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FREQUENCY ANALYSIS OF A PARALLEL FLAT PLATE-TYPE STRUCTURE IN STILL WATER, PART I: A MULTI-SPAN BEAM

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This paper is part I of a two part paper. The model used in part I is a multi-span wide beam in water contained by a rigid water trough. The multi-span beam is used to imitate a typical substructure of a parallel flat plate-type structure. There exists a narrow channel between the lower surface of the wide beam and the upper surface of the bottom plate of the water trough. By using the small parameter expansion method, the added mass and damping coefficients for a typical cross-section of a plate-fluid-plate system are deduced. Moreover, by means of the added water mass and damping coefficients, the free vibration frequencies of the simple model of a multi-span wide beam are analyzed in the first part of the paper. In the second part of this paper, the free vibration frequencies of a complex model of a parallel flat plate-type structure will be analyzed.

1. INTRODUCTION

The assemblies of parallel flat plates are used in the core designs of some research and power reactors including the Engineering Test Reactor, the Materials Testing Reactor and Shipping Atomic Power Station [1]. The flat plates are separated by narrow channels full of a coolant water. In these structures, the inertial and viscous coupling effect of the coolant water, which is expressed as added mass and damping, is usually strong when the structural components vibrate, a direct result of which is that the free vibration frequencies of the structure decrease greatly. Hence, it is necessary to analyze the coupling effect of the water on the frequencies of the structure in the designs of plate-type fuel elements.

The inertial coupling effect of fluid on the free vibration of a structure consisting of circular (or hexagonal or square) cylinders has been successfully studied. For the case of circular cylinders, Chen [2] studied the virtual mass matrix of fluid by using classical ideal flow theory and their free vibration characteristics. Similar work was carried out in references [3–7]. Paidoussis [8] studied the virtual masses of clusters of parallel cylinders in liquid contained by an outer channel and their free vibration characteristics by means of both potential flow theory and fluid finite element method. For the case of hexagonal cylinders, Fujita [9] used the Euler equation to determine the added mass coefficient. A similar study can be found in reference [10] which was for the case of both square and hexagonal cylinders.

When the viscous coupling effects are considered, the analysis becomes more complex. This situation was investigated in some work [11–13] in which both theoretical and experimental methods were used. Using the model of a hexagonal cylinder oscillating inside a hexagonal cavity filled with an incompressible viscous fluid, Wilson [14] gave a

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closed-form analytical expression representing added mass and damping coefficients by means of an approximate thin gap equation derived from the Navier–Stokes equations, which were solved by an asymptotic expansion in terms of a non-dimensional frequency parameter. A main restrictive condition was that the frequency of the hexagonal cylinder was lower.

Generally, it is difficult to analyze the frequencies of a complex plate-type structure in water because there are the coupling effects between water and structure. It is the aim of this paper to develop a simplified method for surmounting such a difficulty. The key to analyzing the frequency of the complex plate-type structure lies in determination of added water mass and added water damping. In part I of this paper, a simple model is used to obtain the added mass and damping of a typical structure of plate-fluid-plate. In part II of this paper [15], the added mass and damping are extended to form the added mass matrix and damping matrix of a complex plate-type structure. As a simple model, the free vibration frequencies of a multi-span wide beam in a rigid water trough, which is used to imitate a typical substructure of a parallel flat plate-type structure, are analyzed in this part of the paper. There is a narrow channel between the lower surface of the beam and the upper surface of the bottom plate of the water trough. By means of the method similar to that in reference [14], the coupling forces of water are considered when the beam simply vibrates. Here, the two term expansion of the stream function for the fluid in the channel is used instead of the one term expansion, which eliminates the restrictive condition of low frequency in reference [14]. Moreover, the varying tendency between the frequencies of the beam and the gaps is studied. The values of calculated frequency are compared with those of measured frequency.

