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FIRST ASSESSMENT OF THE SCALING PROCEDURE FOR THE EVALUATION OF THE DAMPED STRUCTURAL RESPONSE

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1. INTRODUCTION

The renewed interests concerning the applicability of Statistical Energy Analysis (SEA) have led to a strong effort in order to assess definitively the capability of the predictive vibroacoustic methodologies when working in the high frequency ranges. Furthermore, the Finite Element Method (FEM) is also being investigated to clarify if and how it could allow an analysis for decreasing wavelengths. The high frequency range is such that the response of a vibroacoustic system is no longer dominated by that of a single mode but, conversely, the direct and the coupling loss factor will govern the amplitude of the vibrational and acoustic levels. A specific analysis of the literature shows the guidelines outlined in the next paragraphs.

1.1. *SEA extension*

Researchers involved with the development of the SEA technique are defining refined wave-based approaches in order to make available efficient tools for the evaluation of the coupling loss factors. At increasing wavelengths the applicability of SEA has been demonstrated even if the increasing of the confidence intervals of the data could not always allow an engineering usage of these latter [1].

Furthermore an iterative SEA procedure, called Advanced Statistical Energy Analysis, has been proposed [2] in order to solve the *tunnelling* problem. This phenomenon occurs when the energy exchange among structural subsystems that are not directly in contact has to be predicted: the standard SEA is not able to account for it, over-predicting the energy levels, while the proposed ASEA is able to converge onto the real behaviour.

1.2. *FEM extension*

The fundamental problem is related to the computational costs. In fact, the FEM represents the best numerical solution (for sake of brevity: it is an approximation of a differential problem with the required boundary conditions) but the *spatial mesh is frequency dependent*. Also the average of an adequate modal content does not always allow a significant extension in the frequency domain [3]. Significant attempts have been successfully made by using the adsorbing waves inside a finite element approach. This research is still ongoing [4].

1.3. *Novel approaches*

These approaches try to circumvent the spatial limitations of the deterministic approaches, using a transformation of the solution domain [5]. Efficient theoretical comparisons and promising results have appeared in the literature; also however, the applicability of the proposed approaches has still to be demonstrated for more complicated structures (multicomponent analysis): for example, N-plate assemblies. A unified look at

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the problem [5] has been given in order to encompass all the research activities in a proper scheme.

2. BASIS OF THE SCALING PROCEDURE

Fundamentally, what follows here is concerned with the second of the previously outlined guidelines: the aim is related to the possibility of defining a scaled finite element model able to represent the energy exchange for increasing excitation frequency.

The basic idea is to reduce the extension of the spatial dimensions not involved in the energy transmission, to keep the same finite element mesh and also enforcing an augmented damping level to obtain finally the same energy levels of the original model.

The starting point of the work can be summarized by the following general question: *is it possible to evaluate the response of a structural system by using a scaled model in a way to perform the analysis with a smallest test model?* The answer will depend upon the quality of the information one would get.

If the natural frequencies and mode shapes are needed, the properties of the material could be properly defined together with the geometrical dimensions in such a way as to obtain an efficient scaling of the test model. For example, the undamped longitudinal natural frequencies of a free–free rod, length L, could be evaluated by using the relationship

$$f_i = ic_l/(2L) \quad \text{with} \quad i \in \{0, 1 \ldots, N\}. \tag{1}$$

In the laboratory one could use a rod of reduced length, say L^*, and made of different material such that the ratio cl/L remains the same:

$$c_l^*/L^* = c_l/L \Rightarrow f_i = f_i^*. \tag{2}$$

Obviously, the same boundary conditions have to be kept. Note that the wave speed of the pure longitudinal waves, c_l, depends upon the material properties: $c_l = (En/\rho)^{1/2}$.

