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EFFECTS OF LAMINATIONS ON THE VIBRATIONAL BEHAVIOUR OF ELECTRICAL MACHINE STATORS

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An electrical machine stator is a laminated thick cylinder. For simplicity, a homogeneous thick cylinder has often been used as a model of a stator for the purposes of vibrational analysis. Experiments show that the vibrational behaviour of a stator is quite different from that of an homogeneous thick cylinder. Among all of the vibrational modes, predicted by using the homogeneous cylinder model, only a few of them are the modes of a realistic stator, and most of the predicted modes are ones which cannot be traced experimentally. In this paper, based on the analytical results for the vibrations of solid annular plates, and the experimental results for the vibrations of laminated annular plates under a range of clamping pressures, it is found that the vibrational behaviour of a laminated thick cylinder is dominated by that of an individual cylinder lamination. Laminations have significant effects on the vibrations of a cylinder and the effects are mode type dependent. As either the number of laminations or the clamping pressure increases, all of the transverse vibrational modes of a laminated cylinder are eliminated, leaving only the in-plane vibrational modes—that is, the pure radial modes—as the only ones which persist. A laminated stator exhibits mainly pure radial vibrational characteristics, which are quite different from those of an homogeneous thick cylinder.

1. INTRODUCTION

In order to design a quiet electrical machine, an understanding of the vibrational behaviour of a stator is essential. This is because the vibrations, and consequently the noise, generated by an electrical machine can be reduced to a very large extent if electromagnetic forces produced during machine operation are not allowed to excite resonance in the stator. For this reason, several analyses have been developed over the years. A stator is composed of a number of components, such as the stator core, teeth, windings and frame etc. Some parts—for example, the windings and frame—are not continuous parts of the stator body, which makes the vibrational analysis difficult. Hence, in the past, much attention has been paid to how to simplify the stator structure in order to establish an analytical model. While some investigators have treated the teeth and windings merely as additional masses [1, 2], others have treated them as cantilevers with enhanced lateral vibrations at tooth resonance [3], or have not accounted for such resonance at all [4]. In some analyses, the frame was treated rigorously [1, 2, 4]. On the other hand, several investigators have not accounted for the presence of the frame [3], and others have suggested that the frame could be entirely left out of the calculations since its contribution was only small [5].

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There are also some combined analytical and experimental studies on the vibrational behaviour of stators. In reference [6], using a homogeneous thick cylinder as a model, the authors developed an analysis based on the three-dimensional theory of elasticity. The energy method, together with the principles of Rayleigh–Ritz, was used to derive the general frequency equation of a stator. The analysis incorporated the effects of teeth, windings and frame. However, the experimental investigations conducted to validate the analysis, which were reported in references [7, 8], were limited to models of short length. In more recent publications [9, 10], the same analytical method was applied to the free vibrations of a simple and smooth homogeneous thick cylinder, the overall dimensions of which were those of the stator of an actual four-pole, 550 V, 125 HP induction motor. The same method was then applied to the vibrational analysis of the stator of the actual induction motor by the same authors in references [11, 12]. In reference [13], the effects of windings, frame and impregnation upon the resonant frequencies of the same stator were investigated. A comprehensive investigation of the effects of core clamping pressure, windings, wedges, teeth, impregnation, temperature, etc., on the vibrational behaviour of the stators of two types of motors, a 630 kW ring-core type stator and a 2100 kW segmented-core type stator, was reported in reference [14].

The vibrational analysis of a stator is a difficult problem. This is not only because a stator is composed of several parts, but also because a stator is a laminated structure in order to prevent eddy current losses. A stator is actually a laminated thick cylinder or a laminated annular plate. Mechanical vibrations can be considered as a fluctuation between elastic potential energy and kinetic energy in a structure. A continuous elasticity is a necessary condition for this energy fluctuation. For a stator, laminations cut off the elastic connection between individual laminations, and make the transfer of elastic forces between them impossible. Therefore, the vibrational behaviour of an actual stator should be quite different from that of an homogeneous thick cylinder. A typical example can be seen in references [9–13]. For the homogeneous thick cylinder, all of the predicted vibrational modes were verified by experiment with good frequency accuracy, but for the actual laminated stator, of all of the predicted vibrational modes, only a few were the actual modes of the stator. Most of the predicted modes were therefore described as false modes. The frequency errors of some modes were as high as 30%. This phenomenon reveals that a significant difference exists between the vibrations of a laminated stator and those of the homogeneous thick cylinder model. Hence, in order to explain this phenomenon, and to predict the vibrational modes of stators correctly, the effects of laminations upon the vibrations of a laminated thick cylinder need to be investigated.

In this paper, the mode classification of homogeneous thick cylinders as proposed in reference [15] is briefly first introduced. The mode classification is based on the three-dimensional mode shapes of thick cylinders, and can be used to explain the relationship between the vibrational modes of a laminated cylinder and those of an homogeneous cylinder. Therefore, this mode classification is helpful in the understanding of the actual vibrational modes of stators. In order further to investigate the nature of the actual modes, the vibrations of solid annular plates were investigated using the finite element method. The effects of laminations on the vibrations of a series of laminated annular plates were subsequently investigated using experimental modal analysis. Based on the new mode classification, the vibrations of annular plates and the effects of laminations on the vibrations, an understanding of the differences between the vibrations of a laminated thick cylinder and the vibrations of an homogeneous thick cylinder can be obtained. The vibrational behaviour of realistic stators can then be explained.

