Journal of Sound and Vibration (1997) **202**(5), 736–738

COMMENTS ON "THE ROLE OF EIGENVECTORS IN AEROELASTIC ANALYSIS"

R. H. Plaut

The Charles E. Via, Jr. Department of Civil Engineering, Virginia Polytechnic Institute and State University, Blacksburg, VA 24061-0105, *U.S.A.*

(*Received* 2 *August* 1996)

The recent letter by Pidaparti and Afolabi [1] discussed the potential use of eigenvectors to predict the onset of flutter in non-conservative systems. An alternative application of the eigenvectors for the same purpose is described here.

Consider a system governed by the matrix equation of motion

$$\mathbf{M}\ddot{\mathbf{x}}(t) + (\mathbf{U} - p\mathbf{L})\mathbf{x}(t) = 0, \tag{1}$$

where $\mathbf{M}$ and $\mathbf{U}$ are symmetric, positive-definite matrices, $\mathbf{L}$ is not symmetric, and p is a *load parameter*. It is assumed that the system is stable until p reaches the flutter load p_f, at which two vibration frequencies coalesce and coupled-mode flutter occurs. If one considers the motion $\mathbf{x}(t) = \mathbf{c}\exp(\mathrm{i}\omega t)$, equation (1) becomes

$$(\mathbf{U} - p\mathbf{L} - \omega^2\mathbf{M})\mathbf{c} = 0 \tag{2}$$

and the adjoint system can be written as

$$(\mathbf{U} - p\mathbf{L}^{\mathrm{T}} - \omega^2\mathbf{M})\mathbf{d} = 0, \tag{3}$$

where $\mathbf{d}$ can be interpreted as the left eigenvector of equation (2).

At $p = p_f$, the right and left eigenvectors of system (2) will be denoted $\mathbf{c} = \mathbf{c}_f$ and $\mathbf{d} = \mathbf{d}_f$, respectively. Define the function

$$g(\mathbf{c}, \mathbf{d}) = \mathbf{d}^{\mathrm{T}}\mathbf{M}\mathbf{c}. \tag{4}$$

At the flutter load p_f [2],

$$g(\mathbf{c}_f, \mathbf{d}_f) = 0. \tag{5}$$

Hence the magnitude of $g(\mathbf{c}, \mathbf{d})$, which is a function of the eigenvectors, may be used as a measure of the closeness of the system to the onset of flutter.

Two simple examples are considered. The first is a double pendulum subjected to a tangential tip load P. The bars have equal lengths b and equal masses m at their ends, and rotational springs with equal stiffnesses k act at the base and the internal hinge. If the co-ordinates are the angles of the bars from the vertical, if ω^2 is replaced by the non-dimensional quantity $\omega^2 mb^2/k$, and if $p = Pb/k$, then the matrices are [3]

$$\mathbf{M} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}, \qquad \mathbf{U} = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}, \qquad \mathbf{L} = \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}. \tag{6}$$

The flutter load is $p_f = 2$. For $p < 2$, if the eigenvectors corresponding to the lower frequency are normalized by $\mathbf{c}^{\mathrm{T}}\mathbf{c} = 1$ and $\mathbf{d}^{\mathrm{T}}\mathbf{d} = 1$, then the variation of $g(\mathbf{c}, \mathbf{d})$ as a function of p is shown in Figure 1. As flutter is approached, the magnitude of g decreases and approaches zero.

0022–460X/97/200736 + 03 \$25.00/0/sv960819

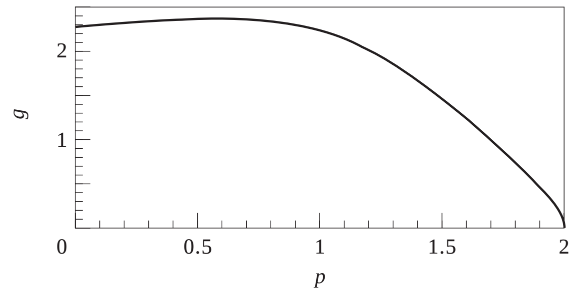

Figure 1. Variation of g as a function of non-dimensional load p.

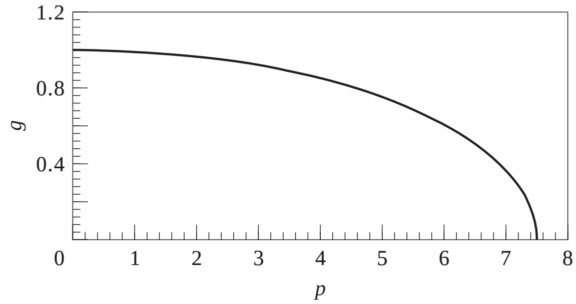

Figure 2. Variation of g as a function of non-dimensional dynamic pressure p.

The second example involves flutter of a simply-supported panel with infinite aspect ratio, subjected to a supersonic flow over one surface. Galerkin's method is applied, using the first two sine functions satisfying the boundary conditions [4, 5]. The resulting matrices are

$$\mathbf{M}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},\qquad \mathbf{U}=\begin{bmatrix}1 & 0\\ 0 & 16\end{bmatrix},\qquad \mathbf{L}=\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}, \tag{7}$$

where p is a non-dimensional dynamic pressure parameter and ω is a non-dimensional frequency. In this case, $p_f = 7{\cdot}5$. With the same choice and normalization of **c** and **d** as in the first example, the resulting plot of $g(\mathbf{c}, \mathbf{d})$ is presented in Figure 2. The magnitude of g provides a measure of the *distance* from the onset of flutter.

Some continuous, non-conservative systems exhibiting coupled-mode flutter obey an instability criterion similar to equation (5), involving eigenfunctions of both the system and its adjoint [6]. Hence the type of procedure described here may also be applicable for continuous systems.

REFERENCES

1. R. M. V. Pidaparti and D. Afolabi 1996 *Journal of Sound and Vibration* **193**, 934–940. The role of eigenvectors in aeroelastic analysis.
2. K. Huseyin and R. H. Plaut 1973 *Journal of Applied Mechanics* **40**, 175–180. The elastic stability of two-parameter nonconservative systems.
3. H. Ziegler 1968 *Principles of Structural Stability*. Waltham, Massachusetts: Blaisdell.
4. J. M. Hedgepeth 1957 *Journal of the Aeronautical Sciences* **24**, 563–573, 586. Flutter of rectangular simply supported panels at high supersonic speeds.
5. R. L. Bisplinghoff and H. Ashley 1962 *Principles of Aeroelasticity*. Wiley: New York.
6. R. H. Plaut 1972 *AIAA Journal* **10**, 967–968. Determining the nature of instability in nonconservative problems.

AUTHORS' REPLY

R. M. V. Pidaparti and D. Afolabi
Department of Mechanical Engineering, Purdue University, Indianapolis, IN 46202, USA

(*Received* 16 *September* 1996)

We thank Professor Plaut for his interest in our paper. The function defined in terms of right and left eigenvectors by Professor Plaut seems to yield qualitatively similar results, in the pre-flutter regime, to the angle between the eigenvectors as defined in our paper. In general, it seems that the eigenvectors may have a role to play in predicting the onset of flutter in aeroelastic systems.