Journal of Sound and Vibration (1997) **203**(2), 358–361

AN ATTEMPT TO MODEL THE EFFECT OF A STRESS CONCENTRATION FIELD ON THE LOWER NATURAL FREQUENCIES OF STRUCTURAL ELEMENTS

V. Sonzogni

INTEC (CONICET-U.N.L.) and Facultad Regional Santa Fe (UTN), 3000-Santa Fe, Argentina

AND

P. A. A. Laura

Institute of Applied Mechanics (CONICET) and Department of Engineering, Universidad Nacional del Sur, 8000-Bahia Blanca, Argentina

(*Received* 14 *August* 1996, *and in final form* 19 *September* 1996)

1. INTRODUCTION

Consider first the simple case of a rectangular plate subjected to a uniaxial state of in-plane stress and simply supported along the edges; Figure 1(a). The determination of the natural frequencies of the structural element constitutes a straightforward problem within the realm of the classical theory of vibrating plates which is solved in a rather elementary fashion substituting the displacement amplitude

$$W(x, y) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} b_{nm} \sin \frac{n\pi x}{a} \sin \frac{m\pi y}{b} \tag{1}$$

in the time-independent Lagrange-Sophie Germaine partial differential equation

$$D\nabla^4 W - N_x\, \partial^2 W/\partial x^2 - \rho h \omega_{nm}^2 W = 0, \tag{2}$$

where the ω_{nm}^2's are the circular natural frequencies of the system.† If due to operational reasons, e.g., passage of conduits, ducts or electrical cables, one places a circular hole at

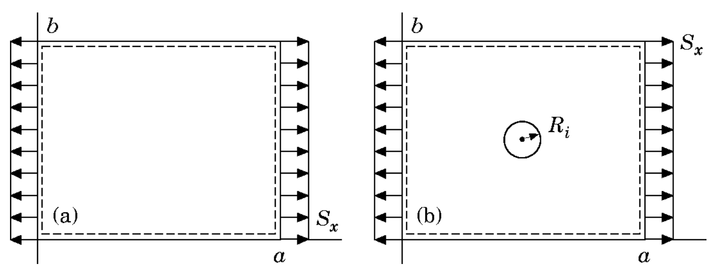

Figure 1. Vibrating plate under study: (a) solid plate, $N_x = S_x$, $N_y = N_{xy} = 0$; (b) plate with a central circular hole; $N_x = N_x(x, y)$, $N_y = N_y(x, y)$, $N_{xy} = N_{xy}(x, y)$.

† As it is customary in the vibrations field the frequency parameters will be expressed in dimensionless form: $\Omega_{nm} = \sqrt{\rho h/D}\,\omega_{nm} a^2$.

0022–460X/97/220358 + 04 $25.00/0/sv960817

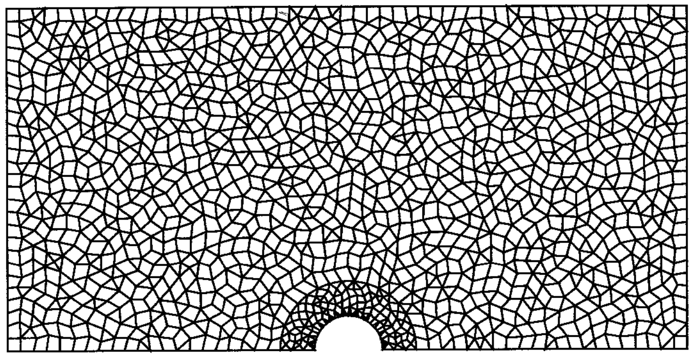

Figure 2. Typical finite element net used in the present study ($R_i/a = 0{\cdot}05$).

the plate center, see Figure 1(b), the determination of the stress field is accomplished solving the corresponding plane stress problem governed by Airy's equation [1]

$$\nabla^4 U = 0 \tag{3}$$

and appropriate boundary conditions.

Once the stress field is evaluated one must substitute the expressions for $N_x(x, y)$, $N_y(x, y)$, and $N_{x,y}(x, y)$, in the vibrating plate equation which now reads

$$D\nabla^4 W - \left(N_x \frac{\partial^2 W}{\partial x^2} + 2N_{xy}\frac{\partial^2 W}{\partial x \partial y} + N_y \frac{\partial^2 W}{\partial y^2}\right) - \rho h \omega_{nm}^2 W = 0. \tag{4}$$

An exact analytical determination of the natural frequency coefficients seems out of the question and one must defer to approximate analytical or numerical procedures.

The present study deals with accurate determinations of the first five natural frequency coefficients of simply supported square plates for two different sized-holes and a rather wide range of values of applied stresses using a well known finite element code [2]. The main goal of this study is to determine the effect of a severe stress concentration field upon the natural frequencies of a basic vibrating structural element.

