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FREQUENCY ANALYSIS OF A PARALLEL FLAT PLATE-TYPE STRUCTURE IN STILL WATER, PART II: A COMPLEX STRUCTURE

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This paper is part II of a two part paper. The model used in this part is a parallel flat plate-type structure in a rigid water trough or rigid rectangular tube. A narrow channel exists between any two adjacent plates of the structure. The motion equations of the plate-type structure vibrating in water are obtained by extending the method for a typical cross-section of plate-fluid-plate system presented in the first part of this paper. The computational frequencies of the structure vibrating both in air and in water are compared with those measured by a resonance test. The results show that the local frequencies of the minor plate-type beams of the structure decrease much more strongly than those of the structure vibrating in water. Moreover, the varying tendency of frequencies of the structure with two different water boundary conditions is discussed.

1. INTRODUCTION

In part I of this paper [1], the added water mass and damping coefficients for a typical cross-section of plate-fluid-plate system and the distributions of the fluids reaction forces on a plate-type beam are obtained by using the small parameter expansion method slightly different from that used in reference [2]. Here, the dynamical characteristics of a more complex structure with a group of parallel flat plate-type beams in a rigid water trough or a rigid rectangular tube are studied. There are narrow channels between the flat plate-type beams. The narrow channels are filled with coolant water. The finite element method is used to obtain the mass and stiffness matrices of the structure and the method presented in the first part of this paper [1] is extended to obtain the added mass and damping matrices of the water in the channels.

2. DESCRIPTION OF THE MODEL

The model used here consists of one main plate-type beam and $2N_1$ minor plate-type beams with the same geometrical and material characteristics. The minor plate-type beams are located symmetrically on the upper and lower sides of the main plate-type beam. The geometrical integrity between these beams is maintained by five retaining blocks. The structure is supported by two linear springs and two torsional springs. Figure 1 shows the structure when $N_1 = 5$. Two types of water boundary conditions are considered in this paper: one is a rigid water trough, and the other is a rigid rectangular tube.

In reference [3], the dynamical characteristics of the structure were calculated by using two methods. In one of the methods, only thin plate elements were used to obtain the

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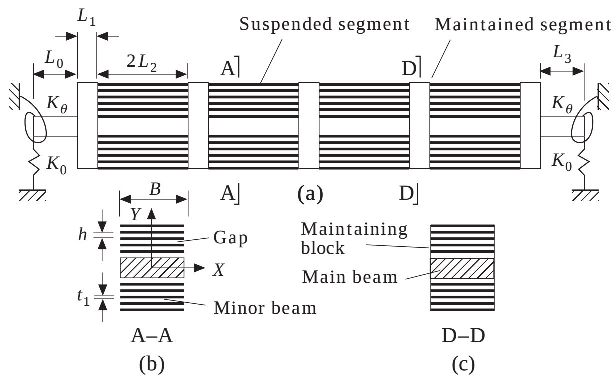

Figure 1. Sketch of the mechanical model of a parallel flat plate-type structure. (a) Structure supported by two linear springs and two torsional springs. (b) Typical cross-section of a suspending segment. (c) Typical cross-section of a maintained segment.

computational model of finite element of the structure, and in the other only pure plane bending beam elements were used to obtain the computational model. The results showed that the calculated frequencies and modal shapes of the two methods were almost the same for the lower order frequencies, and the rotational frequencies of the structure were related to higher order ones. In the case of dynamical analysis for engineering, the simple plane bending beam elements can be used to replace the more complex thin plate elements, and the rotational motion of the structure can be ignored for the parallel flat plate-type structure used here.

There are two basic assumptions: (a) The cross-section of each plate-type beam contained in the structure is rigid in the wide direction (X direction in Figure 1(b)) and the simple harmonic vibrations of the cross-sections are parallel to the Y direction; (b) Because all plate-type beams in each maintained segment of the structure (see Figure 1(a) and (c)) are held together by a retaining block, a maintained segment is considered as a combinative beam element with an equivalent bending stiffness.

