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PASSING THROUGH THE "ELASTIC WAVE BARRIER" BY A LOAD MOVING ALONG A WAVEGUIDE

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An alternative way of passing through the characteristic wave velocity in a waveguide is investigated by a uniformly moving object along an elastic system with a change in parameters such that a transition from subcritical into supercritical object motion is taking place. To demonstrate this way of passing, a uniformly moving constant load along an infinite elastically supported string with a sudden change in density is studied. The results of this investigation are compared with the more usual way of passing by a fast accelerating load along a homogeneous infinite string on the same foundation. For both cases transient solutions for the string vibrations are derived showing the wave processes, which are a combination of the main features of Mach radiation and transitional radiation in the alternative case and Mach radiation and radiation due to a non-uniform load motion in the usual case. It is shown that the alternative case provides a slightly smaller amplification in displacements than the usual case. The alternative way can be more practical since it is known that the faster the critical velocity is passed the smaller the dynamical amplification will be. Evidently a sudden change in the elastic parameters is much more simply organised than a large acceleration of the train.

1. INTRODUCTION

When a moving object passes through the elastic wave barrier (the characteristic wave velocity in a waveguide) the wave field generated by the object is getting very powerful. A well known example is an aeroplane passing through the sound barrier. This phenomenon can also play an important role at the introduction of high speed trains in certain parts of Europe where the subsoil is soft and the critical surface wave velocity is of the order of 42 m/s ($\approx$150 km/h) [1]. High speed trains have to pass the critical velocity or large investments have to be made to improve the subsoil so as to enlarge the critical velocity along the entire track. In this paper the process of passing through the critical velocity is discussed, since in this way the investments for the track improvements may substantially be reduced. The usual way of passing through the critical velocity is by a fast acceleration of the moving object [2, 3]. In the present paper an alternative way of passing is investigated: i.e., by a uniformly moving object along a system with a sudden change in parameters such that a transition from subcritical into supercritical object motion takes

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place. This way can be more practical since it is known that the faster the critical velocity is passed the smaller the dynamical amplification will be [2, 3].

Evidently local changes in the elastic parameters are much more simply organized than a large acceleration of a train and the comfort of the passengers is more convenient, because they will not feel the effect of the horizontal acceleration due to uniform motion of the train. Further for this sudden change in the parameters, investments are much lower than in the case of enlarging the critical velocity along the entire track, since only the subsoil of a small part of the track has to be improved (near railway stations for acceleration and deceleration).

To investigate the alternative way of passing a uniformly moving constant load along an infinite elastically supported string with a sudden change in density is studied. When the string density is chosen in a proper way the motion along the first part of the string is subcritical and along the second part supercritical. During the transition from sub into supercritical motion the load radiates elastic waves. The resulting radiation process is compared with the radiation process due to the passing of a fast accelerating (abrupt velocity change) load along a homogeneous string on the same elastic foundation.

To visualize both radiation processes transient solutions for the string vibrations are calculated showing a combination of the main features of Mach radiation [4] and transitional radiation [5] in the alternative case and Mach radiation and radiation due to a non-uniform load motion [6] in the usual case. To compare the transitional processes of both cases the parameters are chosen such that the maximum steady state displacements before and after the transition are the same. It is shown that the alternative case then provides a slightly smaller transient amplification in displacements than the usual case.

2. MODEL AND GENERAL TRANSIENT SOLUTIONS IN LAPLACE DOMAIN

First a general problem is considered from which the solutions for both the usual case and the alternative case can be taken. Therefore a constant vertical load moving along an infinite string on an elastic foundation is considered. It is supposed that at $x = 0$ the string mass per unit length (ρ) and the load velocity (v) change abruptly, see Figure 1. In order to have a transition from sub into supercritical motion for $x < 0$ the load velocity is assumed smaller than the wave velocity in the string and for $x > 0$ larger than the wave velocity.