2. MODEL AND ITS MOTION EQUATIONS

The complex plate-type structure consists of one main plate-type beam and $2N_1$ pieces of minor plate-type beams with the same geometrical and material characteristics. The minor plate-type beams are symmetrically placed on the upper side and the lower side of the main plate-type beam. The geometrical integrity, between these beams is maintained by five retaining blocks. The structure is supported by two linear springs and two torsional springs. Figure 1 shows this structure when $N_1 = 5$. The simple model used here is a hinged four-span beam on the bottom plate of a rigid water-trough, which is used to imitate a minor plate-type beam of the complex structure. There is a torsional spring with stiff

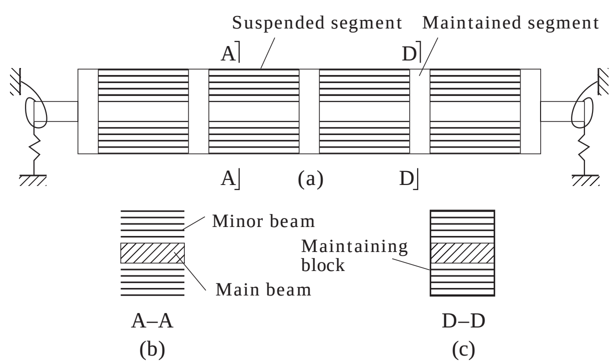

Figure 1. Sketch of the mechanical model of a parallel flat-plate-type structure. (a) Structure supported by two linear springs and two torsional springs. (b) Typical cross-section of a suspending segment. (c) Typical cross-section of a maintaining segment.

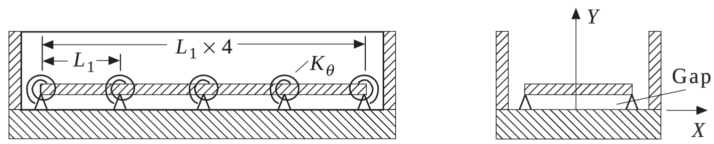

Figure 2. Sketch of the mechanical model of a hinged four-span beam. K_θ is the stiffness coefficient of torsional springs, L_1 the length of one span of the beam. (a) The beam in a rigid water trough. (b) Sketch of the cross-section and the gap between the lower surface of the beam and the upper surface of bottom plate of the rigid water-trough.

coefficient K_θ at each hinged point. The model is shown in Figure 2(a) and its cross-section is shown in Figure 2(b). There is a narrow channel filled with water between the lower surface of the beam and the upper surface of bottom plate of the rigid water-trough, its cross-sectional gap is shown in Figure 2(b). The free surface of the water is in the same plane as the upper surface of the four-span beam.

Obviously, the supporting conditions of the simple model are different from those of a minor plate-type beam in the whole structure, but it is the purpose to obtain the added water mass and the added water damping of a typical cross-section of plate-fluid-plate system as shown in Figure 3. Hence, the difference of supporting conditions of both models is ignored in the following analysis.

Two basic assumptions are made: (a) the beam is rigid in the wide direction (X direction in Figure 2(b)) and the simple harmonic vibration of the cross-section of the beam is parallel to the Y direction; (b) the water motion occurs only in the X–Y plane and the water is treated as an incompressible viscous fluid.

For assumption (a), the structure only consists of plane bending beam elements if the finite element method is used to describe the structure motion in air. Its motion equations can be expressed in the form

$$\mathbf{M}_0\ddot{y} + \mathbf{K}_0\mathbf{y} = 0, \tag{1}$$

where $\mathbf{M}_0$ is the mass matrix, $\mathbf{K}_0$ the stiffness matrix, $\mathbf{y}$ is a general displacement column, $(\dot{\ })$ denotes differentiation with respect to time. Because the free surface of water is in the same plane as upper surface of the beam, the water reaction forces on the upper surface of the beam are very small and are simply taken to be zero when the beam is vibrating. Hence, the water reaction forces on the beam only result from the water in the channel. The motion equations of the structure in water can be expressed in the form

$$\mathbf{M}_0\ddot{y} + \mathbf{K}_0 y = \mathbf{F}, \tag{2}$$

where $\mathbf{F}$ is water reaction force column. For frequency analysis, $\mathbf{F}$ is expressed in the form

$$\mathbf{F} = -\mathbf{C}_d\dot{y} - \mathbf{M}_d\ddot{y}, \tag{3}$$

where $\mathbf{C}_d$ and $\mathbf{M}_d$ are added mass matrix and added damping matrixes respectively.

