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THE CONTROL OF THE OPERATING CONDITION OF A SUBHARMONIC VIBROMACHINE

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The dynamics of a vibration machine with piecewise linear elastic ties under external harmonic excitation is investigated. The possibility of controlling the value and asymmetry sign of the subharmonic regime by smooth variation of the frequency of the harmonic excitation force is shown. The expediency of practical application of the non-linear effect revealed is substantiated (the reverse of vibroconveying, rather destruction of the conveying effect, in horizontal vibroforming machines). The results of the theoretical study are confirmed by experiments with a physical model.

1. INTRODUCTION

At present subharmonic vibromachines, the working heads of which perform subharmonic vibrations, are widely used in industry (vibroshakers and vibroconveyers with electromagnetic excitation [1], vibroforming machines and vibrocompactors with inertial excitation [2] etc.). Their uses are due to certain characteristic properties of the subharmonic regime (complex vibration spectrum, essential asymmetry of time response, predominance of low frequency harmonic components in vibration spectrum, etc.) [2–4], which ensure more effective operation of vibromachines.

There are some vibration manufacturing methods, which require an excitation to change the sign of the asymmetry of vibration regimes (reverse of vibroconveying, vibromixing, etc.). Familiar techniques intended for changing of the vibration asymmetry sign need biharmonic excitation [5, 6], which essentially complicates the design of the machine. In this paper a new approach to the control of vibration asymmetry is considered, based on excitation of the subharmonic oscillations of the working head of the vibromachine.

2. MATHEMATICAL MODEL

The vibromachine model to be analyzed is a single-degree-of-freedom vibratory system with piecewise linear elastic elements (see Figure 1). Working head 1, mounted on elastic supports 2 (compound springs), is considered as a perfectly rigid body. The elastic limiter 3 is located with initial clearance Δ relative to the working head. A harmonic force $P \sin(\omega t + \varphi)$ is generated by the external vibroexciter 4.

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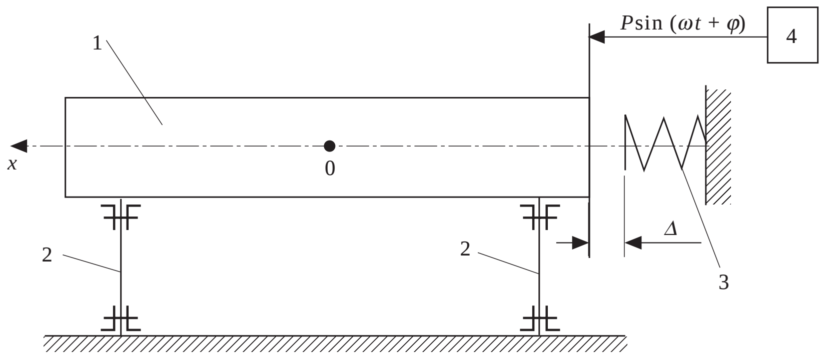

Figure 1. Vibromachine model considered in dynamic analysis: 1, working head of vibromachine; 2, elastic supports; 3, elastic limiter; 4, vibroexciter.

Under the assumption that the vertical displacement of the working head is small in comparison with the horizontal one, the differential equation of vibrations of the working head can be represented as

$$m\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + b\frac{\mathrm{d}x}{\mathrm{d}t} + \left\{\begin{matrix} k_1x,\ x \geqslant -\Delta \\ (k_1 + k_2)x + k_2\Delta,\ x < -\Delta \end{matrix}\right\} = P\sin(\omega t + \varphi), \tag{1}$$

where x is the co-ordinate of the working head, m is the mass of the working head, b is its damping coefficient, k_1 is the stiffness coefficient of the main elastic supports, k_2 is the stiffness coefficient of elastic limiter; P and ω are the amplitude and frequency of the external harmonic excitation and φ is the initial phase angle.

