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DESCRIPTION OF AN ARBITRARY MULTI-AXIAL LOADING PROCESS FOR NON-LINEAR VIBRATION ISOLATORS

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1. Introduction

Existing non-linear stiffness and damping description methods are suitable for obtaining the general contours of symmetric hysteresis loops, which often can be approximated by linear sections, and this situation has not changed for about 30–40 years [1–3]. However, stiffness and damping of many vibration isolators (for example, the all-metal German STOP-CHOC or Russian DKU, VP, used in aerospace technology) have large non-linearity and non-symmetry (Figure 1). It is necessary during the arbitrary loading process to also take into account the history of loading. For calculation simplicity, it is necessary to model the process using the least number of coefficients.

First, an all-metal vibration isolator with dry friction, in which the hysteresis loop is independent of vibration frequency is considered. The most simple description of the load-deflection behavior utilizes Masing's principle, originally proposed for the description of elasto-plastic materials [4]. In accordance with this principle, one can represent an arbitrary loading process using only duplication, translation and rotation of the primary loading curve. However, this principle is not applicable to systems with large non-linearity and non-symmetry of hysteresis loops. The non-linear transformations allow one to use Masing's principle for this systems will be found.

2. Non-linear transformations

Masing's principle is suitable for a system containing spring and constant friction elements [5]. Many real hysteresis loops of vibration isolators are near the loop of the system with non-zero stiffness without friction K and with displacement dependent friction force $H = cU$ (where U is displacement, Ca is a coefficient) (Figure 2). This system will be transformed in accordance with Masing's principle.

In order to eliminate the element K, one can subtract the loop middle line $L(U)$ from the loop contour and obtain the co-ordinate $F^* = F - L(U)$ (Figure 3).

To eliminate the "friction displacement" dependency, one can divide F^* by the friction force $H(U)$. Thus, in the new coordinate system, U, $F^{**} = F^*/H(U)$, a constant friction element with friction force equal to unity (Figure 4) is obtained.

After this transformaion, process A–D, B–G and C–E are non-parallel, and one requires an additional transformation of these processes, namely a similarity transformation in the direction of the U-axis. Its initial point coincides with the projection of the initial point of the process on the U-axis. Its coefficient is inversely proportional to the segment $a(U)$, i.e., the change in U between the initial point of the process and the point where the process intersects the U-axis. In the new co-ordinates,

$$U_a^* = |U - U_a|/a(U_a) \qquad (\text{or } U_b^*,\ U_c^*);\ F^{**},$$

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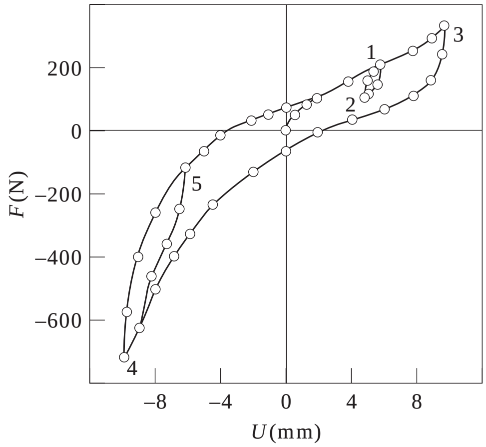

Figure 1. Hysteresis loop of DKU 48-2/22 vibration isolator and arbitrary loading process 0–1–2–1–3–4–5–4.

Masing's principle is fulfilled (Figure 5).

One can also see in Figure 2 that triangles CMN and BPQ are similar, $H(U)/a(U) = \text{constant}$, and it is sufficient to know only the function $H(U)$ for the transformation in the U-axis direction.

