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THE METHOD OF THE MINIMUM SUM OF SQUARED ACOUSTIC PRESSURES IN AN ACTIVELY CONTROLLED NOISE BARRIER

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1. Introduction

Reference [1] and [2] described an application of active control of sound diffracted by a barrier. The basis of the application is the cancellation of the sound pressure at multiple points on the top edge using multiple secondary sources. The results of numerical simulations and experiments have indicated that: i) actively controlled noise barriers work effectively when the interval of the points of cancellation on the diffraction edge is less than half of the wavelength, and ii) that the attenuation increases when the secondary sources are nearer to the primary source, and iii) that the attenuation also increases when more secondary sources are used. In practice, however, the strategy of increasing the number of secondary sources to improve the effectiveness has its limits. A great number of secondary sources will make actively controlled noise barriers complex and expensive. Thus a more effective method is called for.

This paper describes an alternate theoretical model of using several secondary sources to minimize the sum of squared acoustic pressures at several suppressed points on the top edge of the barrier. By adopting the same assumptions as Reference [2], the numerical simulations of both models under the same conditions were made so as to compare the effectiveness directly. The results suggest that: i) the number of the suppressed points can be more than that of the secondary sources, that is, with the same number of secondary sources, this model is more effective than that proposed by Omoto and Fujiwara [2], and ii) by arranging secondary sources properly, e.g. with an arc-type arrangement, the effectiveness can be improved apparently. These will make actively controlled noise barriers practical.

2. Description of the theoretical model

The approximate solution for the diffracted sound field by the half-plane shows that the sound pressure at the vicinity of the diffraction edge has a dominant effect. Ideally, diffracted sound would be suppressed by cancelling the sound pressure uniformly in the region. However, this method is difficult to implement. Instead of cancelling the sound pressure [2], the sum of squared acoustic pressures at a number of points (suppressed points) on the diffraction edge was minimised using some secondary sources. The assumptions are: i) the ground was absorbing, and ii) the barrier was rigid, and iii) the primary and the secondary sources worked in single frequency. The line-type arrangement of secondary sources has been shown in Figure 1 of reference [2], while the arc-type arrangement is shown in Figure 1. The arc is a part of a circle that centers at the primary source. The interval between two vicinal secondary sources is equal to that of two

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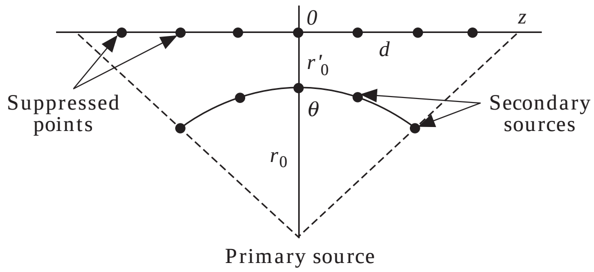

Figure 1. Arc-type arrangement of secondary sources.

suppressed points. One can obtain the sound pressure at an arbitrary suppressed point, p_l

$$P_l = P_{0l} + A^T \Phi_l \tag{1}$$

where

$$P_{0l} = A_0\, e^{ikr_{0l}}/kr_{0l}, \qquad A^T = [A_1, A_2, \ldots, A_L],$$
$$\Phi_l^T = [e^{ikr_{1l}}/kr_{1l}, e^{ikr_{2l}}/kr_{2l}, \ldots, e^{ikr_{Ll}}/kr_{Ll}]. \tag{2–4}$$

A_0 and A_j represent the complex strength of the primary and the jth secondary source, respectively. r_{0l} corresponds to the distance from the primary source to the lth suppressed point, and r_{jl} corresponds to the distance from the jth secondary source to the lth suppressed point. k is the wavenumber of the sound radiated by sources.

From the equations above, one can derive the sum of squared acoustic pressures at all N suppressed points as

$$J = \sum_{l=1}^{N} P_l P_l^* = J_0 + A^T B^* + A^H B^* + A^T Z A^*, \tag{5}$$

where

$$B = \sum_{l=1}^{N} P_{0l}^* \Phi_l, \qquad Z = \sum_{l=1}^{N} \Phi_l \Phi_l^H. \tag{6, 7}$$

H denotes the Hermitian transpose of the vector A (or ϕ_l^H) and is simply the complex conjugate of A^T (or ϕ_l^H).

Equation (5) is a quadratic cost function. It has a similar form to that of total power output from a given source distribution derived by Nelson *et al*. [3].