2. VIBRATIONAL MODES OF HOMOGENEOUS THICK CYLINDERS

Although a stator is a laminated structure, for simplicity, in the past, a homogeneous thick cylinder was often used as a model of a stator for vibrational analysis. A mode classification of cylinders, utilizing the two categories of symmetric longitudinal modes and antisymmetric longitudinal modes, was also often used in the analysis. This mode classification was applied in references [9, 10], in which both the analytical and experimental models were the same simple and smooth homogeneous thick cylinder, the overall dimensions of which were those of the stator of an actual four-pole, 550 V, 125 HP induction motor. All of the predicted modes, both the symmetric longitudinal modes and an equal number of the antisymmetric longitudinal modes, were verified with good frequency accuracy by experiment. The same investigative method was used by the same authors in references [11, 12]; here the analytical model was an homogeneous thick cylinder model, and the experimental model was the realistic stator of the four-pole, 550 V, 125 HP induction motor. In this case, none of the predicted antisymmetric longitudinal modes could be traced, as was also the case with most of the predicted symmetric longitudinal modes. Only a few of the predicted modes were the actual modes of the stator. Hence, the classification into either symmetric or antisymmetric longitudinal modes, while it is an effective classification for an homogeneous cylinder, is not an effective way for classifying the vibrations of stators or laminated thick cylinders. This is because whether a predicted mode is an actual mode or a false mode cannot be determined by this grouping. In fact, it can be seen from references [11–13] that the actual modes can only be identified from the predicted modes by experiment. Therefore, a different mode classification of thick cylinders, which is more detailed, is needed to identify the actual modes analytically.

A new mode classification of homogeneous thick cylinders, based on the three-dimensional mode shapes, was proposed in reference [15]. According to this mode classification, all the vibrational modes of homogeneous thick cylinders can be classified into six categories: pure radial modes, radial motion with shearing modes, extensional modes, circumferential modes, axial bending modes and global mode, as shown in Figure 1. These six types of modes can be further subdivided into two different types categories: pure radial modes; and non-pure radial modes, which include the latter five categories. The difference in the pure radial modes and the non-pure radial modes is that the natural frequencies and mode shapes of the former are only dependent upon the radial dimensions of the cylinder, and are independent of the axial length. For the pure radial modes, there are no nodes in the longitudinal direction. The natural frequency and mode shape of a given order of pure radial mode are the same for a cylinder and a plate if their radial dimensions are the same. Annular plates and thick cylinders can be considered as an extension of one another for this kind of mode. For the non-pure radial modes, the natural frequency and mode shape of a given order mode are dependent upon both the radial dimensions and the axial length of the thick cylinder. The mode shapes of these modes are much more complicated than those of the pure radial modes.

Using the new mode classification, the vibrational modes of the same stator investigated in references [11–13] are investigated in this paper. The radial dimensions of the stator are shown in Figure 2, and the length of the stator was 523 mm. The material constants used in the calculations were as follows: for silicon steel, modulus of elasticity, $E = 207$ GPa; Poisson ratio, $\nu = 0{\cdot}28$; and the equivalent density of the stator model $\rho = 7290$ kg/m^3 [12]. In Table 1 are given the new mode types of the stator together with the experimental results, and the results from the homogeneous cylinder model. From Table 1 it can be seen

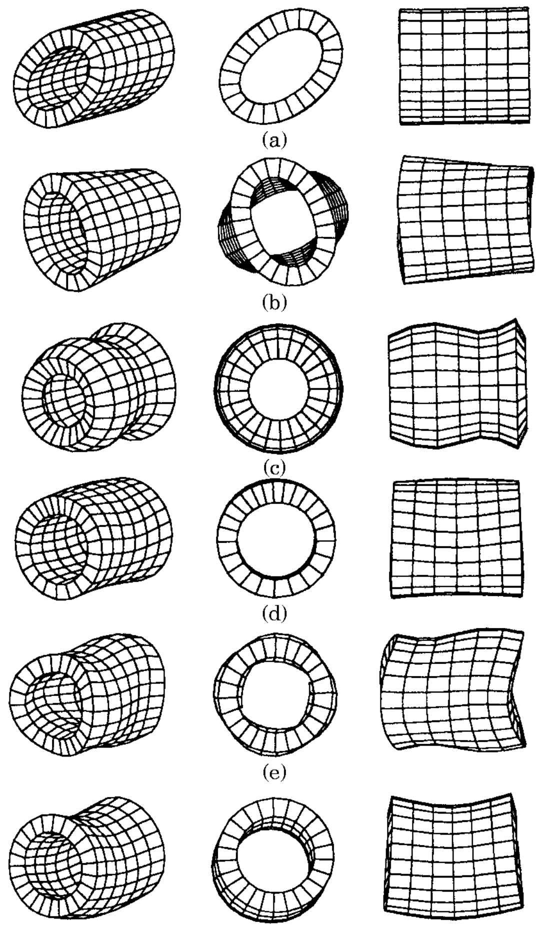


Figure 1. Typical modes of homogeneous thick cylinders: (a) pure radial mode; (b) radial motion with radial shearing mode; (c) extensional mode; (d) circumferential mode; (e) axial bending mode; (f) global bending mode.

that all the pure radial modes are the actual modes of the stator, and the so-called "false modes" are the non-pure radial modes. Hence, this mode classification is most useful in the vibrational analysis of stators. Using this mode classification, it is possible to predict the actual modes of a stator analytically when an homogeneous thick cylinder is used as a model of the stator.