The analysis will proceed as follows. First the expressions describing the string displacements for $t < 0$ will be derived and these expressions (for $t \to 0$) will be used as the initial conditions for deriving the general solution in the Laplace domain for $t \geqslant 0$. Then the inverse Laplace transforms will be taken for the two cases which will be compared: (1) the usual case, for which the string is homogeneous and the load velocity is changed;

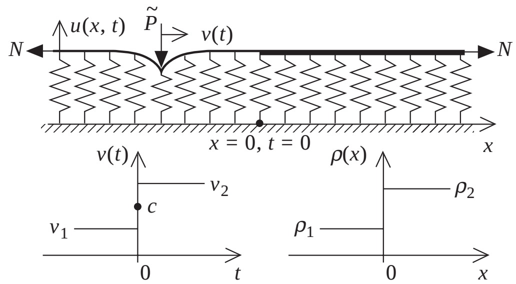

Figure 1. Model, velocity jump and abrupt change in string density at $x = 0$ and $t = 0$.

(2) the alternative case, for which the string is inhomogeneous (jump in ρ) and the load motion is uniform.

The governing equations describing the vertical vibrations of the string for (see Figure 1) are

$$U_{tt}^{(1)} - \rho^2 c_2^2\, U_{xx}^{(1)} + h_2^2\, \rho^2 U^{(1)} = P\rho^2\, \delta(x - v_1\, t) \quad \text{for} \quad x < 0,$$
$$U_{tt}^{(2)} - c_2^2\, U_{xx}^{(2)} + h_2^2\, U^{(2)} = 0 \quad \text{for} \quad x > 0,$$
$$U^{(1)}(0, t) = U^{(2)}(0, t), \qquad U_x^{(1)}(0, t) = U_x^{(2)}(0, t) \tag{1}$$

where

$$c_{1,2} = \sqrt{N/\rho_{1,2}}, \quad v_1 < c_1, \quad h_2^2 = \kappa/\rho_2, \quad P = \tilde{P}/\rho_2 \quad \text{and} \quad \rho^2 = \rho_2/\rho_1.$$

In equations (1) $U^{(n)}(x, t)$ are the vertical string displacements, N is the string tension, κ the stiffness of the elastic foundation per unit length, ρ_n the string mass per unit length, v_n the load velocity, c_n the wave velocity in the string and $\tilde{P}$ is the constant load (the sub- and superscripts (1) denote the string parameters for $x < 0$ and the sub- and superscripts (2) the string parameters for $x > 0$).

The string displacement for $x < 0$ is represented as the following sum of the eigenfield moving with the load $U_e^{(1)}(x, t)$ [5, 6] and a transient solution $U^{(tr)}(x, t)$ which originates as the load is moving near the string inhomogenity:

$$U^{(1)}(x, t) = U_e^{(1)}(x, t) + U^{(tr)}(x, t),$$

with

$$U_e^{(1)}(x, t) = -\frac{P\rho}{2h_2\sqrt{\rho^2 c_2^2 - v_1^2}} \exp\left(\frac{\rho h_2\, |\, x - v_1\, t\, |}{\sqrt{\rho^2 c_2^2 - v_1^2}}\right). \tag{2}$$

Substitution of equations (2) into equations (1) yields

$$U_{tt}^{(tr)} - \rho^2 c_2^2\, U_{xx}^{(tr)} + h_2^2\, \rho^2 U^{(tr)} = 0 \quad \text{for} \ x < 0, \qquad U_{tt}^{(2)} - c_2^2\, U_{xx}^{(2)} + h_2^2\, U^{(2)} = 0 \quad \text{for} \ x > 0,$$

$$U^{(2)}(0, t) - U^{(tr)}(0, t) = \frac{-P\rho}{2h_2\sqrt{\rho^2 c_2^2 - v_1^2}} \exp\left(\frac{\rho h_2\, v_1\, t}{\rho^2 c_2^2 - v_1^2}\right),$$

$$U_x^{(2)}(0, t) - U_x^{(tr)}(0, t) = \frac{P\rho^2}{2(\rho^2 c_2^2 - v_1^2)} \exp\left(\frac{\rho h_2\, v_1\, t}{\sqrt{\rho^2 c_2^2 - v_1^2}}\right). \tag{3}$$

