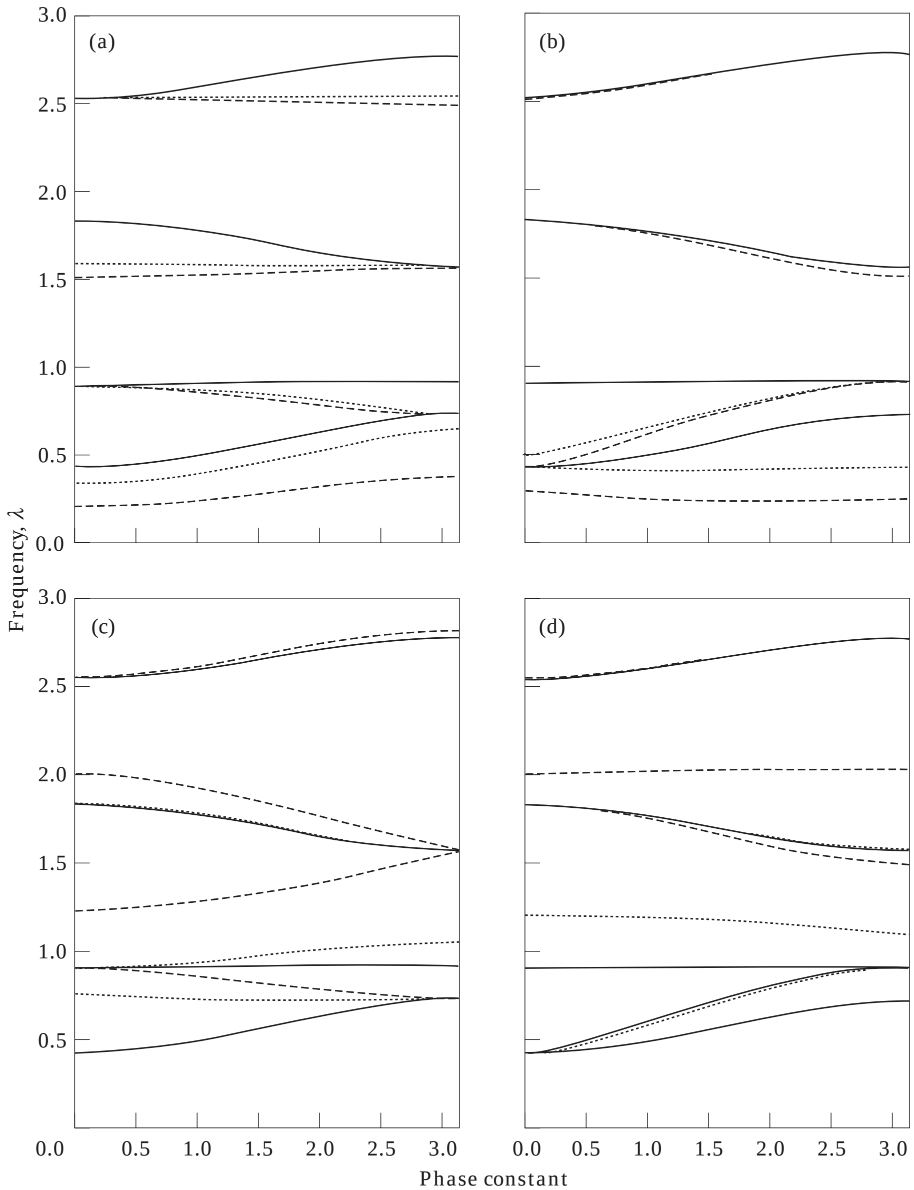

Figure 6. The effects of stiffners on pass-bands of periodically stiffened beams. ——, Stiffened beam with M_t, I_r, K_t and K_r, beam (e). (a) ····, $10M_t$; -----, $100M_t$. (b) ····, $10I_r$; -----, $100I_r$. (c) ····, $10K_t$: -----, $100K_t$. (d) ····, $10K_r$; -----, $100K_r$.

of the bounding frequencies of the first band moves up until it reaches the original bounding frequency of the second band. The other end remains unchanged, which is determined by the nature of the motion at that end. Thus the whole band is close to that of beam (d) or beam (c), respectively. In all these four cases, the higher the orders of the bands are, the less the bands are affected by the changes. For example, the fourth band is shifted insignificantly by the increase of M_t. This is because the motions in the higher order bands are dominated by the vibrations of beam segments between stiffeners and therefore are not sensitive to the changes.