This similitude allows basically the use of a rod of different (reduced) length and made of different material, such that the natural frequencies and the mode shapes are identical to those of the original system. From a numerical point of view, the same reduction could be performed with the finite element method too, but in this case one has to evaluate the effect of modifying the wave speed, so as to properly model the wavelength (number of degrees of freedom). If the wave speed remains unchanged, the natural frequencies will move, for a length reduction, to higher values while the mode shapes should be computed over the reduced domain: they will be again identical.

For the evaluation of the damped response of a structural system or a structural–acoustic one, the problem is quite different. In fact it is really complicated to use a modal solution as a primary reference since the parameters can be found only with difficulty. In respect to wave propagation in non-reverberant systems some considerations could be made.

In a chain of one-dimensional non-reverberant systems, the amount of energy, say $E1$, of the first system that reaches the boundary is proportional to $E1^* \exp(-m\pi)$, where m is the modal overlap factor of the first system. The reflected energy, upon using the previous hypothesis, will be $E1^* \exp(-2m\pi)$. For high values of the modal overlap factor† (i.e. greater than 0·5) the systems are *not reverberant*. This means that for values of the modal overlap sufficiently high, the dynamic response is independent of the phase of various components while the dynamic response will depend on the constant modal overlap factor of each system, and the transmission coefficients among the subsystems.

† Modal overlap factor: $m = \omega\eta n$ where n is the modal density.

Now, a finite element solution is considered. By adopting a mesh for the 1-D domain, the maximum frequency represented inside the system has also been fixed; this frequency is associated with the wavelength at which at least five (or seven) grid points are available for the representation of the wave. For smaller wavelengths the representation will no longer be possible.

Using the same mesh for the scaled 1-D domain, obtained by multiplying all the linear dimensions by α(with $\alpha < 1$), one gets an increase of the modal density n. In fact for 1-D system in which travelling longitudinal waves, is $n = c_l/L$. To keep the same modal overlap factor in the scaled model, the damping loss factor has to be divided by α.

The transmission coefficients for a 1-D system do not depend on the lengths of the systems, so the scaled finite element model can be assembled simply by performing the following operations: the linear dimensions have to be reduced by α, and the damping has to be augmented by $1/\alpha$. Hence by using the scaled finite element model it will be possible to determine the energy exchange at increasing excitation frequency, for growing values of the modal overlap factor.

It is interesting to compare results obtained in this latter way with the SEA standard results, as these can be considered as a reference solution for the energy exchange.

The first selected reference solution is the response of a simple rod. The list of parameters is as follows: length, L; section A; Young's Modulus E_n; mass ρAL; density ρ; semi-infinite impedance $Z = \rho A c_l$; internal loss factor η; excitation force $F = F_0(e^{-j\omega t})$. By using the classical SEA relationships, the energetic balance of a single wave system can be represented by the equation [6]

$$P_{INPUT} = E\eta\omega. \tag{3}$$

For a 1-D structural system the total energy is $E = \frac{1}{2}mv^2 = \frac{1}{2}\rho ALv^2$ and the input power is $P_{INPUT} = \frac{1}{2}|F|^2$ Real $(1/Z)$. Solving equation (3) for the averaged squared velocity gives

$$v^2 = |F|^2[\eta\omega Z\rho AL]^{-1}. \tag{4}$$

For longitudinal waves, the rod is excited at one end and $Z = \rho A c_l$, so that equation (4) can be written as

$$v^2 = |F|^2[\eta\omega\rho A c_l \rho AL]^{-1}. \tag{5}$$

This can be defined as the *original response*. Now, consider a new rod, with * denoting the related symbols. The response of this rod is

$$v^{*2} = |F^*|^2[\eta^*\omega\rho^{*2}A^{*2}c_l^*L^*]^{-1}. \tag{6}$$

This can be defined as the *scaled response*. Imposing $c_l = c_l^*$, $A = A^*$, $F = F^*$, one obtains

$$v^2 = v^{*2} \Rightarrow \eta L = \eta^* L^*. \tag{7}$$

If the scaling function is linear in the parameter α, $L^* = \alpha L$, the scaled model will furnish the same energetic levels of the original one if

$$\eta/\alpha = \eta^*. \tag{8}$$

The damping of the scaled system has to be $1/\alpha$ times the geometrical reduction ($\alpha < 1$) of the original length of the rod (e.g.: the damping of the reduced rod has to be greater than that of the original one). It should be noted that other parameters also could be introduced to scale the model: the choice is not unique. It is useful at this point to view