In the suspended segments of the structure (see Figure 1(a)), there are relative displacements between the plate-type beams. In order to conveniently analyze the fluid reaction forces on the beams, the middle point of each plate-type beam in the suspended segments of the structure is taken as a nodal point, as shown in Figure 2. Thus the structure consists only of bending elements. If the finite element method is used to obtain the motion equations of the structure vibrating in air, the number of degrees of freedom of the structure is $N_A = 2 \times (12 + 4 \times (2N_1 + 1))$ and the motion equations have the form

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{K}\mathbf{q} = 0, \tag{1}$$

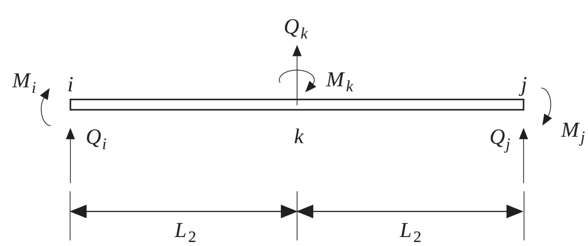

Figure 2. Nodal points of one plate-type beam in a suspended segment.

where **K** is the stiffness matrix, **M** the diagonal mass matrix, **q** the general displacement column and ($\dot{}$) denotes differentiation with respect to time. Generally speaking, the values of the elements corresponding to the rotational displacements in the mass matrix are small and the effects of the rotary inertia on the lower order frequencies of the structure are not important. The mass condensation method [4] is applied to reduce the mass matrix and the stiffness matrix. The general displacement column **q** is divided into the form

$$\mathbf{q} = (\mathbf{y}^{\mathrm{T}} \quad \boldsymbol{\theta}^{\mathrm{T}})^{\mathrm{T}}, \tag{2}$$

where **y** is the transversal displacement column and **θ** the rotational displacement column. Then the mass matrix and the stiffness matrix in equation (1) can be partitioned accordingly, with the result being

$$\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{bmatrix} y \\ \theta \end{bmatrix} = \omega^2 \begin{bmatrix} M_{11} & \\ & M_{22} \end{bmatrix} \begin{bmatrix} y \\ \theta \end{bmatrix}. \tag{3}$$

The reduced motion equations are

$$\mathbf{K}_1 y = \omega^2 \mathbf{M}_1 y, \tag{4}$$

where $\mathbf{K}_1 = K_{11} - K_{12}K_{22}^{-1}K_{21}$, $\mathbf{M}_1 = M_{11} + K_{12}K_{22}^{-1}M_{22}K_{22}^{-1}K_{21}$ and ω is the frequency. It is obvious that $\mathbf{K}_1$ and $\mathbf{M}_1$ remain symmetrical. The eigenvalues and the normalized eigenvectors of equation (4) are

$$\omega_i^2, \quad \boldsymbol{\Psi}_i, \qquad i = 1, 2, \ldots, N, \tag{5}$$

where

$$N = N_A/2, \qquad \boldsymbol{\Psi}_i^{\mathrm{T}}\mathbf{K}_1\boldsymbol{\Psi}_i = \omega_i^2, \qquad \boldsymbol{\Psi}_i^{\mathrm{T}}\mathbf{M}_1\boldsymbol{\Psi}_i = 1.$$

3. MOTION EQUATIONS OF THE STRUCTURE VIBRATING IN WATER

When a structure vibrates in water, there is a coupling effect between the structure and the water. The water reaction forces on the structure give expression to the coupling effects. In the parallel flat plate-type structure shown in Figure 1, there is a narrow channel filled with water between any two adjacent plate-type beams, the typical cross-section of which is shown in Figure 3. In part I [1], a four-span beam is used to imitate a minor plate-type beam of the parallel flat plate-type structure and the distribution of the reaction forces of the water in such a typical channel is obtained based on the two-dimensional fluid model. Generally speaking, the fluids are three-dimensional for the model discussed here. Because the structure is placed in still water, the longitudinal influx approximates to zero. It is evident that any one of the plate-type beams is close to two channels at most. One channel is on the upper surface of the beam and the other on the lower surface of the beam. Hence,

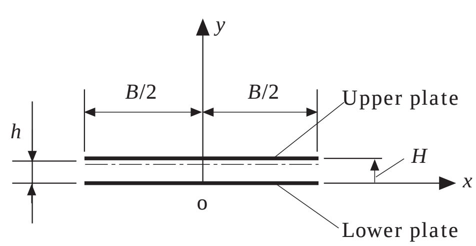

Figure 3. Cross-section of typical plate-fluid-plate structure.