7. CONCLUSIONS

The relations between the proposed method and the theory of infinite periodic structures have been discussed. It has been shown that the proposed method is in fact a very useful approximate approach for calculating not only the natural frequencies and modes of finite periodic structures but also the pass-bands of infinite periodic structures as a function of the phase constant.

The example calculations and discussions based on various periodic beam structures have shown the influence of stiffeners on the pass-bands of the structures. These discussions provide some useful indications of the possibilities and limitations of tuning a pass-band of a periodically stiffened structure by means of tuning the properties of the stiffeners. It has shown that it is possible to use the stiffeners to change the location and width of the pass-bands of a periodic system. For example, the pass-bands of beam (a) can be narrowed by adding I_r and K_r. The bounding frequencies of a pass-band corresponding to $\mu = 0$ and $\mu = \pi$ are controlled by either the rotational inertia and stiffness or the mass and transverse stiffness of the stiffeners. They may be increased or decreased by changing the corresponding properties of the stiffeners. However, significant changes to them may only be achieved for the lower order pass-bands, say the first two bands, by means of changing the properties of stiffeners. This is because the motions in the higher order bands are dominated by inter-stiffener motions and are close to those in a periodically sliding supported structure.

REFERENCES

1. J. Wei and M. Petyt 1997 *Journal of Sound and Vibration* **202**, 555–569. A method of analyzing finite periodic structures, part 1: theory and examples.
2. D. J. Mead 1975 *Journal of Sound and Vibration* **40**, 1–18. Wave propagation and natural modes in periodic systems, I: mono-coupled system.
3. R. M. Orris and M. Petyt 1974 *Journal of Sound and Vibration* **38**, 223–236. A finite element study of harmonic wave-propagation in periodic structures.
4. N. S. Bardell and D. J. Mead 1989 *Journal of Sound and Vibration* **134**, 55–72. Free vibrations of orthogonally stiffened cylindrical shell, part II: discrete general stiffeners.
5. G. Sen Gupta 1970 *Journal of Sound and Vibration* **13**, 89–101. Natural flexural waves and normal modes of periodically supported beams and plates.
6. D. J. Mead 1970 *Journal of Sound and Vibration* **11**, 181–197. Free wave propagation in periodically supported infinite beams.
7. J. Wei 1995 *Ph.D. Thesis*, *University of Southampton*. Modelling of fuselage/floor structures and associated cabin acoustics for the prediction of propeller induced interior sound fields.

APPENDIX A: THE EXPRESSIONS FOR A PERIODICALLY SLIDING SUPPORTED BEAM

The assumed displacement in this case is

$$w(x) = \sum_{m=0}^{\infty} w_m \cos(m\pi x/L), \tag{A1}$$

where $w(x)$ is the beam flexural displacement and L is the length of the beam. m indicates the number of half cosine waves in the shape of the corresponding prescribed function, and w_m is an unknown coefficient associated with the function. The mass and stiffness matrices are diagonal, and the diagonal elements are, for $m_i \neq 0$,

$$M_i = 0{\cdot}5\rho LA[1 + (I/A)(m_i\pi/L)^2], \qquad K_i = 0{\cdot}5\rho LEI(m_i\pi/L)^4 \tag{A2a, b}$$

and if $m_i = 0$,

$$M_i = \rho L A, \qquad K_i = 0, \tag{A3a, b}$$

where ρ is the beam mass density. A and I are the cross-sectional area and the second moment of area of the beam, respectively. E is Young's modulus.

The equivalent constraints in this case are

$$\sum_{i=1}^{\infty} m_i w_{m_i} (-1)^{i-1} = 0, \qquad \text{for } m_1 = 1, 2, 3, \ldots, N_b - 1. \tag{A4}$$

with m_i, $i = 1, 2, \ldots, \infty$, defined by the coupling relationships given in equation (12) of reference [1]. For the coupling groups of $m_1 = N_b$ and $2N_b$, no constraint is required. The $m = 0$ term is in the coupling group of $m_i = 2N_b$ if stiffeners are also added at the supported points.