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EVALUATION OF THE FUNDAMENTAL VIBRATION FREQUENCY OF A BENDING PLATE BY USING AN ITERATIVE APPROACH

R. S. Chen

P.O. Box 284, *Department of Computer Science*, *Nanjing University*, *Nanjing*, *Jiangsu* 210093, *People's Republic of China*

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The fundamental vibration frequency of a bending plate is investigated by using an iterative approach. The biharmonic operator in the equation is reduced by performing the Laplace operator twice, and the finite difference method is used to solve the problem. The frequency and the deflection mode are adjusted and changed simultaneously in the iteration. The boundary conditions for the clamped edge case and free edge case are satisfied implicitly. A convergent tendency has been found in the solution, and high accuracy has been proved in the example problem. Numerical examples are given to demonstrate the efficiency of the method.

1. INTRODUCTION

Previously, the buckling loading of a compression bar with a varying cross-section was investigated by using the shooting method[1]. Clearly, the problem belongs to the eigenvalue problem of an ordinary differential equation. In this paper, the fundamental vibration frequency of a bending plate is studied. In fact, the vibration problem of a bending plate can be reduced to an eigenvalue problem of a partial differential equation. An iterative approach which is like the shooting method in reference [1] is used to solve the problem. However, the suggested approach is not a simple replication of the shooting method in the ordinary differential equation. This can be seen from the following analysis.

For the natural frequency of a rectangular plate, much work has been done by using various methods. Some work in this field is based on the solution of a differential equation [2–3]. Several of them depend on the appropriate choice of the deflection function. In addition, by minimizing the Rayleigh quotient with respect to relevant coefficients in the deflection function, the eigenvalue equation is obtainable [4–7]. In fact, it is not easy to use the above mentioned methods to solve the frequency problem with a complicated boundary condition, as in the case where one part of an edge is free and the remainder is simply supported. By using a conforming rectangular element, the finite element solution for the frequency problem was also suggested [8]. Generally, it is time consuming to assemble the relevant matrix and to evaluate the eigenvalue using the finite element method.

In this paper, the fundamental vibration frequency of a bending plate is investigated by using an iterative approach. In the analysis, the biharmonic operator is reduced by performing the Laplace operator twice, and the finite difference method is used to solve the problem. This can considerably reduce the effort of derivation and computation. The frequency and the deflection mode are adjusted and changed simultaneously in the

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iteration. The boundary conditions for the clamped edge case and free edge case are satisfied implicitly. In the derivation and the actual computation, the method only depends on the solution of finite difference equation and the numerical integration. Comparatively speaking, the mentioned work can be easily performed. Finally, several numerical examples are given to demonstrate the use of the suggested iterative process.

2. ANALYSIS

The problem investigated is illustrated by a rectangular plate in free vibration (Figure 1). It is well known that the governing equation for the deflection mode $w(x, y)$ takes the form [2, 3].

$$\nabla^2\nabla^2 w(x, y) = (\rho h/D)\omega^2 w, \qquad \nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2, \tag{1}$$

where $D = Eh^3/12(1 - v^2)$ denotes the rigidity of the bending plate, ρ the density of materials, h the thickness of plate, E the Young's modulus of elasticity, v the Poisson ratio, and ω the fundamental vibration frequency.

One assumes that three edges (OB, BC and AC) in Figure 1 are simply supported, and the boundary conditions take the form

$$w|_{x=0} = 0, \qquad \nabla^2 w|_{x=0} = 0, \qquad 0 \leqslant y \leqslant b \quad \text{along edge OB}, \tag{2a, b}$$

$$w|_{x=a} = 0, \qquad \nabla^2 w|_{x=a} = 0, \qquad 0 \leqslant y \leqslant b \quad \text{along edge AC}, \tag{2c, d}$$

$$w|_{y=b} = 0, \qquad \nabla^2 w|_{y=b} = 0, \qquad 0 \leqslant x \leqslant a \quad \text{along edge BC}, \tag{2e, f}$$

In addition one edge, OA, in Figure 1 is assumed to be clamped,

$$w|_{y=0} = 0, \qquad \partial w/\partial y|_{y=0} = 0, \qquad 0 \leqslant x \leqslant a \quad \text{along edge OA in the clamped case.} \tag{2g, h}$$