By the substitution $y = x/\Delta$ and $\tau = \omega_1 t = (\sqrt{k_1/m})t$, equation (1) can be transformed into the dimensionless form, more manageable for analysis,

$$\frac{\mathrm{d}^2y}{\mathrm{d}\tau^2} + \beta\frac{\mathrm{d}y}{\mathrm{d}\tau} + \left\{\begin{matrix} y,\ y \geqslant -1 \\ \hline (1 + k_2/k_1)y + k_2/k_1,\ y < -1 \end{matrix}\right\} = p\sin(\nu\tau + \varphi), \tag{2}$$

where $\beta = b/\sqrt{k_1 m}$ is the dimensionless damping coefficient and $p = P/(k_1\Delta)$ and $\nu = \omega/\omega_1$ are the dimensionless amplitude and frequency of the external harmonic excitation.

Equation (2) was solved on a special purpose analogue computer designed in Riga Technical University [2]. The principle of operation of the computer is based on the direct analogy method, which is described in more detail in reference [7].

3. ANALYSIS OF VIBRATIONS

During the investigations the influences of the parameters of the external excitation (P, ω, φ) on the asymmetry coefficient of the subharmonic vibrations has been analyzed. The asymmetry coefficient is quantified by the ratio $\ddot{x}^-/\ddot{x}^+ = \ddot{y}^-/\ddot{y}^+$, where $\ddot{x}^+$ and $\ddot{x}^-$ are the peak values of vibroacceleration of the working head in its forward and backward motions along the x direction, respectively and $\ddot{y}^+ = \ddot{x}^+/(\omega_1^2\Delta)$ and $\ddot{y}^- = \ddot{x}^-/(\omega_1^2\Delta)$ are the dimensionless peak values of the vibroaccelerations $\ddot{x}^+$ and $\ddot{x}^-$. The asymmetry of subharmonic regime is conventionally considered as positive if $\ddot{y}^-/\ddot{y}^+ > 1$ and as negative if $\ddot{y}^-/\ddot{y}^+ < 1$.

As follows from the analysis of the subharmonic solutions of equation (2), there is a singular point on the amplitude-frequency characteristic (AFC) of subharmonic vibrations of the 1/2 order. At this point the time response of the subharmonic regime has become symmetric in acceleration ($\ddot{y}^- = \ddot{y}^+$), but with the asymmetry in displacement y and velocity $\dot{y}$ remaining. What is more, smooth variation of frequency ω in the vicinity of this singular point causes the reversal of the asymmetry sign from positive ($\ddot{y}^-/\ddot{y}^+ > 1$) to negative ($\ddot{y}^-/\ddot{y}^+ < 1$), and vice versa.

As an example, Figure 2(a) shows the AFC of subharmonic oscillations of the 1/2 order and the corresponding graph of asymmetry ratio $\ddot{y}^-/\ddot{y}^+$ versus frequency v. These graphs have been plotted for the following values of the main parameters of equation (2): $p = 15$, $\beta = 0{\cdot}02$, $k_2/k_1 = 3$. Besides, the quantities of half-swing of the displacements $y_0 = (x^+ + x^-)/(2\Delta)$ are laid off as amplitudes on this AFC. As additional information the time responses $\ddot{y} = f(\tau)$ for points d, e and c of the AFC are presented. It is seen that asymmetry ratio $\ddot{y}^-/\ddot{y}^+$ of the subharmonic regime reaches its positive maximum on the upgoing curve *ae* of the AFC (point *d*) and its negative maximum on the downgoing curve *be* of the AFC (point *c*). But at the resonance point *e* of the AFC the time response $\ddot{y} = f(\tau)$ of the subharmonic regime of 1/2 order has become symmetric ($\ddot{y}^- = \ddot{y}^+$); at the same time the asymmetry in the displacement y and velocity $\dot{y}$ remains.