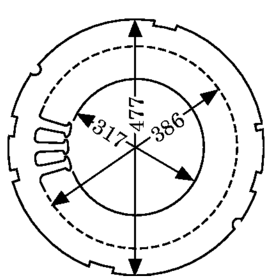


Figure 2. The dimensions of the four-pole, 550 V, 125 HP induction motor.

TABLE 1

Mode types of the realistic stator of the four-pole, 550 V, 125 HP induction motor

Mode, n	Homogeneous cylinder* f_{cal} (Hz)	Realistic stator* f_{exp} (Hz)	m	Mode type
0	3397	3338	0	Pure radial mode
	3405	#		Extensional mode
	3664	#		Extensional mode
1	2463	#		Axial bending mode
	2932	#		Global beam mode
	4576	4512	0	Axial bending mode
2	553	524	0	Pure radial mode
	691	#		Radial shearing mode
	2248	#		Radial shearing mode
	3784	#		Axial bending mode
3	1483	1393	0	Pure radial mode
	1673	#		Radial shearing mode
	2604	#		Radial shearing mode
	4458	#		Axial bending mode
4	2670	2470	0	Pure radial mode
	2858	#		Radial shearing mode
	3609	#		Radial shearing mode
	5172	#		Axial bending mode
5	4022	3533	0	Pure radial mode
	4184	#		Radial shearing mode
	4872	#		Axial bending mode
6	5473	4197	0	Pure radial mode
	6249	#		Radial shearing mode

#, False mode, could not be found by experiment.
* From Reference [12].

3. VIBRATIONAL MODES OF SOLID MODELS USING FEM

Since a stator can be considered as a set of laminated annular plates, the vibrational analysis of an individual annular plate should be the study basis for the vibrations of the plate assembly. The vibrations of an annular plate can be classified into either in-plane vibrations or transverse vibrations. When a number of annular plates are pressed together under a clamping pressure to form a laminated cylinder, the boundary conditions for each plate are no longer the same as those of an isolated freely supported plate. The effects of these boundary condition changes on the in-plane vibrational modes, and on the transverse vibrational modes, are quite different. In order to study these effects, and also to avoid missing any mode by experiment, it is appropriate first to investigate the natural frequencies and mode shapes of solid models analytically. The finite element method and the software, ANSYS [16], were used to perform the calculations. The analytical models are shown in Figure 3(a). The radial dimensions of all the models are the same: inside radius $R_i = 139{\cdot}5$ mm, outside radius $R_o = 190{\cdot}0$ mm; however, their axial thicknesses are different. The axial thickness of model 1 is $H = 28{\cdot}5$ mm. All of the models were made from mild steel, and the material constants used in the calculations were as follows:

modulus of elasticity, $E = 210$ GPa; density, $\rho = 7860$ kg/m^3; Poisson ratio, $\nu = 0{\cdot}30$. The equation of motion used was

$$\mu U_{i,mm} + (\lambda + \mu) U_{m,mi} + F_i = \rho U_{i,tt}, \tag{1}$$

where U is the displacement component, F is the applied force, λ is the Lamé coefficient, μ is the shear modulus of elasticity, ρ is the material density, and i, $m = x$, y, z. If the weighted residual method is applied to Equation (1), then

$$\int_\Omega W[\mu U_{i,mm} + (\lambda + \mu) U_{m,mi} + F_i - \rho U_{i,tt}]\, \mathrm{d}\Omega = 0. \tag{2}$$

The corresponding equation of motion for free vibrations is

$$\mathbf{M}\ddot{\mathbf{U}} + \mathbf{K}\mathbf{U} = 0, \tag{3}$$

where $\mathbf{M} = \int_\Omega \rho W U_i \, \mathrm{d}\Omega$, $\mathbf{K} = \int_\Omega [\mu W_{,m} U_{i,m} + (\lambda + \mu) W_{,i} U_{m,m}]\, \mathrm{d}\Omega$, and where $\mathbf{M}$ is the mass matrix, $\mathbf{K}$ is the stiffness matrix and W is a weighting function. In order to obtain both the in-plane vibrational modes and the transverse vribational modes at the same time, all of the plates were modelled as a collection of three-dimensional block elements with eight nodes and three translational degrees of freedom per node. All of the models were divided into 48 elements in the circumferential direction, since calculations showed that once the element number was over 48, the improvement in the frequency accuracy associated with increasing the element number was very limited in the investigated frequency range 0–10 KHz. Householder's method and the frequency shift technique were used in the mode extraction process.