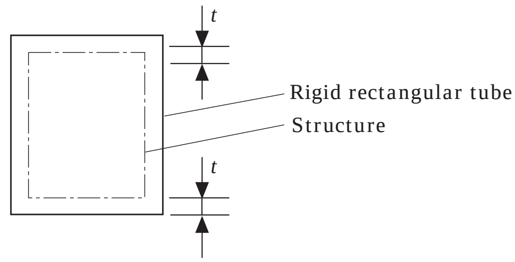

Figure 4. Cross-section sketch of the structure in water contained by a rigid rectangular tube.

the water reaction forces on the beam are equal to the sum of the reaction forces of the water in the upper channel and those of the water in the lower channel. By using the distribution of the total water reaction forces, the lumped force on each nodal point of the structure can be obtained. Then, the motion equations of the structure vibrating in water can be obtained by placing the lumped forces in the right side of equation (4). Of course, the water boundary conditions of the cross-section of a four-span beam at the exits are slightly different from those of the parallel flat plate-type structure. As only an approximation is required, the differences are ignored in the following analysis.

3.1. WATER BOUNDARY OF A RIGID RECTANGULAR TUBE

The cross-section of the structure and the tube is shown in Figure 4. In this case, there are another two channels, one of which is between the upper outline of the structure and the upper rigid side of the tube, and the other between the outline of the structure and the lower rigid side of the tube.

3.1.1. *The water reaction forces on the plate-type beams of the suspended segments*

By means of a small parameter expansion method, the distribution of the fluid reaction force on the beam along its longitude direction has been deduced in reference [1] for the typical cross-section of plate-fluid-plate system shown in Figure 3, which is

$$f = -c_{ad}\dot{H} - m_{ad}\ddot{H}, \qquad c_{ad} = \mu(B/h)^2(B/h + 1{\cdot}5), \qquad m_{ad} = \rho B^2(B/10h + \tfrac{11}{40}), \quad (6, 7)$$

where μ is the viscous coefficient of fluid, ρ the density of fluid, H the relative displacement between the two plates and $(\dot{\ })$ represents differential with respect to time. If the absolute displacement of the upper plate in Figure 3 is y_u and that of the lower plate is y_d, the relative displacement $H = y_u - y_d$. For the three beams shown in Figure 5, the fluid reaction forces on the middle beam result from the fluid in the upper gap and the fluid in the lower gap. Using equation (6), the distribution of fluid reaction forces on the middle beam along its longitude direction has the form

$$f_m = -c_{ad}(\dot{y}_m - \dot{y}_d) - m_{ad}(\ddot{y}_m - \ddot{y}_d) - c'_{ad}(\dot{y}_m - \dot{y}_u) - m'_{ad}(\ddot{y}_m - \ddot{y}_u),$$

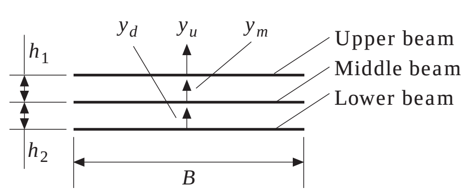

Figure 5. Local cross-section of three adjacent plate-type beams.

or

$$f_m = (\omega \mathbf{c}_d - \omega^2 \mathbf{m}_d)\mathbf{w}_m, \tag{8}$$

where

$$\mathbf{m}_d = [m'_{ad} \;\; -(m_{ad} + m'_{ad}) \;\; m_{ad}], \qquad \mathbf{c}_d = [c'_{ad} \;\; -(c_{ad} + c'_{ad}) \;\; c_{ad}],$$

$$\mathbf{w}_m = \{\mathbf{y}_u \quad \mathbf{y}_m \quad \mathbf{y}_d\},$$

c_{ad} and m_{ad} are the results of equation (7) when $h = h_2$, c'_{ad} and m'_{ad} are the results of equation (7) when $h = h_1$. The distribution is shown in Figure 6. Equation (8) shows that the fluid force distribution is associated with the displacements of the three plate-type beams. Using the normalized modal shapes ψ_i (see equation (5)) of the structure vibrating in air, the displacement column $\mathbf{w}_m$ can be expanded as

$$\mathbf{w}_m = \sum_{k=1}^{N} \psi_k^m(z)\xi_k, \tag{9}$$

where $\psi_k^m(z)$ are the distributions of the components of kth modal shape ψ_k corresponding to $\mathbf{w}_m$ in the longitude direction, N is the number of the modes and ξ_k are constants. The distributions $\psi_k^m(z)$ can be calculated by using the method of third-order spline interpolation. Substituting in equation (8) yields

$$f_m = (\omega \mathbf{c}_d - \omega^2 \mathbf{m}_d)\mathbf{\Psi}^m(z)\xi \tag{10}$$

where

$$\mathbf{\Psi}^m(z) = [\Psi_1^m \quad \Psi_2^m \quad \cdots \quad \Psi_N^m], \qquad \xi = \{\xi_1, \xi_2, \ldots, \xi_N\}.$$