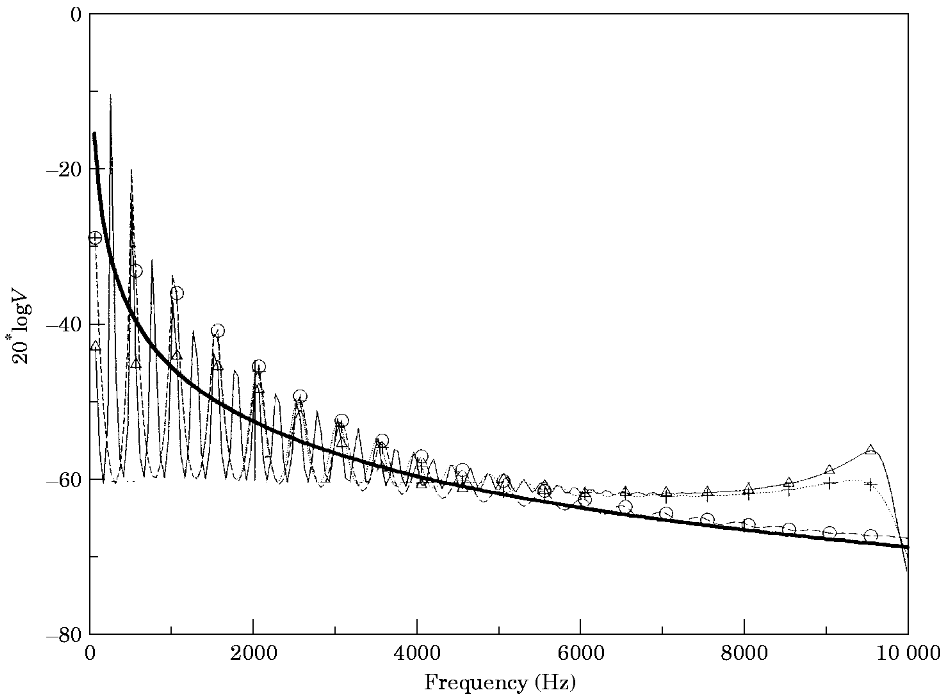

Figure 1. Squared velocity of the rod, —△—, Original F.E. model; · · ·+· · ·, scaled F.E. model; --○--, refined F.E. model; ——, SEA.

the results outlined by equations (7) and (8) from the finite element point of view: (i) the properties of the material are the same, the longitudinal wave speeds in both the models are the same; (ii) the length is simply reduced; (iii) the boundary conditions are the same; (iv) the damping loss factor has to be scaled inversely to the length of the domain. The finite element model of the scaled rod will be identical to the original one, except for the damping; it is necessary only in order to reduce the number of degrees of freedom. The energy evaluated by using the scaled model should be the same as that of the original one. The natural frequencies extracted from the scaled model will be shifted by the parameter $1/\alpha$. The results of the approach are shown in Figure 1, for three selected finite element models, which have the characteristics shown in Table 1. The results shown in Figure 1 are encouraging. The scaled FEM solution fits the SEA one starting 4 kHz. This result justifies the hypothesis of a non-reverberant domain as described above.

TABLE 1

The finite element characteristics of the simple rod

	Itemized as	Length (mm)	Loss factor	No. of F. E.
First F.E. model	Original	$L = 10\,000$	$\eta = 0{\cdot}04$	60
Second F.E. model	Scaled ($\alpha = 1/2$)	$L^*\alpha$	η/α	30
Third F.E. model	Refined ($\alpha = 1/2$)	$L^*\alpha$	η/α	60
Common parameter:	Section $A = 10\,000$ mm^2, $E = 7000$ kp mm^{-2}, $\rho = 2{\cdot}7 \times 10^{-10}$ kp s mm^{-4}, unit excitation amplitude.			