It can be seen from the analytical results that the vibrational modes of an annular plate are either in plane vibrational modes or transverse vibrational modes. The in-plane modes can further be divided into pure radial modes and circumferential modes. For the former, the median surface remains unstretched; for the latter, the median surface is stretched, as

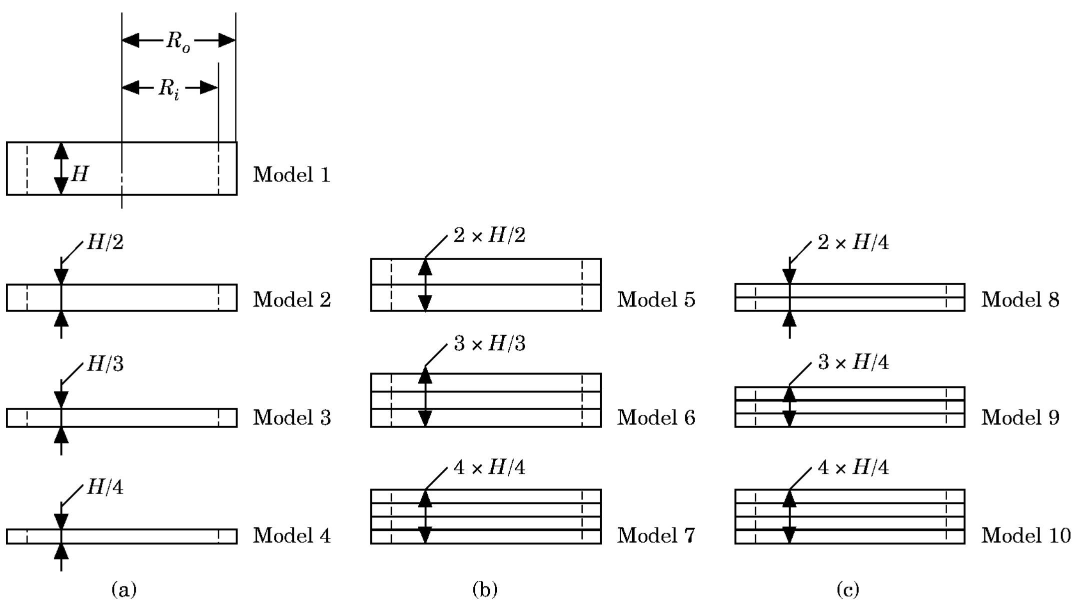
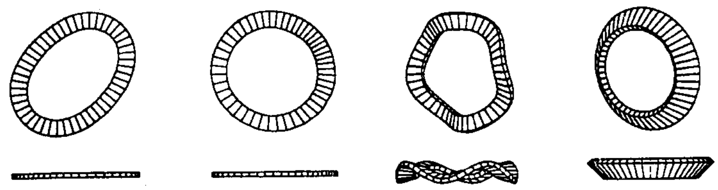


Figure 3. Analytical and experimental models: (a) solid models; (b), (c) laminated models.



(a) (b) (c) (d)

Figure 4. Typical modes of annular plates: (a) in-plane pure radial mode; (b) in-plane circumferential mode; (c) transverse mode with nodal radii; (d) transverse mode with a nodal circle.

can be seen in Figures 4(a) and 4(b), respectively. The transverse modes can further be divided into transverse modes with nodal radii and transverse modes with a nodal circle, as shown in Figures 4(c) and 4(d), respectively. The natural frequencies of the solid models are plotted in Figure 5. It can readily be seen in Figure 5 that the frequency of a given transverse mode is dependent upon the model thickness; generally the thicker the model is, the higher is the frequency. The frequency of a given in-plane mode does not vary with the thickness of the model. Because of the frequency independence of the in-plane modes, it can be appreciated that the frequency of a given in-plane pure radial mode of an annular plate, and the frequency of the same order pure radial mode of a homogeneous thick cylinder, are the same if they both have the same radial dimensions. Hence, the in-plane pure radial vibrational modes of annular plates, and the pure radial modes of homogeneous thick cylinders can be considered as the same kind of modes.

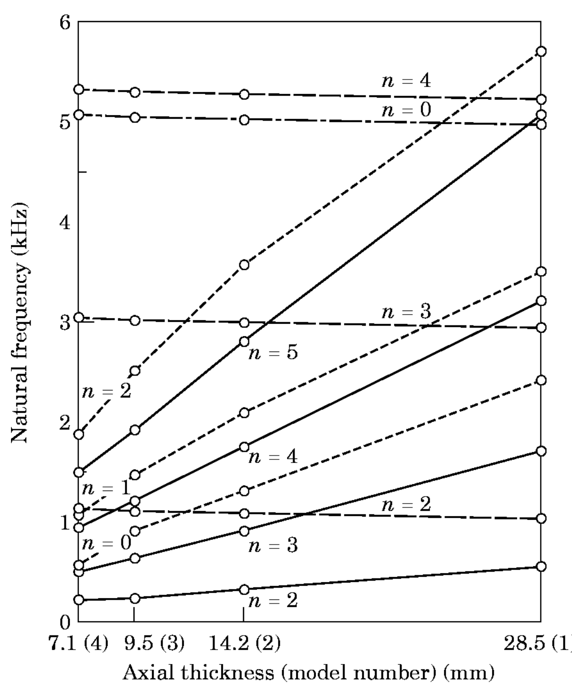


Figure 5. The variation in the resonant frequency of the different modes, with axial thickness, for solid models 1, 2, 3 and 4. — — —, In-plane pure radial modes; —-—, in-plane circumferential modes; ——, transverse modes with nodal radii; - - - - -, transverse modes with a nodal circle.