Seeking the solutions of the differential equations of system (3) in the form

$$U^{(2),(tr)}(x, t) = A^{(2),(tr)}(x) \exp\left(\frac{\rho h_2\, v_1\, t}{\sqrt{\rho^2 c_2^2 - v_1^2}}\right)$$

and accounting for the vanishing of the string displacements for $x \to \pm\infty$ one obtains the following expressions:

$$U^{tr}(x, t) = C^{(tr)} \exp\left(\frac{\rho h_2\, v_1\, t + h_2\, \rho x}{\sqrt{\rho^2 c_2^2 - v_1^2}}\right), \tag{4}$$

$$U^{(2)}(x, t) = C^{(2)} \exp\left(\frac{\rho h_2\, v_1\, t}{\sqrt{\rho^2 c_2^2 - v_1^2}} - \frac{h_2\, x}{c_2} \sqrt{1 + \frac{\rho^2 v_1^2}{\rho^2 c_2^2 - v_1^2}}\right). \tag{5}$$

The unknown constants $C^{(2)}$ and $C^{(tr)}$ can be determined by substitution of expressions (4) and (5) into the conditions at $x = 0$ from equations (3). This gives the expressions for the string displacements for $t < 0$ as

$$U^{(1)}(x,t) = -\frac{P\rho}{2h_2\sqrt{\rho^2c_2^2 - v_1^2}}\exp\left(\frac{-\rho h_2|x - v_1t|}{\sqrt{\rho^2c_2^2 - v_1^2}}\right)$$
$$+ \frac{P\rho(\sqrt{\rho^2c_2^2 + v_1^2(\rho^2 - 1)} - \rho c_2)}{2h_2\sqrt{\rho^2c_2^2 - v_1^2}(\sqrt{\rho^2c_2^2 + v_1^2(\rho^2 - 1)} + \rho c_2)}\exp\left(\frac{\rho h_2 v_1 t + h_2\rho x}{\sqrt{\rho^2c_2^2 - v_1^2}}\right), \quad (6)$$

$$U^{(2)}(x,t) = \frac{-P\rho^2c_2}{h_2\sqrt{\rho^2c_2^2 - v_1^2}(\sqrt{\rho^2c_2^2 + v_1^2(\rho^2 - 1)} + \rho c_2)}$$
$$\times \exp\left(\frac{\rho h_2 v_1 t}{\sqrt{\rho^2c_2^2 - v_1^2}} - \frac{h_2 x}{c_2}\sqrt{1 + \frac{\rho^2v_1^2}{\rho^2c_2^2 - v_1^2}}\right). \quad (7)$$

One can now derive the transient string vibrations for $t \geqslant 0$. Since at the end of the analysis some expressions have to be evaluated numerically it is convenient to introduce dimensionless variables and parameters. Introducing a time variable $\tau = h_2 t$, a space variable $y = (h_2/c_2)x$ and the parameters $\alpha_{1,2} = v_{1,2}/c_2$ (ratio of load velocities and the characteristic velocity c_2) one obtains the following set of equations describing the transient vibrations of the string for $\tau \geqslant 0$:

$$(1/\rho^2)U^{(1)}_{\tau\tau} - U^{(1)}_{yy} + U^{(1)} = 0 \qquad \text{for } y < 0,$$
$$U^{(2)}_{\tau\tau} - U^{(2)}_{yy} + U^{(2)} = -T\,\delta(y - \alpha_2\tau) \qquad \text{for } y > 0,$$
$$U^{(1)}(y,0) = -A\exp(by), \qquad U^{(2)}(y,0) = -A\exp(-y\sqrt{1 + \alpha_1^2b^2}),$$
$$U^{(1)}_\tau(y,0) = \alpha_1 A\sqrt{1 + \alpha_1^2b^2}\exp(by), \qquad U^{(2)}_\tau(y,0) = -\alpha_1 Ab\exp(-\sqrt{1 + \alpha_1^2b^2}y),$$
$$U^{(1)}(0,\tau) = U^{(2)}(0,\tau), \qquad U^{(1)}_y(0,\tau) = U^{(2)}_y(0,\tau), \quad (\tau \neq 0). \quad (8)$$