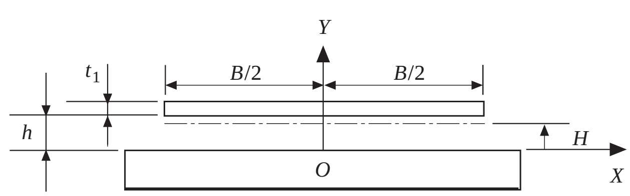

Figure 3. Co-ordinate system. t_1 is the thickness of a cross-section of the beam, h the gap width, B the width of cross-section of the beam and H the distance between the lower side of the cross-section of the beam and the upper side of cross-section of the bottom plate of the rigid water trough when the beam is vibrating.

3. DETERMINATION OF WATER REACTION FORCES

The co-ordinate system is shown in Figure 3, in which t_1 denotes the thickness of cross-section of the beam, B the width of cross-section of the beam and h the width of the gap. From basic assumption (a), the motion of the beam relative to the bottom of the rigid water-trough can be represented by H as

$$H = h + A \sin \omega t, \tag{4}$$

where A is the amplitude of simple harmonic vibration of the beam, ω frequency and t time.

In reality, the motion of the water in the gap is three-dimensional. Because the four-span beam is placed in still water, the longitudinal influx approximates to zero. Thus, a two-dimensional model of the water is used and the motion equations of the water in the gap are

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0, \qquad \frac{\partial U}{\partial t} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{1}{\rho}\frac{\partial p}{\partial X} + v\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right),$$

$$\frac{\partial V}{\partial t} + U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{1}{\rho}\frac{\partial p}{\partial Y} + v\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right), \tag{5}$$

where U is the fluid velocity component in X direction, V the fluid velocity component in the Y direction, p is pressure, ρ is fluid density, μ the fluid viscous coefficient and $v = \mu/\rho$. Introducing the stream function Ψ

$$U = -\partial\Psi/\partial Y, \qquad V = \partial\Psi/\partial X \tag{6}$$

and adopting the following non-dimensional variables and parameters

$$x = X/B, \quad y = Y/h, \quad \tau = \omega t, \quad R = \omega AB/v, \quad u = \epsilon U/\omega A, \quad v = V/\omega A,$$
$$\psi = \Psi/\omega AB, \quad \epsilon = h/B, \quad \beta = A/h, \quad \eta = H/h = 1 + \beta \sin\tau,$$

equation (5) can be rewritten into the form

$$\mathrm{D}/\mathrm{D}\tau\nabla^2\psi = -(\beta/R)\nabla^4\psi. \tag{7}$$

where

$$\nabla^2 = \epsilon^3\frac{\partial^2}{\partial x^2} + \epsilon\frac{\partial^2}{\partial y^2}, \qquad \nabla^4 = \epsilon^4\frac{\partial^4}{\partial x^4} + 2\epsilon^2\frac{\partial^4}{\partial^2 x\,\partial^2 y} + \frac{\partial^4}{\partial y^4},$$

$$\frac{\mathrm{D}}{\mathrm{D}\tau} = \frac{\partial}{\partial\tau} + \beta\frac{\partial\psi}{\partial y}\frac{\partial}{\partial x} - \beta\frac{\partial\psi}{\partial x}\frac{\partial}{\partial y}.$$

The boundary conditions for the velocities corresponding to equation (7) are

$$-\frac{\partial\psi}{\partial y} = \frac{\partial\psi}{\partial x} = 0, \quad y = 0; \qquad -\frac{\partial\psi}{\partial y} = 0, \qquad \frac{\partial\psi}{\partial x} = \frac{1}{\beta}\frac{d\eta}{d\tau}, \qquad y = \eta. \tag{8}$$

For the case of infinite parallel plates, the pressure at the exits of the cross-section of the gap could be taken as constant. For the case of the finite plates shown in Figure 3, the pressure at the exits of $X = \pm B/2$ (see Figure 3) will simply be taken as a function of time, $p_1(t)$.