3. TWO SYSTEMS

The SEA relationship for an assembly of two subsystems is such that the energy ratio, when the first system is excited and the second system is the receiver, is given by

$$\frac{E_2}{E_1} = \frac{n_2}{n_1}\frac{1}{(\eta_2/n_{21}+1)} \Rightarrow \frac{v_2^2}{v_1^2} = \frac{(\rho AL)_1}{(\rho AL)_2}\frac{\eta_2}{\eta_1}\frac{1}{(\eta_2/n_{21}+1)}. \quad (9)$$

Here n denotes the modal density, η_i the damping loss factor, and η_{ij} the coupling loss factor. For the evaluation of the separate energy levels, it has to be remembered that

$$E_1 = \frac{P_1}{\omega}\frac{(\eta_2+\eta_{21})}{(\eta_1+\eta_{12})(\eta_2+\eta_{21})-\eta_{21}\eta_{12}} = \frac{\frac{1}{2}|F|^2\,\mathrm{Real}\{1/Z_1\}}{\omega}\frac{(\eta_2+\eta_{21})}{(\eta_1+\eta_{12})(\eta_2+\eta_{21})-\eta_{21}\eta_{12}}. \quad (10)$$

By using equations (9) and (10) the energy level of the receiving system can be easily evaluated. The scheme now considered is related to two in-line rods. The same terminology is used as before with the addition of an index. The modal densities and the coupling loss factors are exposed explicitly. One has then

$$\frac{v_2^2}{v_1^2} = \frac{(\rho AL)_1}{(\rho AL)_2}\left(\frac{L_2}{c_{l2}}\right)\left(\frac{c_{l1}}{L_2}\right)\left(\frac{\eta_2\omega}{(\tau_{12}/2\pi n_2})+1\right)^{-1}, \quad (11)$$

with

$$\tau_{12} = 4(\sqrt{Z_1/Z_2}+\sqrt{Z_2/Z_1})^{-2} = 4(\sqrt{(\rho Ac_l)_1/(\rho Ac_l)_2}+\sqrt{(\rho Ac_l)_2/(\rho Ac_l)_1})^{-2}.$$

Assuming now for the sake of simplicity that the rods are of the same material, one has

$$\frac{v_2^2}{v_1^2} = \frac{1}{\sigma}\left(\frac{\eta_2\omega}{(\tau_{12}2\pi n_2)}+1\right)^{-1}, \quad (12)$$

with

$$\tau_{12} = 4(\sqrt{\sigma}+\sqrt{1/\sigma})^{-2} \qquad \text{and} \qquad \sigma = A_2/A_1.$$

The transmission coefficient τ depends only upon the ratio, σ, of the two rods [6]. The scaled model of the two rods will be such that all remain unchanged except for the damping and the rod lengths. In particular

$$L_1^* = \alpha L_1 \qquad \text{and} \qquad L_2^* = \alpha L_2. \quad (13)$$

Note also that, $n_i \propto L_i/c_{li}$ and $\eta_{21} = (\tau_{12}/2\pi\omega n_2)$

$$\text{and} \qquad n_{21}^* = (\tau_{12}/2\pi\omega n_2^*) = (\tau_{12}/2\pi\omega n_2\alpha) \Rightarrow \eta_{21}^* = \eta_{21}(1/\alpha).$$

The energy ratio

$$\frac{v_2^2}{v_1^2} = \frac{(v_2^*)^2}{(v_1^*)^2} \Rightarrow \eta_2 n_2 = \eta_2^* n_2^* \Rightarrow \eta_2 L_2 = \eta_2^* L_2^* \Rightarrow \eta_2^* = \eta_2(1/\alpha). \quad (14)$$