Here

$$v_1 < c_1, \qquad v_2 > c_2, \qquad T = P\rho^2/h_2c_2, \qquad b = \rho/\sqrt{\rho^2 - \alpha_1^2},$$
$$A = \frac{T}{\sqrt{\rho^2 - \alpha_1^2}(\sqrt{\rho^2 + \alpha_1^2(\rho^2 - 1)} + \rho)}.$$

The initial conditions of equations (8) are obtained by taking the limit of expressions (6) and (7) for $t \to 0$. Applying the Laplace transform,

$$V^{(n)}(y,s) = \int_0^\infty U^{(n)}(y,\tau)\exp(-s\tau)\,d\tau,$$

to equations (8) results in the following set of ordinary differential equations:

$$V^{(1)}_{yy} - (1 + s^2/\rho^2)V^{(1)} = (A/\rho^2)(s - \alpha_1\sqrt{1 + \alpha_1^2b^2})\exp(by),$$
$$V^{(2)}_{yy} - (1 + s^2)V^{(2)} = A(s - \alpha_1 b)\exp(-\sqrt{1 + \alpha_1^2b^2}y) + (T/\alpha_2)\exp(-sy/\alpha_2),$$
$$V^{(1)}(0,s) = V^{(2)}(0,s), \qquad V^{(1)}_y(0,s) = V^{(2)}_y(0,s). \quad (9)$$

The general solutions of equations (9), accounting for the proper behaviour for $y \to \pm\infty$, are

$$V^{(1)} = C_1 \exp\left(y\sqrt{1+s^2/\rho^2}\right) + D_1 \exp(by), \tag{10}$$

$$V^{(2)} = C_2 \exp\left(-y\sqrt{1+s^2}\right) + D_2 \exp\left(-y\sqrt{1+\alpha_1^2 b^2}\right) + E_2 \exp(-sy/\alpha_2), \tag{11}$$

where

$$D_1 = -A\,\frac{s-\alpha_1\sqrt{1+\alpha_1^2 b^2}}{s^2-\alpha_1^2 b^2}, \quad D_2 = \frac{-A}{s-\alpha_1 b}, \quad E_2 = -T\,\frac{\alpha_2 d^2}{s^2+(d\alpha_2)^2} \quad \text{with } d = \frac{1}{\sqrt{\alpha_2^2-1}},$$

and provided that $\mathrm{Re}\,(\sqrt{1+s^2}) > 0$ and $\mathrm{Re}\,(\sqrt{1+s^2/\rho^2}) > 0$ for $\mathrm{Re}(s) > 0$. The constants C_1 and C_2 can be taken from the conditions at $y = 0$ of equations (9). This gives

$$C_1 = A\,\frac{(b+\sqrt{1+\alpha_1^2 b^2})(s-\alpha_1\sqrt{1+s^2})}{(s^2-\alpha_1^2 b^2)(\sqrt{1+s^2}+\sqrt{1+s^2/\rho^2})} + Td^2\,\frac{s-\alpha_2\sqrt{1+s^2}}{(s^2+\alpha_2^2 d^2)(\sqrt{1+s^2}+\sqrt{1+s^2/\rho^2})}, \tag{12}$$

$$C_2 = A\,\frac{(b+\sqrt{1+\alpha_1^2 b^2})(s+\alpha_1\sqrt{1+s^2/\rho^2})}{(s^2-\alpha_1^2 b^2)(\sqrt{1+s^2}+\sqrt{1+s^2/\rho^2})} + Td^2\,\frac{s+\alpha_2\sqrt{1+s^2/\rho^2}}{(s^2+\alpha_2^2 d^2)(\sqrt{1+s^2}+\sqrt{1+s^2/\rho^2})}, \tag{13}$$

3. COMPARISON OF THE TWO WAYS OF PASSING

Two ways of passing through the elastic wave barrier (the critical velocity c) by a load moving along a string on an elastic foundation are compared: Case 1 (usual case), passing by an accelerating (velocity jump) constant load along a homogeneous string, $\rho^2 = 1 \wedge \alpha_1 < 1 < \alpha_2 \Leftrightarrow \rho = \text{constant} \wedge v_1 < c < v_2$; Case 2 (alternative case), passing by a uniformly moving constant load along a string with a sudden change in density $\alpha_1 = \alpha_2 = \alpha \wedge 1 < \alpha < \rho \Leftrightarrow v = \text{constant} \wedge c_2 < v < c_1$.