3.1. STREAM FUNCTION

Assuming the gap h (see Figure 2) to be small, $\epsilon = h/B$ is a small parameter. Hence, the non-dimensional stream function can be expressed in the expansion

$$\psi = \psi_0 + \epsilon\psi_1 + O(\epsilon^2). \tag{9}$$

Substituting equation (9) into equations (7), (8) yields for the ϵ^0 term

$$\partial^4\psi_0/\partial y^4 = 0. \tag{10}$$

$$-\frac{\partial\psi_0}{\partial y} = \frac{\partial\psi_0}{\partial x} = 0, \quad y = 0; \qquad -\frac{\partial\psi_0}{\partial y} = 0, \qquad \frac{\partial\psi_0}{\partial x} = \cos\tau, \qquad y = \eta \tag{11}$$

and for the ϵ term

$$\frac{\partial^4\psi_1}{\partial y^4} = -\frac{R}{\beta}\frac{\partial^3\psi_0}{\partial\tau\partial y^2} - R\frac{\partial\psi_0}{\partial y}\frac{\partial^3\psi_0}{\partial x\partial y^2} + R\frac{\partial\psi_0}{\partial x}\frac{\partial^3\psi_0}{\partial y^3}, \tag{12}$$

$$-\partial\psi_0/\partial y = \partial\psi_0/\partial x = 0, \qquad y = 0, \eta \tag{13}$$

From equations (11), (12), one can obtain

$$\psi_0 = F_0(\tau) + x[3(y/\eta)^2 - 2(y/\eta)^3]\cos\tau. \tag{14}$$

From equations (12–14),

$$\psi_1 = F_1(\tau) + \left[\frac{Rf_2}{2\beta}\right]y^2 + \left[\frac{Rf_3}{6\beta}\right]y^3 + \left[\frac{Rx\sin\tau}{4\beta\eta^2}\right]y^4 - \frac{R}{\beta}\left[\frac{3x\beta\cos^2\tau}{10\eta^4} + \frac{x\sin\tau}{10\eta^3}\right]y^5$$
$$+\left[\frac{Rx\cos^2\tau}{5\eta^5}\right]y^6 - \left[\frac{2Rx\cos^2\tau}{35\eta^6}\right]y^7 \tag{15}$$

where

$$f_2 = -(11x/10)\sin\tau + (13\beta x/7\eta)\cos^2\tau, \qquad f_3 = (6x/5\eta)\sin\tau - (81\beta x/35\eta)\cos^2\tau.$$

3.2. PRESSURE GRADIENT

Adopting the non-dimensional pressure expression [14]

$$\tilde{p} = p/(\omega\mu AB^2/h^3) \tag{16}$$

and from equation (5), the non-dimensional momentum equations become

$$\frac{\partial\tilde{p}}{\partial x} = -\frac{\partial^3\psi}{\partial y^3} - \epsilon^2\frac{\partial^3\psi}{\partial x^2\partial y} + \frac{\epsilon R}{\beta}\frac{\partial^2\psi}{\partial\tau\partial y} - \epsilon R\left(\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y} - \frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}\right) \tag{17}$$

$$\frac{\partial\tilde{p}}{\partial y} = \epsilon^4\frac{\partial^3\psi}{\partial x^3} + \epsilon^2\frac{\partial^3\psi}{\partial y^2\partial x} - \frac{\epsilon^3 R}{\beta}\frac{\partial^2\psi}{\partial\tau\partial x} + \epsilon^3 R\left(\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x^2} - \frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial x\partial y}\right) \tag{18}$$

Substituting equation (9) into equations (17), (18) respectively, the two term expansions of the pressure gradient can be obtained

$$\frac{\partial \tilde{p}}{\partial x} \approx \frac{12x}{\eta^3}\cos\tau + \epsilon R\left(-\frac{6x}{5\beta\eta}\sin\tau + \frac{81x}{35\eta}\cos^2\tau\right), \qquad \frac{\partial \tilde{p}}{\partial y} \approx 0 \tag{19, 20}$$

In equation (19), the first part of the right side is in phase with the velocity of the beam, the second part in phase with the acceleration of the beam and the third part is lower and higher harmonic. This part can be neglected for frequency analysis of the structure. Equation (19) is rewritten in the form

$$\partial\tilde{p}/\partial x \approx (12x/\eta^3)\cos\tau - (6\epsilon x R/5\beta\eta)\sin\tau \tag{21}$$