The optimal secondary source strengths can be obtained by minimizing the sum of squared acoustic pressures. It is determined by differentiating equation (5) with respect to A and setting the resultant equation equal to zero. This process yields

$$A^* = -Z^{-1}B. \tag{8}$$

The acoustic pressure at the receiver R in the actively controlled sound field is given as

$$P = \sum_{n=0}^{L} A_n P_n, \tag{9}$$

where p_n can be calculated by equation (2) of reference [2].

The effectiveness of the active control can be defined as

$$\Delta L = 20 \log_{10}(P_{off}/P_{on}) \tag{10}$$

where P_{on} is the acoustic pressure at the receiver working with optimal secondary sources, while P_{off} is the value without secondary sources.

Because the ground was assumed to be absorbant as with Omoto's model, the ground effect was not considered. Therefore the comparison of the two models can be made under the same condition. It would be easier to use our method to take the ground effect into account. A detailed discussion is left to further study.

3. NUMERICAL SIMULATION

The effectiveness of active control of the model presented in section 2 was simulated as well as the model proposed by Omoto and Fujiwara [2]. Among the parameters that influence the effectiveness of active control; the range of the controlled region is the most important one. Special attention has been paid to it, and an arc-type arrangement of secondary sources to widen it is proposed.

The conditions of the simulations were the primary source at $(r_0, \theta_0, z_0) = (0{\cdot}5\ \text{m}, 60^\circ, 0{\cdot}0\ \text{m})$, receiver at $(r, \theta, z) = (1{\cdot}0\ \text{m}, 300^\circ, 0\text{–}2{\cdot}0\ \text{m})$, $\theta_0 = \theta_0$, the signal frequency of the primary and the secondary sources $f = 5000$ Hz, and the intervals $d = 0{\cdot}03$ m (to make sure that the active control can work effectively). They are all similar to those used in the numerical simulation of reference [2].

3.1. *The secondary sources on a straight line*

At first, the number of the secondary sources, L, and the number of the suppressed points, N, were assumed to be equal. Using two different models, the effectiveness of the control corresponding to the z position of the receiver for the line-type arrangement of the secondary sources was calculated for two values of L ($L = 5$ and $L = 11$). The results are shown in Figure 2. The dash lines indicate the results of the model of cancelling sound pressure, while the dots show the results of the model of minimizing the sum of squared acoustic pressures. The results of both models under the same conditions agree entirely.

Then, the condition in which the number of the suppressed points was greater than that of the secondary sources was considered. Two different numbers of the secondary sources, $L = 5$ and $L = 11$, using two models were also simulated. For the new model, the numbers of the suppressed points, N, were 11 and 17. The results are shown in Figure 3.

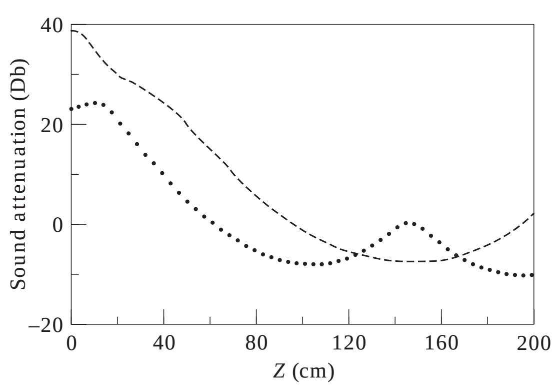

Figure 2. Sound attenuation by active control as a function of z position of the barrier in the diffracted sound field: -----, sound pressure cancellation; · · · ·, minimum sum of squared sound pressures.

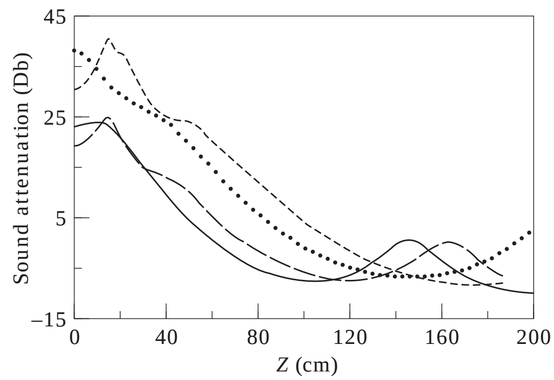

Figure 3. Sound attenuation by active control as a function of z position of receiver in the diffracted sound field. Sound pressure cancellation: ——, $L=5$; · · · ·, $L = 11$; minimum sum of squared sound pressure: — — $L = 5$, $N = 11$; -----, $L = 11$, $N = 17$.