Hence, the lumped fluid forces on the nodal points in Figure 6 can be expressed in the forms

$$F_i = \int_0^{0\cdot5L_2} f_m(z)\,\mathrm{d}z, \qquad F_k = \int_{0\cdot5L_2}^{1\cdot5L_2} f_m(z)\,\mathrm{d}z, \qquad F_j = \int_{1\cdot5L_2}^{2L_2} f_m(z)\,\mathrm{d}z. \tag{11}$$

3.1.2. *The water reaction forces on the plate-type beams of the maintaining segments*

As mentioned above, a maintained segment is considered as a bending beam element with an equivalent bending stiffness. The fluid reaction forces on a maintained segment come from the fluids in the two gaps, one of which is the gap between the upper surface of the maintained segment and the upper boundary of the rigid rectangular tube, and the other is the gap between the lower surface of the maintained segment and the lower

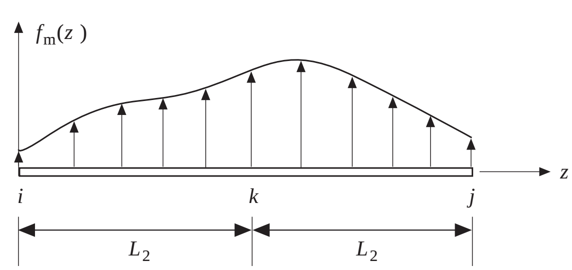

Figure 6. Sketch of the distribution of water reaction forces on a beam in a suspended segment along its longitude direction.

boundary of the rigid rectangular tube. In this case, the distribution of the fluid reaction forces on a maintained segment along its longitude direction has an expression similar to equation (8), but $y_u = y_d = 0$ here. $c_{ad} = c'_{ad}$ and $m_{ad} = m'_{ad}$ are the results of equation (7) when $h = t$. With the same steps as those in section 3.1.1, the lumped fluid forces on two nodal points of a maintaining segment can be expressed in the form

$$F_i = \int_0^{0\cdot 5L_1} f_m(z)\,\mathrm{d}z, \qquad F_j = \int_{0\cdot 5L_1}^{L_1} f_m(z)\,\mathrm{d}z. \tag{12}$$

3.1.3. *The water reaction forces on the two beam elements adjacent to the supporting points of the main plate-type beam*

As shown in Figure 1(a), there is no narrow channel on the up or down sides of the two beam elements near the two supporting points of the main plate-type beam. So, the experiential formula presented in reference [5] is adopted to calculate the fluid reaction forces on the two beam elements. The formula is

$$f_e(z) = \pi\rho\omega^2(B/2)^2 y_e, \tag{13}$$

where y_e is the distribution of displacement of an element along its longitude direction. With the same steps as those in section 3.1.2, the lumped fluid forces on the nodal point are

$$F_i = \int_0^{0\cdot 5L} f_e(z)\,\mathrm{d}z, \qquad F_j = \int_{0\cdot 5L}^{L} f_e(z)\,\mathrm{d}z. \tag{14}$$

In equation (14), $L = L_0$ for the left element and $L = L_3$ for the right element.