3.3. FLUID FORCES ON THE BEAM PER UNIT LENGTH IN LONGITUDE DIRECTION

Integrating equation (21) from x to 1/2, the non-dimensional pressure distribution is

$$\tilde{p} = \tilde{p}_1(\tau) + \left(\frac{12x^2 - 3}{2\eta^3}\right)\cos\tau - \left(\frac{\epsilon R(12x^2 - 3)}{20\beta\eta}\right)\sin\tau \tag{22}$$

where $\tilde{p}_1(\tau)$ is the non-dimensional pressure at the exit of $X = B/2$. It is a small value and is simply taken to be zero. When the amplitude of simple harmonic vibration is small, $\eta = 1 + (A^1/h)\sin\tau \approx 1$. The dimensional form corresponding to equation (22) is

$$p = \frac{6\mu\omega AB^2}{h^3}\left[\left(\frac{x}{B}\right)^2 - \frac{1}{4}\right]\cos\omega t - \frac{3\mu\omega ABR}{5\beta h^2}\left[\left(\frac{x}{B}\right)^2 - \frac{1}{4}\right]\sin\omega t \tag{23}$$

Hence the fluid forces associated with the pressure can be obtained by integrating equation (23) from $-B/2$ to $B/2$

$$F_1 = \int_{-B/2}^{B/2} p\,\mathrm{d}x = -(\mu\omega AB^3/h^3)\cos\omega t + \rho\omega^2 AB^3/10h \sin\omega t$$

or

$$F_1 = -\mu(B/h)^3\dot{H} - (\rho B^3/10h)\ddot{H} \tag{24}$$

The fluid forces associated with the shear stress can be calculated from

$$F_2 = 2\int_{-B/2}^{B/2} \mu \left.\frac{\partial U}{\partial Y}\right|_{Y=H} \mathrm{d}X = \frac{3\mu}{2}\left(\frac{B}{h}\right)^2\dot{H} + \frac{11\rho B^2}{40}\ddot{H} \tag{25}$$

All fluid forces on the beam per unit length in its longitude direction is

$$F_0 = F_1 - F_2 = (-\omega c_{ad} + \omega^2 m_{ad})H \tag{26}$$

where $c_{ad} = \mu(B/h)^2(B/h + 3/2)$ will be called the added water damping coefficient, $m_{ad} = \rho B^2(B/10h + 11/40)$ called the added water mass coefficient. Equation (26) describes a distribution of water reaction forces on the beam. Figure 4 shows the distribution on a bending beam element, in which $2L$ denotes the length of the beam element. Obviously the water reaction force distribution is associated with the displacement of the beam. Assuming $\psi_k(z)$ to be the distribution of kth mode of the beam vibrating in air, the

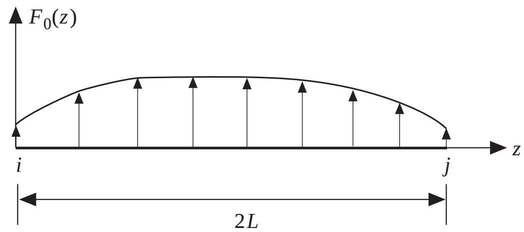

Figure 4. The distribution of the water reaction force on a bending beam element in its longitude direction.

displacement of the beam in water can approximately be expressed in the form

$$H = \sum_{k=1}^{N} \psi_k(z)\xi_k, \tag{27}$$

where N is the number of modal shapes of the beam vibrating in air, and ξ_k is the coefficient. By means of the eigenvectors which result from equation (1), the distribution $\psi_k(z)$ can be calculated by using the third order spline interpolation method. The distributing forces of water reaction on a beam element (Figure 4) are simply lumped on the two nodal points of the beam element. The results are

$$F_i = (-\omega c_{ad} + \omega^2 m_{ad}) \sum_{k=1}^{N} \int_0^L \psi_k(z)\,\mathrm{d}z\xi_k, \qquad F_j = (-\omega c_{ad} + \omega^2 m_{ad}) \sum_{k=1}^{N} \int_L^{2L} \psi_k(z)\,\mathrm{d}z\xi_k, \tag{28, 29}$$