To compare the transitional processes of both cases the parameters are chosen such that (1) the steady state displacements under the load for both cases are the same, (2) the maximum steady state displacements after the transition for both cases are the same and (3) a minimum dynamical amplification for both cases in steady state occurs. This gives the following unique set of parameters: Case 1 (usual); $\alpha_1 = 0{\cdot}972$ and $\alpha_2 = 1.053$ Case 2 (alternative); $\alpha = 1{\cdot}313$ and $\rho = 1{\cdot}35$.

Now the string displacements will be derived for these parameters. By combining equations (10)–(13) the solutions in the Laplace domain of equations (9) for both cases can be written as follows:

Case 1,

$$V^{(1)} = \left\{\frac{Ab\alpha_1}{s^2-\alpha_1^2 b^2}\left(-1+\frac{s}{\alpha_1\sqrt{1+s^2}}\right) - \frac{T\alpha_2 d^2}{2(s^2+\alpha_2^2 d^2)}\left(1-\frac{s}{\alpha_2\sqrt{1+s^2}}\right)\right\}$$

$$\times \exp\left(y\sqrt{1+s^2}\right) - \frac{A}{s+\alpha_1 b}\exp(by), \tag{14}$$

$$V^{(2)} = \left\{\frac{Ab\alpha_1}{s^2 - \alpha_1^2 b^2}\left(1 + \frac{s}{\alpha_1\sqrt{1+s^2}}\right) + \frac{T\alpha_2 d^2}{2(s^2+\alpha_2^2 d^2)}\left(1 + \frac{s}{\alpha_2\sqrt{1+s^2}}\right)\right\}$$

$$\times \exp(-y\sqrt{1+s^2}) - \frac{A}{s-\alpha_1 b}\exp(-by) - \frac{T\alpha_2 d^2}{s^2+\alpha_2^2 d^2}\exp\left(\frac{-sy}{\alpha_2}\right) \quad (15)$$

Case 2,

$$V^{(1)} = \left\{\frac{A(b+\sqrt{1+\alpha^2 b^2})}{s^2-\alpha^2 b^2} + \frac{Td^2}{s^2+\alpha^2 d^2}\right\}\frac{(s-\alpha\sqrt{1+s^2})}{(\sqrt{1+s^2}+\sqrt{1+s^2/\rho^2})}$$

$$\times \exp(y\sqrt{1+s^2/\rho^2}) - A\frac{s-\alpha\sqrt{1+\alpha^2 b^2}}{s^2-\alpha^2 b^2}\exp(by), \quad (16)$$

$$V^{(2)} = \left\{\frac{A(b+\sqrt{1+\alpha^2 b^2})}{s^2-\alpha^2 b^2} + \frac{Td^2}{s^2+\alpha^2 d^2}\right\}\frac{(s+\alpha\sqrt{1+s^2/\rho^2})}{(\sqrt{1+s^2}+\sqrt{1+s^2/\rho^2})}$$

$$\times \exp(-y\sqrt{1+s^2}) - \frac{A}{s-\alpha b}\exp(-\sqrt{1+\alpha^2 b^2 y}) - \frac{Td^2\alpha}{s^2+\alpha^2 d^2}\exp\left(\frac{-sy}{\alpha}\right). \quad (17)$$