4. EXPERIMENTAL STUDY OF LAMINATED ANNULAR PLATES

From Table 1, it can be seen that for a stator, which is a laminated thick cylinder, a great number of the modes which exist in an homogeneous cylinder cannot be found. Compared with a solid structure, the analytical vibrational analysis of a laminated structure with interlaminar slip is much more complicated, because of both the mutual elastic effects of adjacent laminations and the complexity of micro–macro slip between laminations. In the past, several authors have investigated the vibrations of laminated structures, such as frictional joints [17], beams [18, 19], turbine blades [20, 21] and plates [22], but a thorough understanding of the mechanism of the effects of laminations is still limited. Although several simple models of the frictional interface have been proposed, a widely accepted and realistic model of laminations, which is pertinent to the case of the laminated thick cylinder with interlaminar slip, has not been found in the literature. In this paper, an experimental method to investigate the effects of laminations on the vibrations of laminated thick cylinders is reported.

4.1. EXPERIMENTAL MODELS

There were ten experimental models. The radial dimensions of all the ten models were the same, but their axial thicknesses were different, as can be seen in Figure 3. The ten models were divided into three groups. Group (a) had four solid models, 1–4. This group was used to confirm the vibrational modes of annular plates, and to provide a comparative basis for the natural frequencies and mode shapes for groups (b) and (c). In group (b), models 5, 6 and 7 were laminated models, formed by pressing several of either models 2, 3 and 4 together to make the total axial thickness the same as that of model 1. Group (b) was used mainly to compare the vibrational behaviour of each laminated model with that of the corresponding solid model in group (a), in order to investigate the effects of laminations on vibrations. In group (c), models 8, 9 and 10 were formed by pressing two, three or four models 4 together, respectively. Group (c) was mainly used to investigate the effects of lamination number and clamping pressure on the vibrations. Model 10 was exactly the same as model 7. For the convenience of explanation, two different model numbers were used for the same model in groups (b) and (c), respectively.

The contacting surfaces of the laminated models had a fine turned and lapped finish, which was then uniformly abraded with fine glass beads. To connect the contacting surfaces of the middle plates, small holes were made in these plates. In order to obtain an even clamping force over the whole of the contacting surfaces, a fine equalizing circumferential groove was also cut on the contacting surfaces. A soft tape covered the inside and outside seams to form an edge seal. The air between the contacting surfaces was partially withdrawn by means of a vacuum pump, and the difference between the internal and atmospheric pressures was used to provide the clamping force. By changing the amount of vacuum between the contacting surfaces, the clamping pressure could be adjusted within the required range. Figure 6 is the half cross-section of laminated model 7.

The advantage of using atmospheric pressure for clamping rather than bolts, which are used to keep the stator laminations together in practice, is that the localized effects of the bolts can be eliminated allowing for the observation of the effect of laminations alone. For a realistic stator, the bolts may have some small effect on the high order vibrational modes, but the effect on the fundamental pure radial vibrational modes is very limited, because a bolt is much softer than the stator body in the radial direction.

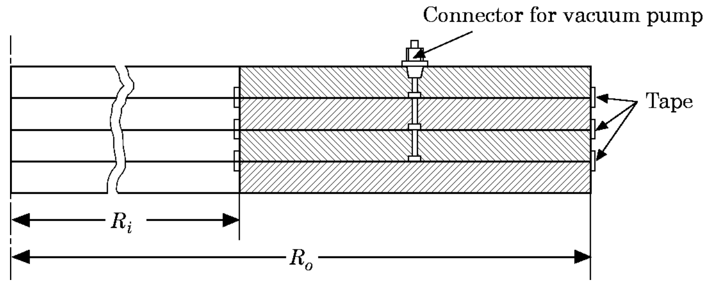


Figure 6. The half cross-section of experimental laminated model 7.

4.2. EXPERIMENTAL SET-UP AND PROCEDURE

A schematic diagram of the experimental set-up is shown in Figure 7. The experimental models were suspended by three rubber strings to provide free boundary conditions. An impact hammer was used as the exciter. A uniaxal accelerometer was used to pick up the response signal. Both the excitation signal and the response signal were fed into the dual channel analyzer to calculate the frequency response function. With a steel tip on the hammer, frequency response functions up to 6·4 KHz could be obtained. Frequency response functions were then transferred to the modal analysis system to extract the mode shapes. The vacuum loading system consisted of a vacuum pump, an accumulator, a control valve and a manometer.

For experimental models 1–4, frequency response functions in both the radial and axial directions were measured at each measurement point. Each model was divided into 16 equal segmental elements, and altogether 128 frequency response functions were measured per model. From the 128 frequency response functions, the corresponding modal parameters were obtained.

For experimental models 5–10, each model was also divided into 16 equal segmental elements. The air between the contacting surfaces of the laminations was evacuated until the pressure fell to 0·9 atm; i.e., the clamping pressure acting on the models was 0·1 atm. This pressure was then maintained until all of the 128 frequency response functions in both the radial and axial directions had been measured. The clamping pressure was then increased in 0·1 atm increments, and the procedure was repeated for the nine different clamping pressures investigated.