Observable changes in the asymmetry sign of the subharmonic vibrations of 1/2 order are caused by characteristic features of the phase relations between harmonic components of the vibration spectrum of the subharmonic regime. As is shown in Figure 2(b), the phase shift $\varphi_{1/2}$ between the subharmonic component and the external excitation force is changed from $\pi/4$ *to* $\pi/2$ (in the case of $\ddot{y}^-/\ddot{y}^+ > 1$) and from $\pi/2$ to $3\pi/4$ (in the case of $\ddot{y}^+/\overline{\ddot{y}}^+ < 1$). But at the resonance point *e* the phase angle $\varphi_{1/2}$ is always equal to $\pi/2$. At the same time the phase angle $\varphi_{2/2}$ of the fundamental harmonic is changed very slightly, if at all ($\varphi_{2/2} \approx 0$). Since the subharmonic component $\ddot{y}_{1/2}$ and fundamental harmonic $\ddot{y}_{2/2}$ dominate in their amplitudes in the vibration spectrum of the subharmonic regime[2–4], the change of phase difference ($\varphi_{1/2} - \varphi_{2/2}$) from $\pi/4$ to $3\pi/4$ predetermines the reversal of the asymmetry sign of the subharmonic regime.

Thus, there is a range of the parameters of the non-linear system with an asymmetric elastic characteristic within which a subharmonic regime of 1/2 order has become symmetric in the acceleration. The bounds of this range can be determined subject to the condition $\varphi_{1/2} = \pi/2$. As an example, Figure 3 shows the curve on the co-ordinate plane p and v, which is the locus of symmetry of the time response $\ddot{y} = f(\tau)$ of the subharmonic vibrations (the graph corresponds to the case $k_2/k_1 = 3$ and $\beta = 0{\cdot}02$).

4. EXPERIMENTAL INVESTIGATIONS

The practicability of the results obtained by the analogue simulation was verified by experiments with a vibration-testing machine. The line diagram of the experimental installation is shown in Figure 4. The working head 1 had the mass $M = 9{\cdot}64$ kg and was mounted on four compound springs with total stiffness $k_1 = 31\,900$ N m^{-1}. Therefore the natural frequency of the system was $\omega_1 = \sqrt{k_1/M} = 57{\cdot}5$ rad/s. The elastic limiter 3 (a cylindrical helical spring with stiffness $k_2 = 95\,700$ N m^{-1}) was located with initial clearance $\Delta = 0{\cdot}5$ mm relative to working head 1. The logarithmic decrement of small free vibrations was $\delta = 0{\cdot}064$ (with collisions). Horizontal vibrations of working head 1 were generated by the imbalanced vibroexciter 4, operated by the direct current motor 5 through the intermediary of the flexible shaft 6. Smooth variation of the motor rotational speed ω was carried out by an autotransformer brought into the electric circuit of motor 5. The static moment of the imbalanced mass was changed between the limits $(mr)_1 = 0{\cdot}0054$ kg m and $(mr)_2 = 0{\cdot}010$ kg m.