Next $L(U)$ and $H(U)$ will be found for a real vibration isolator. For the system shown in Figure 2, $L(U)$ and $H(U)$ are the half-sum and half-difference of loading and unloading processes during sliding in the friction element. The ends of the hysteresis loop are not used for the definition of $L(U)$ and $H(U)$. One can approximately find the length of these sections using Masing's principle. According to this principle, the initial point of the primary loading curve lies on the line where the friction force equal zero. For the system shown in Figure 2, this is the middle line of the hysteresis loop $L(U)$. One can double the displacement U_g from the initial point of the primary loading curve to the coincidence of this curve with the loading process (Figure 2) and subtract this section from the ends of loop. Thus, if one knows the hysteresis loop with the displacement amplitude $A_{\max}$, one can obtain the functions $L(U)$ and $H(U)$ on the interval $[-A_{\max} + 2U_g;\ A_{\max} - 2U_g]$. Therefore, $A_{\max}$ must be as large as possible. If the loading and unloading processes, F_l and F_u, of this large loop, are known, one can determine the non-linear functions $L(U) = (F_l + F_u)/2$ and $H(U) = (F_l - F_u)/2$. To obtain the initial point of the primary

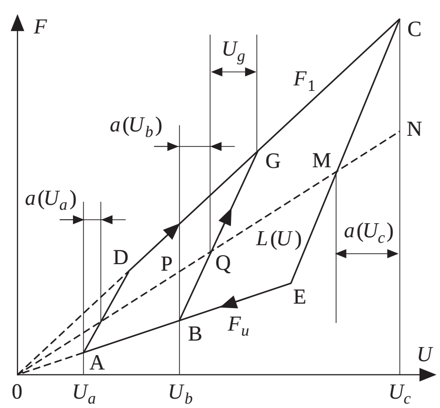

Figure 2. Deformation process of the system with displacement-dependent friction.

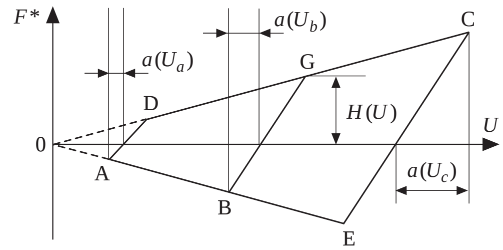

Figure 3. Subtraction of the middle loop line.

loading curve on the middle line, the deformation processes with reducing amplitudes are recommended. Before the approximation of the primary loading curve, it is necessary to subtract $L(U)$ and to divide by $H(U)$.

Every loading and unloading process is described consistently using only duplication, translation and rotation of the primary loading curve. For vibration isolators DKU, functions $L(U)$ and $H(U)$ are approximated with good accuracy using Chebyshev's polynomials of degree three (Chebyshev's polynomials give the highest accuracy towards the ends of segment, where non-linearity is usually more significant). The zero-degree coefficient for the function $L(U)$ equals zero. The transformed primary loading curve is described by the function $R(A') = A'/(A' + r)$, where A' is the primary loading amplitude from the initial point on line $L(U)$ and r is a constant coefficient. Thus, any loading process can be modelled using only eight coefficients.

The compact description provides some possibilities for the modelling of the multi-axial nonlinear arbitrary loading process. If there are some large loops and primary loading curves in the first axis direction U_1 for different loads in the second axis direction U_2, can obtain L, H and R in the direction U_1 as two-coordinate functions:

$$L_1 = \sum_{i,j=1}^{n} l_{1ij} U_1^i U_2^j; \qquad H_1 = \sum_{i,j=1}^{n} h_{1ij} U_1^i U_2^j;$$

$$R_1 = A_1 \Bigg/ \left(A_1 + \sum_{j=1}^{n} r_{1j} U_2^j \right) \quad \text{if } A_1 < U_{1g}; \qquad R_1 = 1 \text{ if } A_1 \geqslant U_{1g}. \tag{1}$$

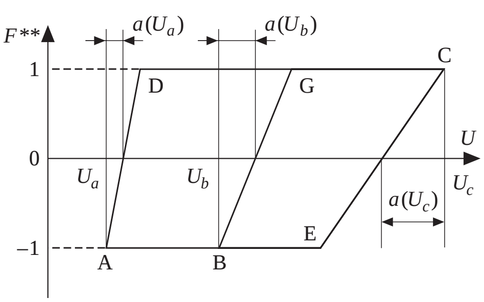

Figure 4. Dividing by friction force.