2. FINITE ELEMENT DETERMINATIONS

The numerical results have been obtained using SAMCEF finite element code with hybrid elements of triangular and rectangular shape (elements type 55 and 56 of the SAMCEF Library). All calculations were performed with a finite element net containing 1694 elements and 1468 nodes. It is important to point out that some verifications were performed taking 2718 elements and 2344 nodes. The eigenvalues determined using the latter net practically agreed with the results obtained using the previously mentioned net. The Poisson ratio was taken equal to 0·30 in all calculations.

3. NUMERICAL RESULTS AND CONCLUSIONS

Table 1 depicts the first five natural frequency coefficients $\Omega_i = \sqrt{\rho h/D}\,\omega_i a^2$, of a solid plate, computed (1) using the finite element method and (2) by means of the exact analytical formulation, for different values of the applied in-plane stress parameter $S_x a^2/D$. The maximum differences are of the order of 0·1%. When no applied stress is acting on the plate, the exact and finite element values agreed to four significant figures (19·74, 49·35, 49·35, 78·96 and 98·70).

TABLE 1

Values of Ω_i in the case of a simply supported solid, square plate subjected to a uniaxial state of stress S_x

Mode		$S_x a^2/D$ 1	5	10	20	50	100
1	a	19·99	20·95	22·10	24·23	29·72	37·11
	b	19·99	20·95	22·10	24·23	29·72	37·10
2	a	49.46	49.86	50.35	51.33	54.14	58.53
	b	49.44	49.85	50.34	51.31	54.12	58.50
3	a	49.76	51.33	53.22	56.81	66.43	79.93
	b	49.77	51.31	53.20	56.79	66.40	79.89
4	a	79.24	80.23	81.46	83.85	90.66	100.98
	b	79.20	80.20	81.42	83.81	90.60	100.91
5	a	98.82	99.02	99.27	99.77	101.26	103.69
	b	98.75	98.95	99.20	99.69	101.16	103.57

a Finite element solution; b exact, analytical results.

TABLE 2

Values of Ω_i in the case of a simply supported square plate with a central circular hole ($R_i/a = 0{\cdot}05$) subjected to S_x

Mode	$S_x a^2/D$ 1	5	10	20	50	100
1	19·93	20·90	22·05	24·20	29·72	37·14
2	49·46	49·86	50·35	51·32	54·15	58·54
3	49·76	51·32	53·20	56·79	66·39	79·86
4	79·10	80·10	81·33	83·74	90·57	100·94
5	98·36	98·60	98·86	99·37	100·85	103·28

Table 2 shows the frequency coefficients when a central hole ($R_i/a = 0{\cdot}05$) is cut in the plate. The eigenvalues are, in general, slightly lower for a given value of $S_x a^2/D$, than those corresponding to a solid plate, except for the first two eigenvalues in the case when $S_x a^2/D = 100$ where they are slightly higher. A similar situation is observed when $R_i/a = 0{\cdot}10$ (Table 3), although now, the value of Ω_1 for $S_x a^2/D = 50$ is also slightly higher in the case of the plate with the circular hole than when the plate is solid.

Clearly, independently of the presence of in-plane stresses, as a hole is placed in the structural element, the plate loses a percentage of mass and flexural rigidity. Depending upon which effect possesses more importance, frequencies will increase, stay constant or decrease. In any case of a simply supported square plate with a central hole the fundamental frequency remains practically constant as the size of the hole increases for $R_i/a < 0{\cdot}15$ when no applied stresses act on the system [3]. This situation is illustrated in Table 4 where the first five values of Ω_i are shown as functions of R_i/a. It becomes apparent in the situation under study that the stress concentration field does not alter significantly, at least the lower natural frequency coefficients; their magnitudes being determined primarily by the value of the applied in-plane stress.

TABLE 3

Values of Ω_i in the case of a simply supported square plate with a central circular hole ($R_i/a = 0{\cdot}10$) subjected to S_x

	$S_x a^2/D$					
Mode	1	5	10	20	50	100
1	19·79	20·28	21·96	24·14	29·74	37·24
2	49·40	49·80	50·30	51·28	54·13	58·56
3	49·70	51·25	53·12	56·68	66·21	79·60
4	78·74	79·76	81·01	83·45	90·39	100·89
5	97·92	98·13	98·38	98·87	100·32	102·70

TABLE 4

Values of Ω_i when no stresses are acting: effect of the concentric circular hole

	R_i/a		
Mode	1	0·05	0·10
1	19·74	19·67	19·53
2	49·35	49·35	49·29
3	49·35	49·35	49·29
4	78·96	78·84	78·49
5	98·70	98·20	97·81

ACKNOWLEDGMENT

The present study has been sponsored by CONICET Research and Development Program (PID-BID 003/92).

REFERENCES

1. S. TIMOSHENKO and J. N. GOODIER 1951 *Theory of Elasticity*. New York: McGraw Hill.
2. SAMCEF User's Manuals. Version 5.1. 1994 Belgium: Samtech and University of Liege.
3. R. D. BLEVINS 1979 *Formulas for Natural Frequency and Mode Shape*. New York: Van Nostrand Reinhold, p. 273.