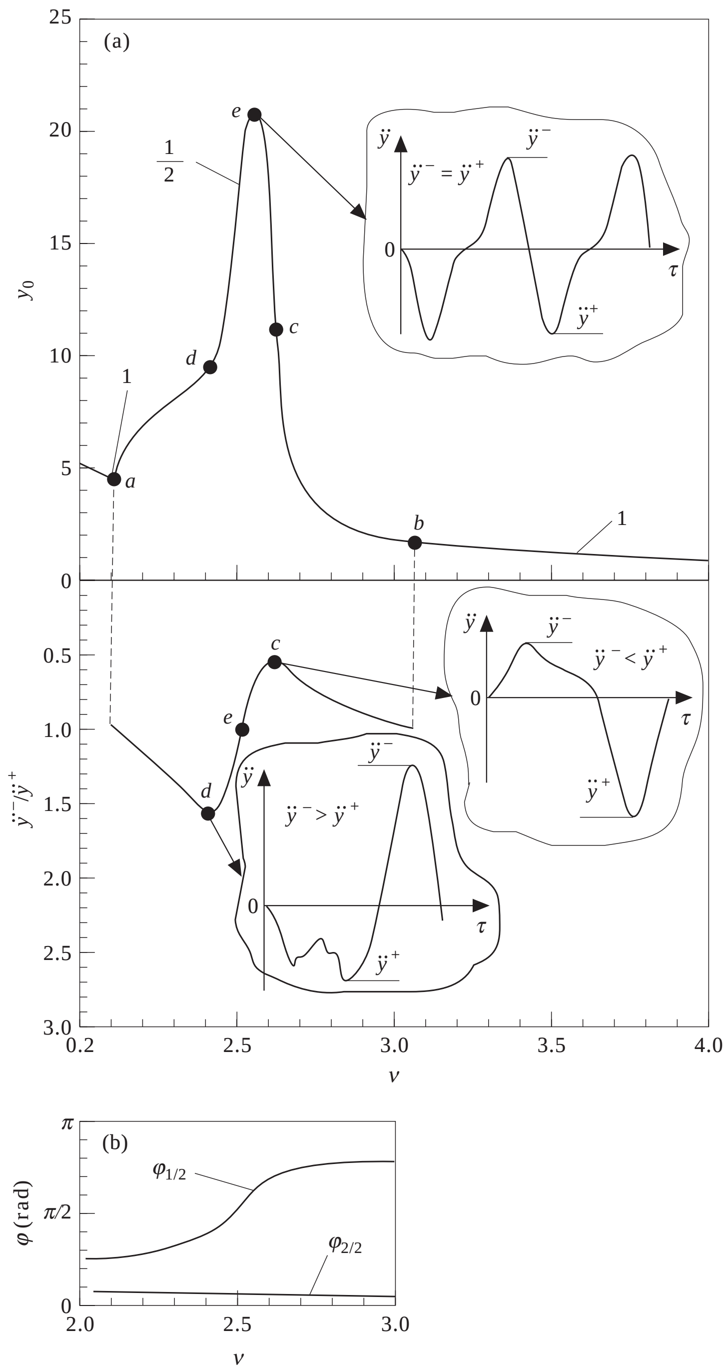

Figure 2. Amplitude-frequency characteristic (a) and phase diagram (b) for subharmonic regime of 1/2 order ($p = 15$, $\beta = 0{\cdot}02$, $k_2/k_1 = 3$).

The rotational speed ω of the imbalanced mass 4 was measured by a Brüel & Kjaer stroboscopic tachometer (type 4913). Measurements of vibration parameters of working head 1 (vibroaccelerations $\ddot{x}^+, \ddot{x}^-$, vibration spectrum, etc.) were made by

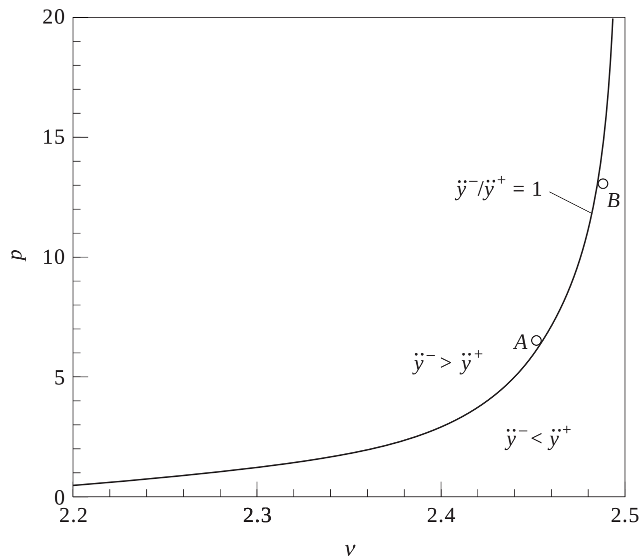

Figure 3. Locus of symmetry of time response $\dot{y} = f(\tau)$ of subharmonic vibrations of 1/2 order ($k_2/k_1 = 3$, $\beta = 0{\cdot}02$). Points A and B correspond to the experimental data: point A for the case $mr = 0{\cdot}0054$ kg m and $\omega = 141$ rad/s; point B for the case $mr = 0{\cdot}01$ kg m and $\omega = 143$ rad/s.