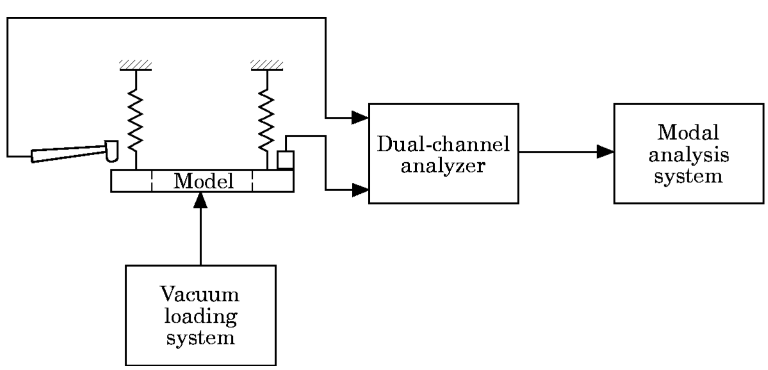


Figure 7. The experimental set-up.

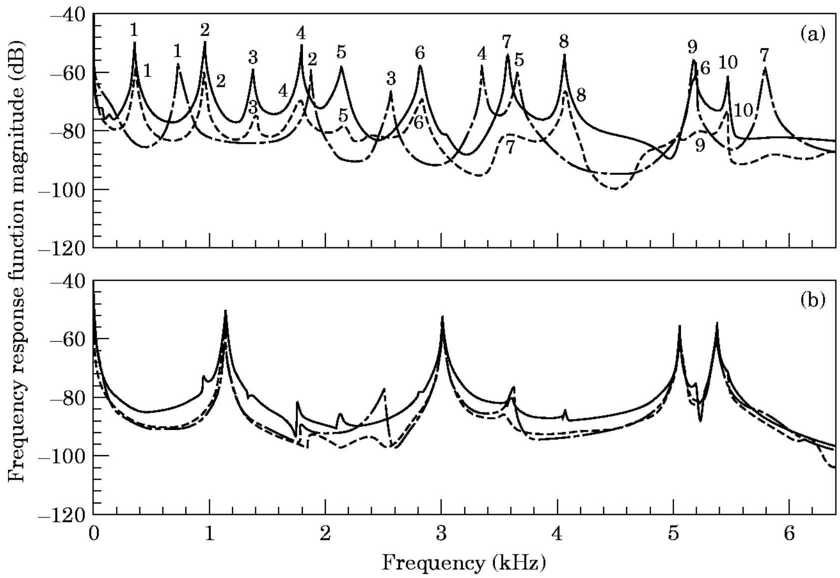


Figure 8. The frequency response functions of models 1, 2 and 5: (a) transverse vibrations; (b) in-plane vibrations. ——, model 1; — - —, model 2; – – – – –, model 5.

4.3. EXPERIMENTAL RESULTS AND DISCUSSION

Since it is not possible to present all of the experimental results here, the most significant findings are summarized as follows.

(1) The results for experimental models 1–4 confirmed that the natural frequencies of the in-plane vibrational modes are independent of the axial thickness of the model, and that they depend only upon the radial dimensions of the model. The natural frequencies of the transverse vibrational modes are dependent upon both the axial thickness and the radial dimensions of the model, and the frequency of a specific mode increases as the axial thickness of the model increases.

(2) The frequency response function magnitudes of model 5 for a clamping pressure of 0·1 atm, measured in the transverse and radial directions respectively, are shown in Figures 8(a) and 8(b). For comparison the frequency response functions of models 1 and 2 are also plotted in the figure. From Figure 8(a) it can be seen that the vibrational behaviour of model 5, in the transverse direction, is quite different from that of model 1 although their dimensions are exactly the same. From Figure 8(a) it can also be seen that the frequency response function of model 5 is close to that of model 2. This means that the vibrational behaviour of laminated model 5 is dominated by that of the individual parts, i.e., model 2, from which it is built. The same conclusions were obtained for models 6 and 7 and models 8–10. In the case of the in-plane modes, shown in Figure 8(b), all models (1, 2 and 5) exhibit the same vibrational behaviour. Again, this was also found to be the case with models 6–10. It can be concluded that the vibrational behaviour of a laminated thick cylinder is dominated by that of an individual lamination from which the laminated cylinder is composed. Therefore, it is suggested that the vibrational behaviour of a realistic stator is dominated by that of an individual stator lamination.

(3) Laminations reduce the magnitudes of the frequency response functions of laminated plates, and the reduction is dependent upon both lamination number and mode type. In Figure 9 are shown the reductions in the frequency response function with clamping pressures for models 8–10, as compared with the solid model 4, from which they are

respectively built. From the figure it can be seen that the magnitudes of the frequency response functions of the laminated models are all smaller than those of the solid model 4. The reduction for each different kind of mode varies: generally, the reductions of the in-plane vibrational modes are less than those of the transverse vibrational modes for each model. From Figure 9 it can be seen that, for the same clamping pressure, as the number of laminations increases, the magnitudes of the frequency response functions for all modes decrease further, and the reductions are mode type dependent. The reductions of the transverse vibrational modes are larger than the reductions of the in-plane vibrational modes. For each model, as the clamping pressure increases, the subsequent reductions in the frequency response functions continue to be mode type dependent. For the in-plane vibrational modes, the magnitude of the frequency response function for a given model does not change with clamping pressure, whereas there is a significant decrease in the case of all the transverse vibrational modes. The same conclusions were obtained for models 6 and 7. Therefore, as the number of laminations or the clamping pressure increases, the transverse vibrational modes start to disappear, but the in-plane vibrational modes are unaffected, and finally the in-plane modes are the only modes that persist. The laminated stator of the four-pole, 550 V, 125 HP induction motor was composed of over 1000 stator laminations. Because of the effects of laminations, the transverse modes disappeared quickly, leaving only the in-plane modes as the ones detectable.