The energy of the scaled source system is thus calculated as

$$E_1^* = \frac{|F|^2\{1/(\rho Ac_l)_1\}}{\omega}\frac{(\eta_2^*+\eta_{21}^*)}{(\eta_1^*+\eta_{12}^*)(\eta_2^*+\eta_{21}^*)-\eta_{21}^*\eta_{12}^*}. \quad (15)$$

Therefore, it is easy to demonstrate that:

$$E_1^* = \alpha E_1, \qquad \tfrac{1}{2}\rho_1 A_1 L_1^*(v_1^*)^2 = \alpha\tfrac{1}{2}\rho_1 A_1 L_1(v_1)^2 \quad (16, 17)$$

TABLE 2

Common properties of the finite element models of the six rod model

	Rods					
	1	2	3	4	5	6
Mass per unit length	1	10	3	7	8	2
Longitudinal wave speed	5000					
Harmonic excitation	Unit value at $x = 0{\cdot}0$ (free end of the first rod)					

but imposing $L_1^* = \alpha L_1$ gives the final result as

$$(v_1^*)^2 = (v_1)^2. \tag{18}$$

4. THE SIX RODS PROBLEMS

For the primary assessment of the scaling procedure, a test-case presented in the literature in 1994 has been used. It represents an assembly of six in-line rods whose properties are reported in Tables 2 and 3. This model is a severe test for the standard SEA approach, because the problem of tunnelling is involved [7]: the problem of the energy exchanges between subsystems that are separate from other subsystems. In reference [7] the feasibility was demonstrated of a new iterative scheme called ASEA, that is able to overcome the 30 dB overprediction in the sixth rod due to the standard SEA; see Figure 2 in which the finite element results are also included.

The starting finite element model has been designed to work up to 3400 Hz by using 600 finite elements; see Table 3. The other finite element model characteristics are reported in Tables 4, 5, and 6. The standard SEA relationships can be easily derived and they are not shown here. In reference [7] the velocity ratios of the rods 3, 4, 5 and 6 to that of the source rod (the first one) have been reported. Here, only the *worst* result is shown, concerning that for the sixth rod.

The efficiency of the scaling procedure is immediately evident. It is possible to obtain the same quality of results by using the SCALED finite element model, while keeping the total number of degrees of freedom, it is possible to extend the frequency validity of the finite element model (see the models REFINED and REFINED2).

Again the scaled FEM results are in good accordance with the analytical one starting from a frequency value where the modal overlap factor becomes high enough and hence such that the domains can be considered non-reverberant. Moreover it must be stressed that the scaled FEM approach can be applied successfully for a system where SEA is not valid. In fact the solution schemes of the two approaches are completely different. In the SEA one does not take into account the energy exchanges among subsystems not directly

TABLE 3

Characteristics of the F.E. model original, ($\alpha = 1$)

	Rods					
Length	1	2	3	4	5	6
Length	23	28	25	24	29	21
Loss factor	0·02					
Number of elements	600					