3.1.4. *The motion equations of the structure vibrating in water*

Using all the forces mentioned above, the fluid force column of the structure can be constructed as

$$\mathbf{F} = -[\omega C_a - \omega^2 M_a]\boldsymbol{\xi}. \tag{15}$$

Substituting equation (15) into equation (4) and premultiplying it by $\boldsymbol{\Psi}^{\mathrm{T}}$, one can express the motion equations of the structure vibrating in water in the form

$$[\boldsymbol{\Lambda} + \omega C_{aa} - \omega^2(\mathbf{I} + M_{aa})]\boldsymbol{\xi} = 0 \tag{16}$$

where

$$\boldsymbol{\Lambda} = \mathrm{diag}\,(\omega_I^2), \qquad C_{aa} = \boldsymbol{\Psi}^{\mathrm{T}} C_a, \qquad M_{aa} = \boldsymbol{\Psi}^{\mathrm{T}} M_a, \qquad \boldsymbol{\Psi} = [\Psi_1, \Psi_2, \ldots, \Psi_N],$$

$\mathbf{I}$ is a matrix and $\boldsymbol{\xi}$ is a column. With the method described in the first part of this paper, equation (16) can be rewritten in the form

$$\begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}_0^{-1}\Lambda & -\mathbf{M}_0^{-1}C_{aa} \end{bmatrix}\eta = \lambda\eta \tag{17}$$

where $\mathbf{M}_0^{-1} = (\mathbf{I} + M_{aa})^{-1}$, $\eta = (\boldsymbol{\xi}^{\mathrm{T}}\ \dot{\boldsymbol{\xi}}^{\mathrm{T}})^{\mathrm{T}}$. The frequencies of the structure vibrating in water are

$$\omega_I^2 = a_I^2/\zeta_I^2, \qquad \zeta_I^2 = a_I^2/(b_I^2 - a_I^2), \qquad I = 1, 2, \ldots, N \tag{18}$$

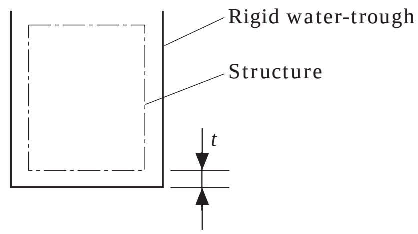

Figure 7. Cross-section of the structure in water contained by a rigid water trough.

where

$$a_I = \text{Re}\,(\lambda_I), \qquad b_I = \text{Im}\,(\lambda_I).$$

3.2. WATER BOUNDARY OF A RIGID WATER TROUGH

The cross-section of the structure and the water trough is shown in Figure 7. In this case, supposing the free surface of the water is in the same plane as the upper surface of the outline of the structure does, the fluid forces on the upper surface of the uppermost plate-type beam of the structure are simply taken as zero, which is the only difference found when compared to the case of the water boundary of a rigid rectangular tube. The remainder fluid forces and analyzing steps are the same as those in section 3.1. It is hence unnecessary to go into details.

4. FREQUENCIES OF THE STRUCTURE AND DISCUSSIONS

There are five minor plate-type beams with the same geometrical and material characteristics on both sides of a main plate-type beam in the computational model used here. The values of the basic parameters are: $N_1 = 5$, $h = 0{\cdot}002$ m, $B = 0{\cdot}0477$ m, $t = 0{\cdot}028$ m, $t_1 = 0{\cdot}002$ m, $L_0 = 0{\cdot}03$ m, $L_1 = 0{\cdot}08$ m, $L_2 = 0{\cdot}081$ m, $L_3 = 0{\cdot}05$ m, $K_\theta = 16{\cdot}84$ Km/rad, $K_0 = 168{\cdot}4$ MN/m. The bending stiffness of the minor plate-type beams is $(EI)_1 = 37{\cdot}4$ Nm^2 and their line density $\rho_1 = 0{\cdot}808$ kg/m. The bending stiffness of the main plate-type beam is $(EI)_2 = 3{\cdot}654$ KNm^2 and its line density $\rho_2 = 10{\cdot}205$ kg/m. The water viscous coefficients is $\mu = 1{\cdot}0041 \times 10^{-3}$ Ns/m^2 and its density $\rho = 1000$ kg/m^3.

4.1. FREQUENCIES OF THE STRUCTURE VIBRATING IN AIR

The first 42 frequencies of the structure vibrating in air are listed in Table 1, in which there is a dense frequency band containing 36 frequencies with a difference of about 1 Hz. One finds that these frequencies in the band are the typical frequencies of local vibrations of the minor plate-type beams by means of the corresponding modes. Figure 8(a) shows one of these typical modal shapes of local vibration. From the first modal shape of the

TABLE 1

Frequencies of the plate-type structure vibrating in air, (Hz)

	ω_1	ω_2	ω_3	ω_4	ω_5–ω_{40}	ω_{41}	ω_{42}
A	68·5	199·8	228·1	230·9	232·6–233·3	252·3	298·5
B	70·8	180·0	212·0	216·0	223·0– —	232·0	—
C	3·2	11·0	7·6	6·9	4·3– —	8·8	—

A, the computational values of frequency; B, the test values of frequency; C, the relative errors of frequency, (%).