To get the string displacements for $t > 0$ inverse Laplace transforms of expressions (14)–(17) have to be determined. The last members in expressions (14) and (16) and the last two members in expressions (15) and (17) are standard functions and can be inverted with the help of tables of integrals (see for example reference [7]). The inverse transforms of the other members of (14)–(17) can be found in reference [8]. Employing the above mentioned transforms yields the solution of equations (8) for Case 1 as

$$U^{(1)} = \frac{2}{\pi}\int_0^1 \left\{\sinh(y\sqrt{1-z^2})\cos(z\tau)\left(\frac{Ab\alpha_1}{z^2+\alpha_1^2 b^2} - \frac{T\alpha_2 d^2}{2(\alpha_2^2 d^2 - z^2)}\right)\right.$$

$$\left. + \cosh(yz)\sin(\tau\sqrt{1-z^2})\left(\frac{Ab}{b^2-z^2} - \frac{Td^2}{2(d^2+z^2)}\right)\right\}\mathrm{H}(y+\tau)\,\mathrm{d}z$$

$$- A\exp(b(y-\alpha_1\tau)), \quad (18)$$

$$U^{(2)} = \frac{2}{\pi}\int_0^1 \left\{\sinh(y\sqrt{1-z^2})\cos(z\tau)\left(\frac{Ab\alpha_1}{z^2+\alpha_1^2 b^2} - \frac{T\alpha_2 d^2}{2(\alpha_2^2 d^2 - z^2)}\right)\right.$$

$$\left. + \cosh(yz)\sin(\tau\sqrt{1-z^2})\left(\frac{Ab}{b^2-z^2} - \frac{Td^2}{2(d^2+z^2)}\right)\right\}\mathrm{H}(\tau-y)\,\mathrm{d}z$$

$$- A\exp(b(y-\alpha_1\tau)) - \{\exp(b(y-\alpha_1\tau))\mathrm{H}(\tau-y)$$

$$+ \exp(-b(y-\alpha_1\tau))\mathrm{H}(y-\alpha_2\tau)\}$$

$$+ \{-A\exp(-b(y-\alpha_1\tau)) + Td\sin(d(y-\alpha_2\tau))\}\{\mathrm{H}(y-\tau)$$

$$- \mathrm{H}(y-\alpha_2\tau)\}, \quad (19)$$

where $\alpha_1 = 0{\cdot}972$, $\alpha_2 = 1{\cdot}053$ and $\rho = 1$. For Case 2 one similarly obtains

$$U^{(1)} = -A \exp(by)\left\{\cosh(\alpha b\tau) - \frac{\sqrt{1+\alpha^2 b^2}}{b}\sinh(\alpha b\tau)\right\}\mathrm{H}(-y-\rho\tau)$$

$$+ A\left\{\frac{2\rho^2}{\pi(\rho^2-1)}\int_1^\rho \frac{f_{11}(z)}{z^2(z^2+\alpha^2 b^2)}\,\mathrm{d}z\right.$$

$$\left.+\frac{2}{\pi}\int_0^1 \frac{f_{12}(z)}{(z^2+\alpha^2 b^2)(\sqrt{1-z^2}+\sqrt{1-z^2/\rho^2}\}}\,\mathrm{d}z\right\}\mathrm{H}(y+\rho\tau)$$

$$+ Td^2\left\{\frac{2\rho^2}{\pi(\rho^2-1)}\int_1^\rho \frac{f_{13}(z)}{z^2(\alpha^2 d^2 - z^2)}\,\mathrm{d}z\right.$$

$$\left.+\frac{2}{\pi}\int_0^1 \frac{f_{14}(z)}{(\alpha^2 d^2 - z^2)(\sqrt{1-z^2}+\sqrt{1-z^2/\rho^2}}\,\mathrm{d}z\right\}H(y+\rho\tau)$$

$$-\frac{A}{2}\left\{\left(1-\frac{\sqrt{1+\alpha^2 b^2}}{b}\right)\exp(-b(y+\alpha\tau))\right.$$

$$\left.+\left(1+\frac{\sqrt{1+\alpha^2 b^2}}{b}\right)\exp(b(y-\alpha\tau))\right\}\mathrm{H}(y+\rho\tau), \tag{20}$$

$$U^{(2)} = -A\left\{\exp(\alpha b\tau - y\sqrt{1+\alpha^2 b^2})\mathrm{H}(y-\alpha\tau) + \exp(-(\alpha b\tau - y\sqrt{1+\alpha^2 b^2}))\mathrm{H}(\tau - y)\right\}$$