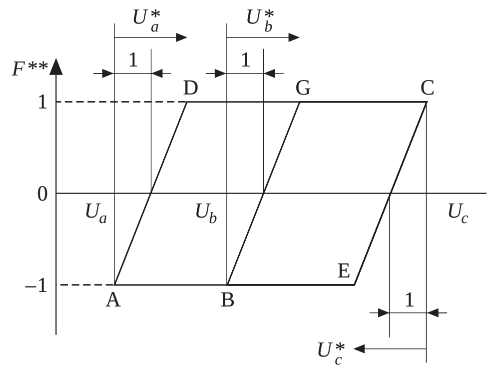

Figure 5. The similarity transformation.

In the same way, one obtains in direction U_2:

$$L_2 = \sum_{i,j=1}^{n} l_{2ij} U_2^i U_1^j; \qquad H_2 = \sum_{i,j=1}^{n} h_{2ij} U_2^i U_1^j;$$

$$R_2 = A_2 \Bigg/ \left(A_2 + \sum_{j=1}^{n} r_{2j} U_1^j \right) \quad \text{if } A_2 < U_{2g}; \qquad R_2 = 1 \text{ if } A_2 \geqslant U_{2g}. \tag{2}$$

Here, A_1, A_2 are the displacement amplitudes in the directions U_1, U_2.

Coefficients l, h and r for the vibration isolator *DKU*-48-2/22 (with the axes U_1 and U_2 being parallel and perpendicular to the vibration isolator axis of symmetry, respectively) are given in Table 1. The arbitrary loading process 0 – 1 – 2 – 1 – 3 – 4 – 5 – 4 (with the angle 45° to the vibration isolator symmetry axis) is shown in Fig. 1. The difference between calculation and experiment is less than 3%, an requires only 25 non-zero coefficients.

TABLE 1

Coefficients for description multi-axis loading process of DKU 48-2/22 *vibration isolator*

Direction	Coefficient	$i\backslash j$	0	1	2	3
	l_{1ij},	1	32·97	0·0	−0·0790	0·0
U_1		2	−5·870	0·0	0·1106	0·0
	N/mm^{i+j}	3	0·8166	0·0	0·0	0·0
		0	47·80	0·0	0·0	0·0
	h_{1ij},	1	−6·151	0·0	0·0	0·0
		2	1·831	0·0	0·0	0·0
	N/mm^{i+j}	3	0·1668	0·0	0·0	0·0
	r_{1j}, N/mm^j		7·501	0·0	0·0689	0·0
	l_{2ij}	1	21·19	0·1442	0·2107	0·01475
U_2		2	0·0	0·0	0·0	0·0
	N/mm^{i+j}	3	0·4039	0·0737	0·0172	0·0
		0	38·38	0·0	0·0	0·0
	h_{2ij},	1	0·0	0·0	0·0	0·0
		2	0·4955	−0·0088	0·0	0·0
	N/mm^{i+j}	3	0·0	0·0	0·0	0·0
	r_{2j}, N/mm^j		3·756	0·2867	−0·0627	−0·0175

However, for a more accurate description of the multi-axial loading process, it is necessary to research the influence of large displacements in the direction of one axis on the loading processes history in the direction of another axis, and to investigate the influence of phase differences of loading in different directions.

If there are some large loops and primary loading lines for different periods of the vibration isolator lifetime, one can obtain description coefficients as time functions, and model arbitrary loading processes during the whole lifetime of the vibration isolator. If the stiffness and damping of the vibration isolator are a function of frequency, we can obtain some large loops and primary loading lines for different frequencies and find coefficients as frequency functions.

3. CONCLUSIONS

Thus, Masing's principle with non-linear transformations gives a general and simple method for the description of the arbitrary loading process of vibration isolators with large non linear stiffness and non-linear damping. It is possible to use this method for precise calculations of dynamic processes in non-linear vibration systems.

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