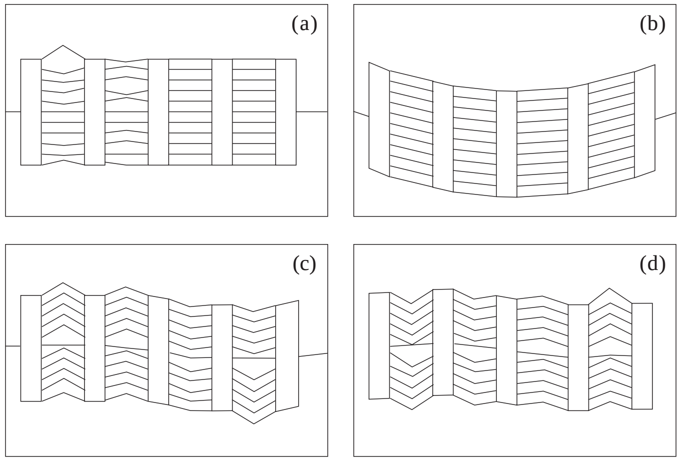

Figure 8. Modal shapes of the structure vibrating in air. (a) Modal shape corresponding to the fifth order calculated frequency, $\omega_5 = 232{\cdot}6$ Hz. (b) Modal shape corresponding to the first order calculated frequency, $\omega_1 = 68{\cdot}5$ Hz. (c) Modal shape corresponding to the second order calculated frequency, $\omega_2 = 199{\cdot}8$ Hz. (d) Modal shape corresponding to the 42nd order calculated frequency, $\omega_{42} = 298{\cdot}5$ Hz.

structure (see Figure 8(b)), one knows that the first frequency is the first bending frequency of the whole structure vibrating. The second to fourth modal shapes are the coupling ones of the whole structure and the minor plate-type beams vibrating, so that their corresponding frequencies are called the "coupling frequencies". Figure 8(c) is the second modal shape. The 41st frequency is a coupling one corresponding to the coupling modal shape of the main plate-type beam and the minor plate-type beams vibrating. The 42nd frequency belongs to the second bending frequency of the main plate-type beam vibrating, the modal shape of which is shown in Figure 8(d). By comparing the computational frequencies with the test ones listed in Table 1, it is evident that the simple frequencies, such as the first and 5–40th frequencies, are more accurate than those of the coupling frequencies. The relative errors of the former are about 4·3% and those of the latter about 11%.

4.2. FREQUENCIES OF THE STRUCTURE VIBRATING IN WATER

First, the free vibrations of the structure with the water boundary of a rigid water trough are discussed. In this case, some typical frequencies of the structure are listed in Table 2. From all the computed values of frequencies, the local frequencies of the minor plate-type beams are incorporated into five frequency bands. Each band of the five bands contains eight frequencies. In Table 2, ω_1 is the first frequency of the first frequency band, the

TABLE 2

Typical frequencies of the plate-type structure vibrating in water, (Hz)

	ω_1 (#)	ω_9 (#)	ω_{17} (# #)	ω_{18} (#)	ω_{26} (#)	ω_{34} (# #)	ω_{35} (#)	
A	47·6	53·2	62·3	66·0	95·6	163·4	168·7	**
A	47·6	53·2	60·9	66·0	95·6	159·8	168·0	***
B	48·8	56·6	61·9	86·9	111·3	—	—	**
C	2·5	6·0	0·6	24·1	14·1	—	—	**

A, the computational values; B, the test values; C, the relative errors, (%). **water contained by a rigid water trough; ***water contained by a rigid rectangular tube; #the local frequencies of minor plate-type beams; # #the frequencies of whole structure.