standard electronic instrumentation: piezoelectric transducer 7, amplifier 8, vibration meter 9, voltmeter 10, spectrum analyzer 11, double-beam oscilloscope 12. Calibration of the vibrometering instrumentation was carried out with the aid of a Brüel & Kjaer calibration exciter (type 4294).

The results of the experiments were the following. Under the given parameters of vibration-testing machine, subharmonic vibrations of 1/2 order were steadily excited within the frequency range from 115 to 174 rad/s. Numerical values of vibroaccelerations $\ddot{x}^+$, $\ddot{x}^-$, acceleration difference $\Delta\ddot{x} = \ddot{x}^- - \ddot{x}^+$ and acceleration ratio $\ddot{x}^-/\ddot{x}^+$, as observed during the variation of frequency ω and static moment mr of the harmonic excitation

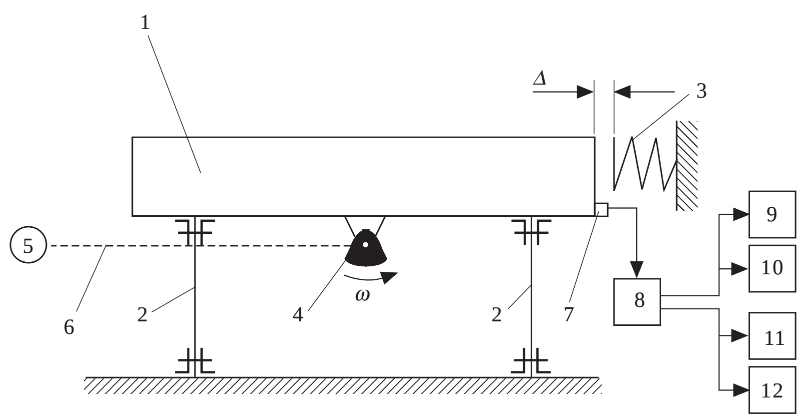

Figure 4. Experimental installation: 1, working head of vibromachine; 2, compound springs; 3, elastic limiter; 4, imbalanced vibroexciter; 5, direct-current motor; 6, flexible shaft; 7, Brüel & Kjaer piezoelectric transducer, Type 4375; 8, Brüel & Kjaer amplifer, Type 2635; 9, vibration meter Model ВШВ-003; 10, Brüel & Kjaer voltmeter, Type 2425; 11, spectrum analyzer, Model SBA-101; 12, double-beam oscilloscope, Model C1-93.

TABLE 1

Results of experiments for the case $mr = 0{\cdot}0054$ kg m

Frequency ω (rad/s)	Amplitude P of excitation (N)	Peak value of acceleration ($\ddot{x}^-$ m s^{-2})	Peak value of acceleration ($\ddot{x}^+$ m s^{-2})	Acceleration difference ($\Delta\ddot{x}$ m s^{-2})	Acceleration ratio ($\ddot{x}^-/\ddot{x}^+$)
115	71·4	8·0	8·0	0	1
120	77·8	11·7	8·6	3·1	1·36
125	84·4	17·0	10·5	6·5	1·62
130	91·3	19·2	11·6	7·6	1·66
135	98·4	21·8	18·1	3·7	1·20
141	107·4	25·3	25·3	0	1
145	113·5	26·3	30·4	−4·1	0·86
150	121·5	21·7	30·0	−8·3	0·72
155	129·7	21·1	27·4	−6·3	0·77
160	138·2	20·9	25·1	−4·2	0·83
165	147·0	18·8	21·0	−2·2	0·90
174	163·5	17·4	17·4	0	1