(4) Laminations have a great effect on the natural frequencies of the transverse vibrational modes, but have no effect on the natural frequencies of the in-plane vibrational modes, as can be seen in Figure 10. The natural frequencies of the in-plane vibrational modes are independent of the thickness of the lamination, the number of laminations and the clamping pressure. The same conclusions were also obtained for models 8, 9 and 10. A similar result was reported in reference [14]. In reference [14] the vibrational modes to two types of motor stators, a 630 kW ring-core type stator and a 2100 kW segmented-core type stator, were investigated. It was found experimentally that when the clamping pressure varied from 0·49 kg/mm^2 to 28·6 kg/mm^2, the natural frequencies of the in-plane modes showed no significant change. From the above, it can be seen that the natural frequencies of the in-plane vibrational modes are independent of both axial thickness, the number of laminations and the axial clamping pressure. They are dependent only upon the radial

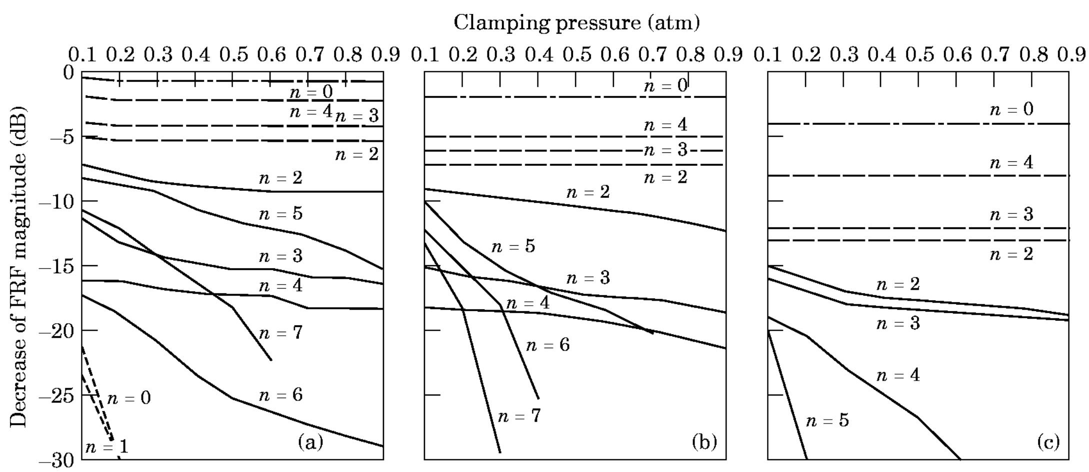


Figure 9. The reduction in the frequency response function magnitude of various modes with clamping pressure: (a) model 8; (b) model 9; (c) model 10. — —, In-plane pure radial modes; — - —, in-plane circumferential modes; ——, transverse modes with nodal radii; – – – – –, transverse modes with a nodal circle.

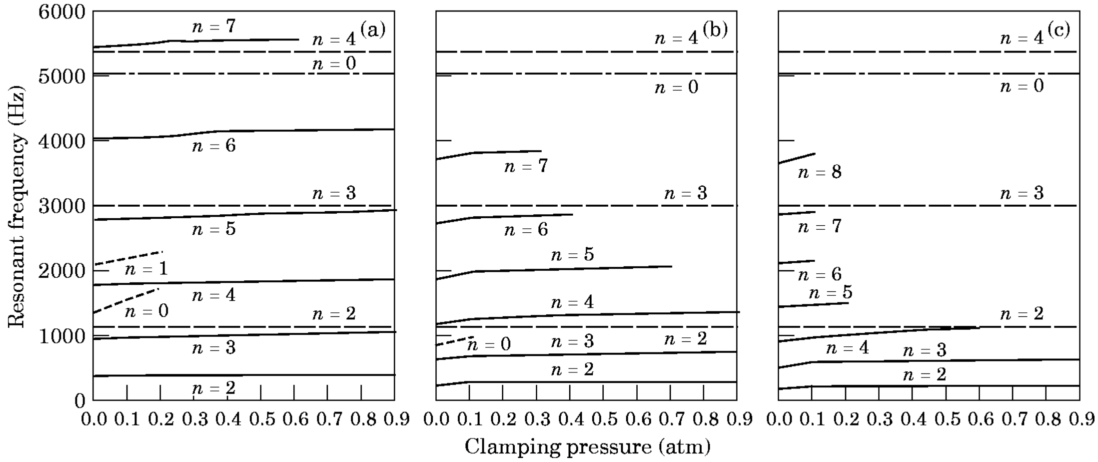


Figure 10. The variation in resonant frequencies of various modes with clamping pressure: (a) model 8; (b) model 9; (c) model 10. ——, In-plane pure radial modes; —-—, in-plane circumferential modes; ——, transverse modes with nodal radii; –––––, transverse modes with a nodal circle.

dimensions of the lamination. Because of this frequency independence, the natural frequencies of the in-plane modes of a solid annular plate, laminated annular plates and an homogeneous thick cylinder are the same if they have the same radial dimensions. This is also the reason why, in the past, an homogeneous thick cylinder could be used as a model of a stator to calculate the natural frequencies of the pure radial modes.