$$+A\left\{\frac{2\rho^2}{\pi(\rho^2-1)}\int_1^\rho \frac{f_{21}(z)\sqrt{1-z^2/\rho^2}}{z^2(z^2+\alpha^2 b^2)}\,\mathrm{d}z + \frac{2}{\pi}\int_0^1 \frac{f_{22}(z)}{(z^2+\alpha^2 b^2)(\sqrt{1-z^2}+\sqrt{1-z^2/\rho^2}}\,\mathrm{d}z\right\}\mathrm{H}(\tau - y)$$

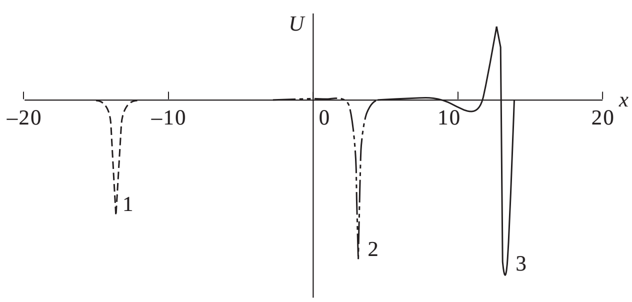

Figure 2. Vibrations for the usual way of passing, case 1.

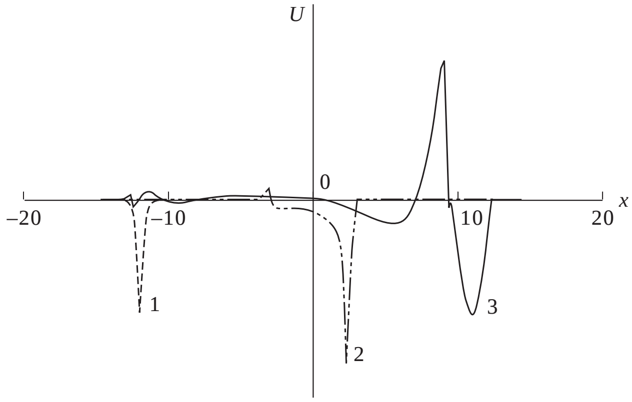

Figure 3. Vibrations for the alternative way of passing, Case 2.

$$+ Td^2\left\{\frac{2\rho^2}{\pi(\rho^2-1)}\int_1^{\rho}\frac{f_{23}(z)}{z^2(\alpha^2 d^2 - z^2)}\,dz - \frac{2}{\pi}\int_0^1 \frac{f_{24}(z)}{(\alpha^2 d^2 - z^2)(\sqrt{1-z^2}+\sqrt{1-z^2/\rho^2})}\,dz\right\}H(\tau - y)$$

$$- \{A \exp(\alpha b\tau - y\sqrt{1+\alpha^2 b^2}) + Td \sin(d(\alpha\tau - y))\}\{H(\alpha\tau - y) - H(\tau - y)\}, \qquad (21)$$

where $\alpha = \alpha_{1,2} = 1{\cdot}313$ and $\rho = 1{\cdot}35$. The functions $f_{ij}(z)$ in equations (20) and (21) are given in the Appendix.

In Figures 2 and 3, the string vibrations for the usual and the alternative case are depicted respectively. The displacements are shown for three specific moments: (1) before the transition, when the eigenfield is moving stationary with the load and no waves are being radiated; (2) during the transition, when radiation is taking place due to a non-uniformly moving load or transitional radiation, respectively [5, 6], (3) after the transition, when the Mach radiation field (steady state shape of the string due to a supercritically moving load) [4] originates. The figures show that both the transitional radiation and the radiation due to a non-uniformly moving object is taking place during the change of the load eigenfield into the Mach field. It is also seen that the transient vibrations are powerful: i.e., maximum transient displacements are larger than the steady state displacements.