force $P = mr\omega^2 \sin \omega t$, are presented in Table 1 and Table 2. It is seen that at $mr = 0{\cdot}0054$ kg m (Table 1) the time response of the subharmonic regime is symmetric in acceleration ($\Delta\ddot{x} = 0$), if $\omega = 141$ rad/s. Subharmonic vibrations with positive asymmetry ($\ddot{x}^-/\ddot{x}^+ > 1$) take place at $\omega < 141$ rad/s, but vibrations with negative asymmetry ($\ddot{x}^-/\ddot{x}^+ < 1$) at $\omega > 141$ rad/s. In addition, small objects located on the working head of the vibromachine were transported in one direction at $\omega < 141$ rad/s and in another direction

TABLE 2

Results of experiments for the case $mr = 0{\cdot}01$ kg m

Frequency ω (rad/s)	Amplitude P of excitation (N)	Peak value of acceleration ($\ddot{x}^-$ m s^{-2})	Peak value of acceleration ($\ddot{x}^+$ m s^{-2})	Acceleration difference ($\Delta\ddot{x}$ m s^{-2})	Acceleration ratio ($\ddot{x}^-/\ddot{x}^+$)
115	132·2	14·7	14·7	0	1
120	144·0	18·4	17·6	0·8	1·05
125	156·2	24·1	21·5	2·6	1·12
130	169·0	31·9	24·5	7·4	1·30
135	182·2	44·4	29·6	14·8	1·50
140	196·0	57·9	36·0	21·9	1·61
143	204·5	52·5	52·5	0	1
150	225·0	36·9	61·5	−24·6	0·60
155	240·2	36·4	53·6	−17·2	0·68
160	256·0	37·1	48·1	−11·0	0·77
165	272·2	35·1	38·7	−3·6	0·91
174	302·8	32·2	32·2	0	1

at $\omega > 141$ rad/s. Similar effects were observed in the case of $mr = 0{\cdot}01$ kg m (Table 2). Only a symmetric subharmonic regime in this case is realized at $\omega = 143$ rad/s.

Experimental data have shown close agreement with theoretical results. As an example, two experimental points A and B of the graph of p versus v are shown in Figure 3. Point A corresponds to the case of $\omega = 141$ rad/s and $P = 107{\cdot}4$ N, but point B to the case of $\omega = 143$ rad/s and $P = 204{\cdot}5$ N. In accordance with calculations, deviation of these two points from theoretical curve $p = f(v)$ is not more than 0·5%.

5. PRACTICAL APPLICATIONS IN ENGINEERING

The results of the analysis can find applications in different fields of vibration engineering. Some possible leads for utilization are as follows.

5.1. REVERSAL OF VIBROCONVEYING OF MATERIAL

In the case of vibroconveying of granular materials (sand, cement, gravel, etc.) low frequency vibration regimes (the frequency range is under 50 Hz) are more effective [8]. But most production-type vibroexciters (inertial, electromagnetic) are not able to excite (without special converters) vibrations with a frequency under 50 Hz. Under such conditions, frequency reduction can be achieved by the tuning of the vibroconveyer on the subharmonic regime [1, 9].

It is proposed to use the non-linear effect of reversal of the asymmetry sign for the realization of vibroconveying reversal in subharmonic vibromachines. The direction of vibroconveying is set by the asymmetry sign of the subharmonic regime (see Figure 2); under the condition of $\ddot{y}^-/\ddot{y}^+ > 1$ conveying goes in one direction, but if $\ddot{y}^-/\ddot{y}^+ < 1$ conveying proceeds in the other direction. A control of the asymmetry sign of the subharmonic regime in a real vibroconveyer is carried out technologically rather simply: by smooth variation of the excitation frequency ω in the vicinity of the resonance point e. Unlike familiar practice [3, 4], the proposed method of reversal is carried out with monoharmonic external excitation. Thanks to this, there are real possibilities of reducing the energy losses and to simplify the design of the vibroconveyer.