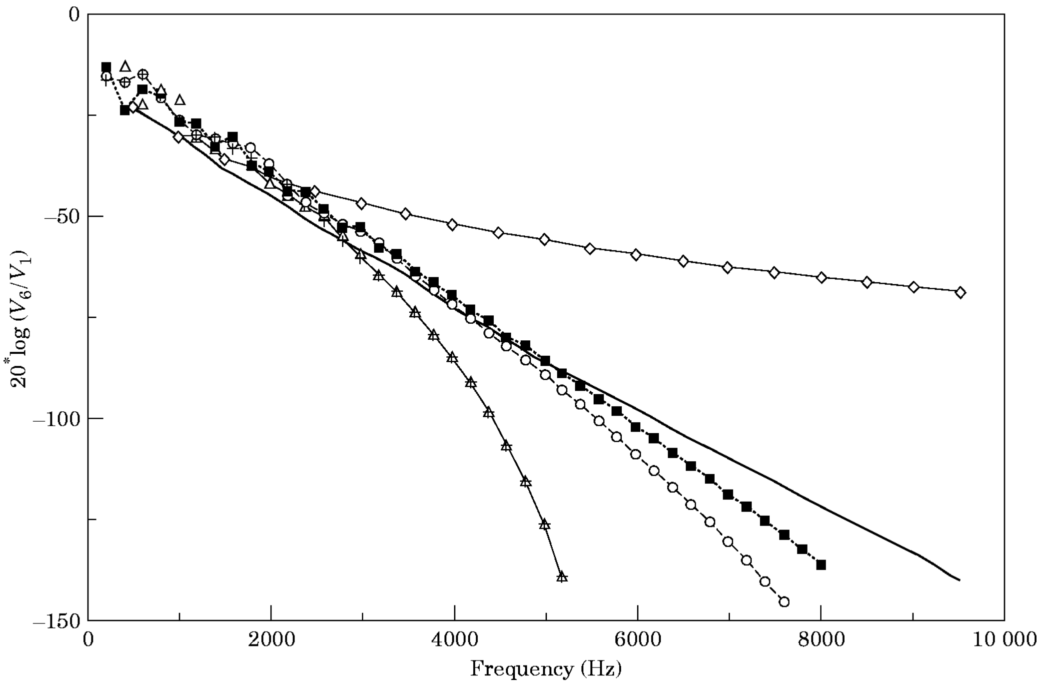

Figure 2. The six rods problem: ratio of the sixth rod velocity to that of the source rod. —◇—, SEA; ——, analytical; · · ·■· · ·, F.E. REFINED2 (601 dofs); · · · ○· · ·, F.E. REFINED (601 dofs); · · · + · · ·, FE SCALED (301 dots); △, FE ORIGINAL (601 dofs).

connected (for example the first rod with the sixth one). In the FEM approach the solution is not developed by splitting the domain into subsystems: the stiffness, mass and damping matrices are assembled and solved for the complete domain.

It is useful to discuss the limits of the scaling procedure presented. Virtually, there are no superior limits, if one satisfies the modal density requirement. In particular

TABLE 4

Characteristics of the first scaled F.E. element model ($\alpha = 1/2$) *scaled*

	Rods					
	1	2	3	4	5	6
Length	23*α	28*α	25*α	24*α	29*α	21*α
Loss factor	$0{\cdot}02/\alpha = 0{\cdot}04$					
Number of elements	300					

TABLE 5

Characteristics of the second scaled F.E. model ($\alpha = 1/2$) REFINED

	Rods					
	1	2	3	4	5	6
Length	23*α	28*α	25*α	24*α	29*α	21*α
Loss factor	$0{\cdot}02/\alpha = 0{\cdot}04$					
Number of elements	600					

TABLE 6

Characteristics of the third scaled F.E. model ($\alpha = 1/3$) REFINED2

	Rods					
	1	2	3	4	5	6
Length	$23*\alpha$	$28*\alpha$	$25*\alpha$	$24*\alpha$	$29*\alpha$	$21*\alpha$
Loss factor			$0{\cdot}02/\alpha = 0{\cdot}06$			
Number of elements			600			

Figure 3 shows the values of the driving point admittance, Y. Note that Real $(1/Z)$ = Real $(Y) = (\pi/2)n(\omega)/M$. Therefore the value of Y corresponds to that of the modal density of the systems. The energy based similitude can be adopted until the modal densities are sufficiently represented. Finally it has to be noted that the finite element velocities have been averaged over the space domain; they were not averaged over the excitation locations and the frequency domain.

Some comments are needed concerning better analysis and the validity and the applicability of the results previously outlined. The whole approach is based upon the analysis of the energy exchange among subsystems.

It is useful to recall some of the basic hypothesis: (i) the boundary conditions do not affect the energetic response; (ii) high modal density and modal overlap factor response; (iii) the subsystems energy exchange is determined by considering semi-infinite systems.