Apparently, the actively controlled region of the model of minimizing the sum of squared acoustic pressure is wider than that of the model of cancelling sound pressure using the same number of the secondary sources, which means that the former method is more effective than the latter. This can be explained by Fresnel diffraction. For the same number of the secondary sources, the former model uses more suppressed points and allows a much wider range of the virtual line source on the top edge of the barrier being actively controlled. The range of controlled region in the diffracted field is therefore wider than that of the latter.

3.2. *The secondary sources on an arc*

From an engineering point of view, an actively controlled noise barrier should be simple and stable. This demands a moderate number of secondary sources. Prior research has shown that the range of controlled region in the diffracted sound field is determined by two parameters: the number of the suppressed points and the range of virtual line source on the top edge of the barrier being controlled (the shadow formed by the secondary sources towards the primary source on the top edge of the barrier). For the model of minimizing the sum of squared acoustic pressures, the number of the suppressed points on the top edge of the barrier can be greater than that of the secondary sources. Adjusting the arrangement of the secondary sources can widen the range of virtual line source on the top edge of the barrier being actively controlled, which also serves to widen the range of actively controlled region in the diffracted sound field.

An arc-type arrangement of the secondary sources as shown in Figure 1 was considered. The radius was 0·3 m, while the other conditions were the same as above. From a geometrical point of view, it is easy to derive that the range of controlled region on the top edge of the barrier (the shadow measured by the angle Θ as shown in Figure 1), using less secondary sources, is nearly equal to that of the line-type arrangement as shown in reference [2] (Figure 1). Thus with the same number of secondary sources the arc-type arrangement is more effective than the line-type arrangement.

Figure 4 shows the simulation results of both arrangements. It is obvious that the effectiveness of the arc-type arrangement is better than that of the line-type arrangement, especially for the model of minimizing the sum of squared acoustic pressures, and for relatively more secondary sources.

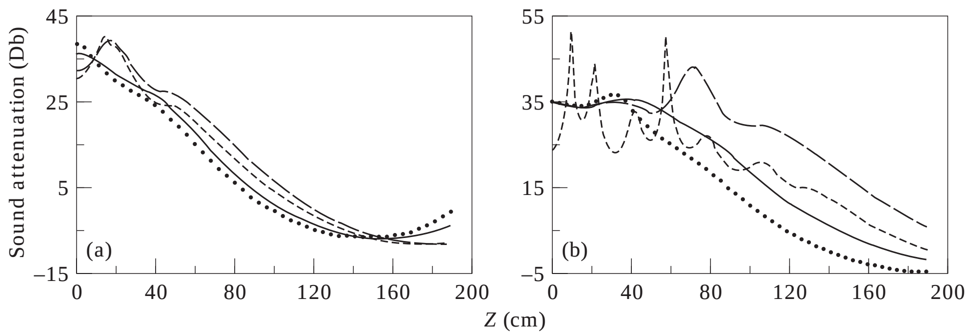

Figure 4. Sound attenuation by active control as a function of z position of receiver in the diffracted sound field. Secondary sources on (a) an arc: ——. $L = N = 11$; — — , $L = 11$, $N = 17$; on a line ·····, $L = N = 11$; – – –, $L = 11$, $N = 17$. Secondary sources (b) on an arc: ——, $L = N = 17$; — — , $L = 17$, $N = 33$; on a line ·····, $L = N = 17$; – – –, $L = 17$, $N = 33$.

4. CONCLUSIONS

On the basis of the simulations and discussions, two conclusions can be drawn:

(1) The model of minimizing the sum of squared acoustic pressures is more effective than the model of cancelling sound pressure [2].

(2) The arc-type arrangement of the secondary sources can apparently improve the effectiveness of active control, especially for the model of minimizing the sum of squared acoustic pressures and relatively more secondary sources.

Although the numerical simulations made here are under a particular assumption of frequency $f = 5$ kHz, a relatively high frequency, so as to compare the two models directly, it does not mean that this model is effective only for high frequencies. The simulations and experiments made by Omoto and Fujiwara [2], have proved that active control can be effective when the intervals of the secondary sources (suppressed points) are less than half of the wavelength. For lower frequencies, with greater wavelengths, the intervals of the secondary sources (and suppressed points) can be increased. Thus, for the same number of secondary sources, the range of the controlled region is wider than that of higher frequencies. It is to be welcomed for future applications.

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