5.2. ELIMINATION OF UNFAVOURABLE VIBROCONVEYING OF CONCRETE MIX IN HORIZONTAL VIBROFORMING MACHINES

At the present time vibroforming machines generating horizontal resonant vibrations of the form are widely used in production of reinforced concrete blocks (predominantly long or plane) [8, 10]. In order to stabilize resonant vibrations, these vibromachines are usually equipped with double-sided elastic limiters, positioned symmetrically relative to the form [11]. The main demerit of horizontal vibroforming machines lies in the occurrence of unfavourable vibroconveying of concrete mix during its vibrocompaction. The vibroconveying effect is caused by a series of different factors, facilitating the disturbance of initial symmetry in the time response of the form vibrations (inevitable if only a small asymmetry in elastic characteristic, skewness of vibrating table, etc.). Vibroconveying of concrete mix has an unfavourable effect on the homogeneity and strength of the concrete blocks produced.

Existing methods of elimination of the vibroconveying effect are based on execution of fine and labour-consuming adjustments, directed at maximal symmetrization in the time response of the form vibrations. But even after adjustments the vibroconveying effect can occur spontaneously during the operation of the vibromachine (for example, due to irregular wear of the contacting surfaces of the form and elastic limiters, relaxation of bolted and riveted joints, etc.). In such cases the vibroconveying of the concrete mix can be eliminated only by isolating the vibromachine and excluding it from the production process; all this inevitably leads to downtime and rise of manufacturing cost.

The procedure for elimination of the vibroconveying effect can be simplified appreciably by the excitation in the vibromachine of subharmonic vibrations of 1/2 order, which are sufficiently effective for vibroforming [2]. Modernization of the original vibromachine into the subharmonic one can be carried out by mounting other main elastic supports having smaller stiffness k_1 and by exclusion from the structure of one elastic limiter. After that, elimination of unfavourable vibroconveying can be achieved by the application of the non-linear property described above to change the asymmetry sign of the subharmonic regime through variation of the excitation frequency. Specifically, the asymmetry of the time response, which causes the occurrence of the vibroconveying of the concrete mix, is balanced by the smooth regulation of the frequency ω in the vicinity of resonance point e of the AFC (see Figure 2). For example, if after assembly and mounting of the vibromachine the concrete mix is moved in the $+x$ direction, then predominant positive vibroacceleration $\ddot{y}^+$ can be balanced by tuning of the system in the part de of the AFC (asymmetry $\ddot{y}^-/\ddot{y}^+ > 1$). On the contrary, if conveying goes in the opposite direction, the vibromachine must be tuned in the part ec of the AFC. The regulation of the frequency ω during this tuning is terminated only after complete disappearance of the vibroconveying effect.

An important advantage of the proposed approach lies in its simplicity and low labour input. The procedure for elimination of the vibroconveying effect actually reduces to a rather simple production operation: regulation of the excitation frequency ω in the vicinity of subharmonic resonance. And, unlike conventional methods, the necessary tuning of the vibromachine can be carried out with the drive motor running. Besides, machine tuning from the main resonance to the subharmonic one results in itself (with other conditions being equal) in reduction of the specific quantity of metal per structure, and decreasing of the noise level and harmful vibrations transmitted into the environment [1].

6. CONCLUSIONS

The results presented in this paper indicate a new way for control of the operating condition of a subharmonic vibromachine. The proposed method of control is based on utilization of a new non-linear phenomenon revealed during the investigation of a vibratory system with a piecewise linear characteristic of the elastic restoring force. The phenomenon is a specific change of the asymmetry sign of the subharmonic regime under the smooth variation of the frequency of a harmonic excitation force. The practicability of utilizing this non-linear phenomenon has been verified by experiments with a vibration-testing machine. The expendiency of practical application of received results in vibration engineering is substantiated (reversal of vibroconveying, destruction of unfavourable conveying of concrete mix in horizontal vibroforming machines).

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