From equations (28), (29), the right side of equation (2) can be constructed. There exists a little difference between the right side and the left hand side of equation (2), which is that the left side is expressed in a physical co-ordinate system, but the right side is in a mode co-ordinate system. The eigenvectors of the beam vibrating in air are used to expand the general displacement column **y**. Equation (2) can then be rewritten in the form

$$(\mathbf{I} + \mathbf{M}_{ad})\ddot{\xi} + \mathbf{C}_{ad}\dot{\xi} + \boldsymbol{\Lambda}\xi = 0 \tag{30}$$

where **I** is a unit matrix, $\mathbf{M}_{ad}$ is the added mass matrix in the mode co-ordinate system, $\mathbf{C}_{ad}$ is the added damping matrix in the mode co-ordinate system, $\boldsymbol{\Lambda} = \mathrm{diag}\,(\omega_I^2)$, ω_I is frequency of the beam vibrating in air and $\xi = \{\xi_1, \xi_2, \ldots, \xi_N\}$.

4. FREQUENCIES OF THE BEAM VIBRATING IN WATER

The following are the values of the main parameters: $B = 0{\cdot}047$ m, $L = 0{\cdot}1625$ m, $h = 0{\cdot}015$ m, $K_\theta = 350$ Nm/rad, $\mu = 1{\cdot}004 \times 10^{-3}$ Ns/m^2, water density $\rho = 1000$ kg/m^3, material density of the beam $\rho_1 = 8400$ kg/m^3 and the elasticity modulus of the beam $E = 9{\cdot}8 \times 10^{10}$ N/m^2.

Equation (30) describes the free vibration of a damped system. Its eigenvalues can be expressed in the form

$$\lambda_i = \zeta_i\omega_i + \mathrm{j}\sqrt{1 - \zeta_i^2}\,\omega_i, \qquad i = 1, 2, \ldots, N \tag{31}$$

where ζ_i is viscous damping factor, ω_i is frequency, $\mathrm{j} = \sqrt{-1}$. Equation (30) can be

TABLE 1

Frequency values of the beam

	Frequencies in air (Hz)				Frequencies in water (Hz)			
	ω_1	ω_2	ω_3	ω_4	ω_1	ω_2	ω_3	ω_4
A	187·4	198·1	213·1	227·1	121·7	128·7	138·5	147·6
B	196·0	207·0	219·0	228·0	118·0	137·0	147·0	152·0
C	4·4	4·3	2·7	0·4	3·1	6·1	6.1	2·9

A, computational values of frequency; B, test values of frequency; C, the relative errors of frequency (%).

rewritten in the form

$$\begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{\Lambda} & -\mathbf{M}^{-1}\mathbf{C}_{ad} \end{bmatrix} q = \lambda q \tag{32}$$

where $\mathbf{M}^{-1} = (\mathbf{I} + \mathbf{M}_{ad})^{-1}$, $q = (\xi^{\mathrm{T}} \ \dot{\xi}^{\mathrm{T}})^{\mathrm{T}}$. Neglecting the N eigenvalues with positive real parts in the $2N$ eigenvalues resulting from equation (32), the remainder eigenvalues are written as

$$\lambda_i = a_i + \mathrm{j}b_i, \qquad i = 1, 2, \ldots, N \tag{33}$$

where $q_i = \mathrm{Re}(\lambda_i)$, $b_i = \mathrm{Im}(\lambda_i)$. Comparing equations (31) and (33) yields

$$\zeta_i\omega_i = -a_i, \qquad \sqrt{1-\zeta_1^2}\,\omega_i = b_i. \tag{34}$$

From equation (34), one can write

$$\zeta_i^2 = a_i^2/(b_i^2 - a_1^2), \qquad \omega_i^2 = (a_i/\zeta_i)^2. \qquad 1 = 1, 2, \ldots, N \tag{35}$$

When the gap $h = 0{\cdot}015$ m, the values of the first four frequencies are listed in Table 1, from which one observes that the maximum value of the relative error between the computational frequencies and the test frequencies of the beam vibrating in air is 4·4%, and the maximum one of the beam vibrating in water is 6·1%. The relative error of the first frequency of the beam vibrating in water is 3.1%. This relative error can satisfy the