5. CONCLUSIONS

The vibrational behaviour of an electrical machine stator is quite different from that of an homogeneous thick cylinder, since a realistic stator is a laminated thick cylinder. Because of the effects of laminations on the vibrations, the vibrational behaviour of a laminated thick cylinder is dominated by that of an individual cylinder lamination. As the number of laminations or the clamping pressure increases, all of the transverse vibrational modes of the laminated cylinder are eliminated. Finally, only the in-plane or the pure radial vibrational modes persist. As a laminated thick cylinder, an electrical machine stator exhibits mainly a pure radial vibrational behaviour.

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REFERENCES

1. E. ERDELYI and G. HORVEY 1957 *Transactions of the American Society of Mechanical Engineers* [*E*] **24**, 39–45. Vibration modes of stators of induction motors.
2. S. P. VERMA and R. S. GIRGIS 1973 *IEEE Transactions* **PAS-92**, 1577–1585. Resonance frequencies of electrical machine stators having encased construction, part I: derivation of the general frequency equation.
3. H. FROHNE 1959 *Ph.D. Thesis*, *Technical University of Hannover*. On the main parameters which determine the noise-level of asynchronous machines (in German).

4. A. Ellison and S. Yang 1971 *Proceedings of the IEE* **118**, 185–190. Natural frequencies of stators of small electrical machines.
5. A. L. Penniman and H. D. Taylor 1941 *AIEE Transactions* **60**, 283–288. Suppression of magnetic vibration and noise of two-pole turbine generators.
6. R. S. Girgis and S. P. Verma 1981 *Proceedings of the IEE* **128**, 1–11. Method for accurate determination of resonant frequencies and vibration behaviour of stators of electrical machines.
7. S. P. Verma and R. S. Girgis 1981 *Proceedings of the IEE* **128**, 12–21. Experimental verification of resonant frequencies and vibration behaviour of stators of electrical machines, part I: model, experimental procedure and apparatus.
8. S. P. Verma and R. S. Girgis 1981 *Proceedings of the IEE* **128**, 22–32. Experimental verification of resonant frequencies and vibration behaviour of stators of electrical machines, part II: experimental investigations and results.
9. S. P. Verma, R. K. Singal and K. Williams 1987 *Journal of Sound and Vibration* **115**, 1–12. Vibration behaviour of stators of electrical machines, part I: theoretical study.
10. R. K. Singal, K. Williams and S. P. Verma 1987 *Journal of Sound and Vibration* **115**, 13–23. Vibration behaviour of stators of electrical machines, part II: experimental study.
11. S. P. Verma, K. Williams and R. K. Singal 1989 *Journal of Sound and Vibration* **129**, 1–13. Vibrations of long and short laminated stators of electrical machines, part I: theory, experimental models, procedure and set-up.
12. K. Williams, R. K. Singal and S. P. Verma 1989 *Journal of Sound and Vibrations* **129**, 15–29. Vibrations of long and short laminated stators of electrical machines, part II: results for long stators.
13. R. K. Singal, K. Williams and S. P. Verma 1990 *Experimental Mechanics* **September**, 270–280. The effect of windings, frame and impregnation upon the resonant frequencies and vibrational behavior of an electrical machine stator.
14. S. Watanabe, S. Kenjo, K. Ide, F. Sato and M. Yamamoto 1983 *IEEE Transactions on Power Apparatus and Systems* **102**, 949–956. Natural frequencies and vibration behaviour of motor stators.
15. H. Wang and K. Williams 1996 *Journal of Sound and Vibration* **191**, 955–971. Vibrational modes of thick cylinders of finite length.
16. ANSYS, 1992 Swanson Analysis Systems Inc. Houston, Texas.
17. C. F. Beards and J. L. Williams 1977 *Journal of Sound and Vibration* **53**, 333–340. The damping of structure vibration by rotational slip in joints.
18. E. H. Dowell and H. B. Schwartz 1983 *Journal of Sound and Vibration* **91**, 255–267. Forced response of a cantilever beam with a dry friction damping attached, part I: theory.
19. E. H. Dowell and H. B. Schwartz 1983 *Journal of Sound and Vibration* **91**, 269–291. Forced response of a cantilever beam with a dry friction damping attached, part II: experiment.
20. C. H. Manq, J. Bielak and J. H. Griffin 1986 *Journal of Sound and Vibration* **107**, 279–293. The influence of microslip on vibration response, part I: A new microslip model.
21. C. H. Manq, J. H. Griffin and J. Bielak 1986 *Journal of Sound and Vibration* **107**, 295–307. The influence of microslip on vibration response, part II: comparison with experimental results.
22. C. F. Beards and D. A. Robb 1980 *International Conference on Recent Advances in Structural Dynamics*, *England*, 749–760. The use of frictional damping to control the vibration of plates in structures.