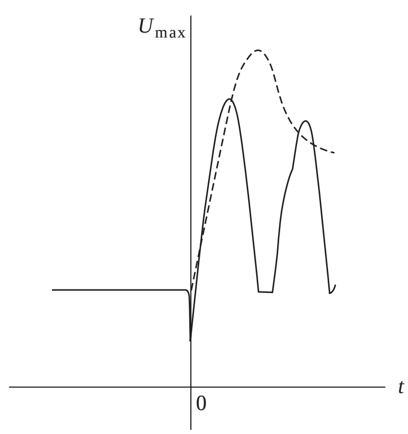

Figure 4. Maximum displacements for both the usual (– – –) and the alternative (——) cases.

Figure 4 shows the maximum displacements as function of time for both cases. It is evident from this figure that the maximum displacements in the alternative case are smaller than in the usual case and that the transient vibrations are finite (even in case for smooth acceleration of the load the displacements are finite [1, 2]). So the alternative way of passing through the elastic wave barrier provides a somewhat smaller dynamical amplification.

4. CONCLUSIONS

In this paper an alternative way of passing through the elastic wave barrier has been investigated: i.e., by a uniformly moving object along an elastic system with a change in parameters such that a transition from subcritical into supercritical object motion is taking place. This way of passing is compared with the more usual way of passing by a fast accelerating object along a homogeneous elastic system. For both cases the transient vibrations are derived. It is shown that the transient vibrations are more powerful than the steady state vibrations and are finite in both cases. By comparing the maximum displacements for both cases it can be seen that the alternative way provides a slightly smaller dynamical amplification than the usual way.

The results of this investigation are the first in research into an alternative way of passing through the elastic wave barrier. The main advantages of this alternative way are that local changes in the elastic parameters are much more simply organized than a large acceleration of a train and that the comfort of the passengers is more convenient, because they will not feel the effect of the horizontal acceleration due to uniform motion of the train.

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APPENDIX

The functions $f_{ij}(z)$ used in equations (20) and (21) are as follows:

$$f_{11}(z) = \sinh\left(y\sqrt{1-z^2/\rho^2}\right)\cos(z\tau)\left\{\left(b+\sqrt{1+\alpha^2 b^2}\right)\sqrt{z^2-1}\left(\alpha\sqrt{z^2-1}-z\right)\right\},$$

$$f_{12}(z) = \cosh\left(y\sqrt{1-z^2/\rho^2}\right)\sin(z\tau)\left\{\left(b+\sqrt{1+\alpha^2 b^2}\right)\sqrt{1-z^2/\rho^2}\left(z-\alpha\sqrt{z^2-1}\right)\right\},$$

$$f_{13}(z) = \{z \cosh(y\sqrt{1 - z^2/\rho^2}) \sin(z\tau) + \alpha\sqrt{1 - z^2} \sinh(y\sqrt{1 - z^2/\rho^2}) \cos(z\tau)\}(b + \sqrt{1 + \alpha^2 b^2}),$$

$$f_{14}(z) = -z \cosh(y\sqrt{1 - z^2/\rho^2}) \sin(z\tau) - \alpha\sqrt{1 - z^2} \sinh(y\sqrt{1 - z^2/\rho^2}) \cos(z\tau),$$

$$f_{21}(z) = \sin(y\sqrt{z^2 - 1} - z\tau)\{(b + \sqrt{1 + \alpha^2 b^2})(\alpha\sqrt{z^2 - 1} - z)\},$$

$$f_{22}(z) = \{z \cosh(y\sqrt{1 - z^2}) \sin(z\tau) + \alpha\sqrt{1 - z^2/\rho^2} \sinh(y\sqrt{1 - z^2}) \cos(z\tau)\}(b + \sqrt{1 + \alpha^2 b^2}),$$

$$f_{23}(z) = \sqrt{1 - z^2/\rho^2} \sin(y\sqrt{z^2 - 1} - z\tau)\{z - \alpha\sqrt{z^2 - 1}\},$$

$$f_{24}(z) = z \cosh(y\sqrt{1 - z^2}) \sin(z\tau) - \alpha\sqrt{1 - z^2/\rho^2} \sinh(y\sqrt{1 - z^2}) \cos(z\tau).$$