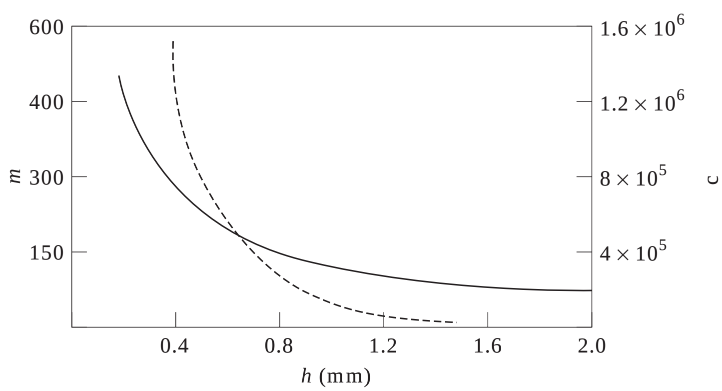

Figure 5. The varying curves of added water mass damping coefficients with gaps. The solid line denotes the added mass coefficient, the dashed line the added damping coefficient. Non-dimensional added mass coefficient $m = m_{ad}/(\rho_1 B t_i)$, non dimensional added damping coefficient $c = c_{ad}/\mu$, h is the gap.

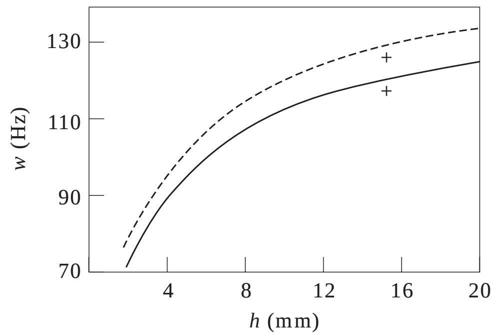

Figure 6. The varying curves of the frequencies of the beam vibrating in water with gaps. Solid line denotes the first frequency, dashed line the second frequency and + the test values of the two frequencies.

needs of dynamical analysis in engineering. The coincidence between the computational values and the test values of frequency demonstrates the correctness of the added water mass and damping matrices.

Now, the varying tendency of added mass, added damping coefficients and frequencies of the beam vibrating in water with the gap width h are examined. Figure 5 shows the varying curve of computational added mass and added damping coefficients with the gap width h, in which the solid line denotes the non-dimensional added mass coefficient $m = m_{ad}/(\rho_i B t_i)$, and the dashed line the non-dimensional added damping coefficient $c = c_{ad}/\mu$. Obviously, the added damping coefficient increases more sharply than the added mass coefficient does when the gap becomes smaller. The reason is that the added damping coefficient is inversely proportional to h^3, but the added mass coefficient is only inversely proportional to h (see equation (26)). Hence, when the gap h is very small, the effects of added damping on the response of a structure with narrow channels become more important than those of added mass. The varying curves of the first two frequencies of the four-span beam vibrating in water with the gap are shown in Figure 6, in which the solid line denotes the first frequency, the dashed line the second frequency and '+' the test values of the two frequencies. Figure 6 shows that the smaller the gap h, the lower the frequencies are. The reason is that the added water mass increases when the gap h decreases which makes the frequencies of the beam drop. Hence, the varying curves of the frequencies of the beam are typical. Moreover, when the gap $h = 0{\cdot}015$ m, there is a 35% decrease of the first frequency of the beam vibrating in water as compared with that in air. When the gap $h = 0{\cdot}002$ m, the frequency decreases by 63%.

5. CONCLUSIONS

By means of the small parameter expansion method similar to that used in reference [14], the added water mass and damping coefficients for a typical cross-section of plate-fluid-plate are deduced. The added water mass and damping are used to analyze the frequency of a hinged four span beam vibrating in water and contained by a rigid water trough. The correctness of added mass and damping coefficients is indirectly confirmed by the coincidence between the computational frequency of the beam and the test one.

The frequencies of the hinged four-span beam vibrating in water depends largely on the gap width between the lower side of the cross-section of the beam and the upper side of the cross-section of the bottom plate of the water trough. The smaller the gap, the larger is the added water mass and the lower are the frequencies of the beam.

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