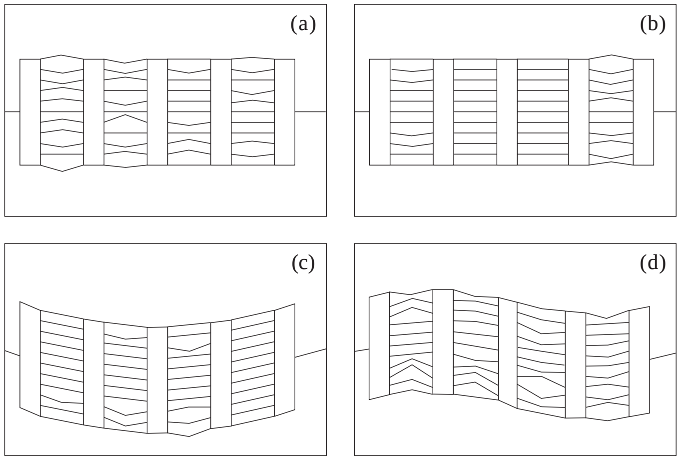

Figure 9. Modal shapes of the structure vibrating in water with the water boundary of a rigid water trough. (a) Modal shape corresponding to the first order calculated frequecy, $\omega_1 = 47{\cdot}6$ Hz. (b) Modal shape corresponding to the ninth order calculated frequency, $\omega_9 = 53{\cdot}2$ Hz. (c) Modal shape corresponding to the 17th order calculated frequency, $\omega_{17} = 62{\cdot}3$ Hz. (d) Modal shape corresponding to the 34th order calculated frequency, $\omega_{34} = 163{\cdot}4$ Hz.

corresponding modal shape of which is shown in Figure 9(a); ω_9 is one of the second band, the corresponding modal shape of which is shown in Figure 9(b); ω_{18} is the one of the third band; ω_{26} is the one of the fourth band and ω_{35} is the one of the fifth band. In Figure 9(a) and (b), the local vibration of the plate-type beam of the structure is obvious, but no bending displacement of the whole structure occurs. Because the added mass and damping depend on the modal shapes of the structure, the difference of local modal shapes of the structure causes the incorporation of the local vibration frequencies of the structure into five frequency bands. ω_{17} and ω_{34} are the first and the second bending frequencies of the whole structure vibrating, the corresponding modal shapes of which are shown in Figure 9(c) and (d), in which the bending displacements of the whole structure are obvious. The test frequencies of the structure with the water boundary of a rigid water trough are also listed in Table 2. It is obvious that the relative errors of the first bending frequency and the frequencies in the first, and second bands are in the range of 6%, but those of the frequencies in the 3rd–5th bands are in the range of 24·1%. It is important to point out that the local frequencies in the first and second bands are lower than the first bending frequency of the whole structure, and the ones in the third and fourth bands are lower than the second bending frequency of the whole structure. These characteristics of the structure vibrating in water are very different from those of the structure vibrating in air. That is to say, the most easily stimulated frequency is not the first bending one of the structure but the local one of the structure when the structure vibrates in water. A comparison of Table 1 and Table 2 shows that the local frequencies in the first frequency band of the structure vibrating in water with the boundary of a rigid water trough decrease by about five times as compared with those of the structure vibrating in air.

Secondly, the free vibrations of the structure with the water boundary of a rigid rectangular tube are briefly discussed. The calculated frequency results are also listed in Table 2. It is obvious that the bending frequencies of the whole structure with the water boundary of a rigid rectangular tube are lower than those of the structure with the water boundary of a rigid water trough. There is no difference between the local frequencies of the structure under the two kinds of water boundary conditions.

5. CONCLUSIONS

The small parameter expansion method is applied to determine the added water mass and damping matrices of a parallel flat plate-type structure with the water boundary conditions of both a rigid rectangular tube and a rigid water trough. This can avoid the problems posed by a system with a large number of degrees of freedom resulting from the application of fluid finite elements. The results show that the method presented in this paper is efficient for the frequency analysis of the parallel flat plate-type structure in liquid contained by a rigid container.

The effects of the added water mass and damping make the local frequencies of the minor plate-type beams of the structure decrease sharply. It means that the local frequencies are most easily stimulated when the structure is in work. In order to ensure the safety of the structure, attention must be paid to such a decrease of the local frequencies of the minor plate-type beams vibrating in water in the design of plate-type elements in water.

The effects of the added water mass and damping on the frequencies of the structure in water contained by a rigid rectangular tube are slightly stronger than those of the structure in water contained by a rigid water trough. Such effects are confined to the frequencies corresponding to the bending vibration of the whole structure and have nothing to do with the frequencies corresponding to local vibration of the minor plate-type beams.

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