The third point could be overcome by using the finite element approach. In fact when using the finite element method the transmission coefficient between the subsystems i and j could be evaluated also in presence of a kth subsystem. The energy based similitude could

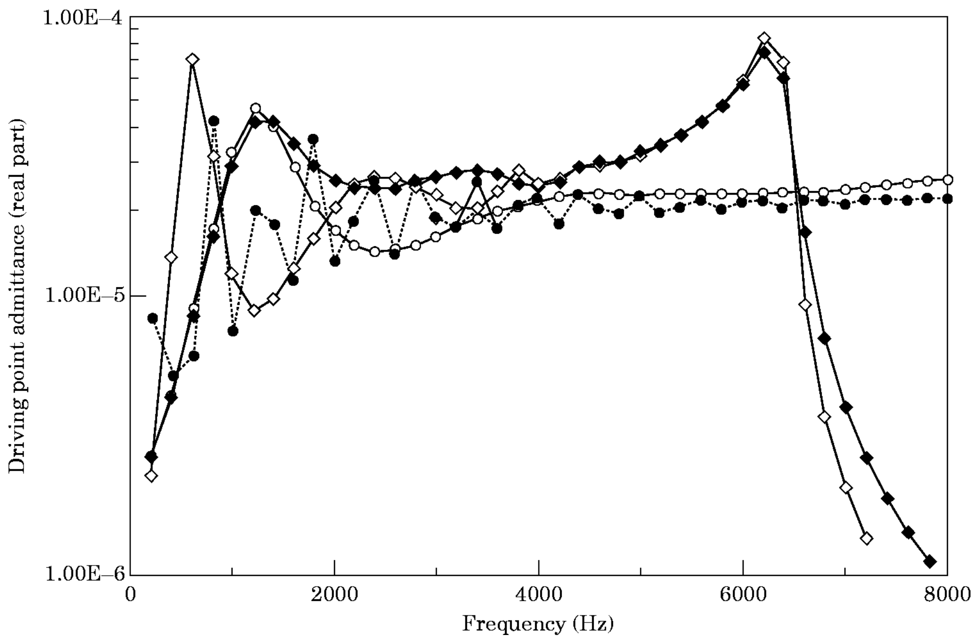

Figure 3. The six rods problem: analysis of the power input for the several finite element models. Theoretical value of the driving point admittance is 1·83 E-5. Finite element models: —◇—, ORIGINAL; ···◆···, SCALED; —○—, REFINED; —●—, REFINED 2.

be so used also for evaluating the transmission coefficients among structural and acoustic subsystems in the presence of the rest of the structure; this should be useful for the frequency region where the subsystems oscillate as finite systems. The next step of the research will be focused toward results for plate assemblies, which are severe test cases for the random transmission characteristics.

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REFERENCES

1. K. H. Heron 1990 *I.S.V.R. Seminar, Institute of Sound and Vibration Research, University of Southampton, Southampton, England*. Power flow investigation on aeronautical structures.
2. K. H. Heron 1994 *Philosophical Transactions Royal Society of London* **A 346**, 501–510. Advanced statistical energy analysis.
3. S. De Rosa, F. Franco, F. Marulo and F. M. Ventriglia 1995 *Proceedings of the XIII National Congress of the Italian Association of Aeronautics and Astronautics, Rome, Italy*, 11–15 *September* **2**, 1281–1293. Experiences of Structural Dynamics and Interior Noise at Medium and High Modal Densities.
4. K. De Langhe 1996 *Ph.D. Thesis, Faculteit der Togepaste Wetenschappen, Department Produktietechnieken, Machinebouw en Automatisering, Leuven, Belgium*. High frequency vibrations: contributions to experimental and computational SEA parameter identification techniques.
5. A. Sestieri and A. Carcaterra 1995 *Invited Lecture, Proceedings of the XIII AIMETA, September, Naples*. Circumventing space sampling limitations in mechanical vibrations.
6. R. H. Lyon 1975 *Statistical Energy Analysis of Dynamical Systems*. Cambridge MA: M.I.T. Press.
7. L. Cremer, M. Heckl and E. E. Ungar 1988 *Structure-Borne Sound*. Berlin: Springer-Verlag. Second edition.

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