Alternatively, in the free traction case of edge OA, the condition (2g, h) should be replaced by

$$\partial^2 w/\partial y^2 + v\,\partial^2 w/\partial x^2|_{y=0} = 0, \qquad \partial^3 w/\partial y^3 + (2 - v)\,\partial^3 w/\partial x^2\,\partial y|_{y=0} = 0,$$

$$0 \leqslant x \leqslant a \text{ along edge OA in the traction free case} \tag{2i, j}$$

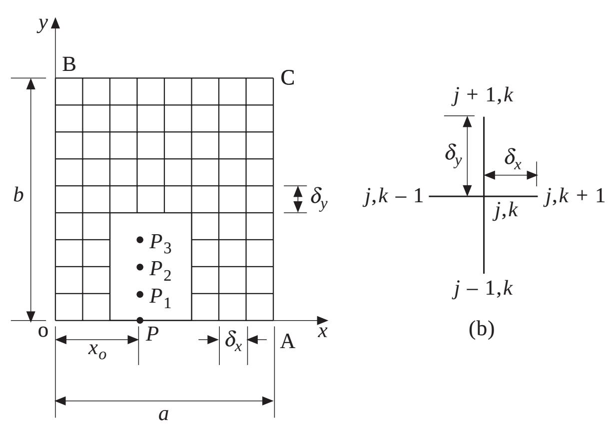

Figure 1. (a) a rectangular bending plate, (b) the center node with four adjacent nodes.

For convenience in derivation, one introduces the function $u(x, y)$ defined by

$$u(x, y) = \nabla^2 w(x, y) \tag{3}$$

Therefore, the governing equation and the boundary conditions can be rewritten as

$$\nabla^2 u(x, y) = (\rho h/D)\omega^2 w(x, y), \qquad \nabla^2 w(x, y) = u(x, y) \tag{4, 5}$$

and

$$w|_{x=0} = 0, \qquad u|_{x=0} = 0, \qquad 0 \leqslant y \leqslant b \quad \text{along edge OB,} \tag{6a, b}$$

$$w|_{x=a} = 0, \qquad u|_{x=a} = 0, \qquad 0 \leqslant y \leqslant b \quad \text{along edge AC,} \tag{6c, d}$$

$$w|_{y=b} = 0, \qquad u|_{y=b} = 0, \qquad 0 \leqslant x \leqslant a \quad \text{along edge BC,} \tag{6e, f}$$

$$w|_{y=0} = 0, \qquad \partial w/\partial y|_{y=0} = 0, \qquad 0 \leqslant x \leqslant a \quad \text{along edge OA in the clamped case.} \tag{6g, h}$$

In the traction free case of edge OA, from (2i, j), the condition (6g, h) should be replaced by

$$\alpha\, \partial^2 w/\partial y^2 + u|_{y=0} = 0, \qquad \beta\, \partial^3 w/\partial y^3 + \partial u/\partial y|_{y=0} = 0$$
$$0 \leqslant x \leqslant a \text{ along edge OA in the free case,} \tag{6i, j}$$

where

$$\alpha = (1 - v)/v, \qquad \beta = -(1 - v)/(2 - v). \tag{7}$$

In the iterative approach, the finite difference method is used. In computation, the rectangular region is divided into a $N_1 \times N_2$ mesh with the side lengths δ_x, δ_y in the x and y directions respectively (Figure 1).

The iterative process is performed by using the following steps:

(1) It is assumed that, after the nth iteration the eigenvalue and the values of the functions $u(x, y)$ and $w(x, y)$ at nodes are known, and they are denoted by $\omega_{(n)}$, $u_{(n)}(x, y)$, $w_{(n)}(x, y)$, respectively.

(2) The second step is to obtain the new values of $u(x, y)$ at the inner nodes. By using the finite difference method, from (4) one has [2]

$$\tilde{u}_{(n+1)j,k} = \delta_x^2\delta_y^2/2(\delta_x^2 + \delta_y^2)\{(1/\delta_x^2)(u_{(n)j,k-1} + u_{(n)j,k+1})$$
$$+ (1/\delta_y^2)(u_{(n)j-1,k} + u_{(n)j+1,k}) - (\rho h/D)\omega_{(n)}^2 w_{(n)j,k}\} \qquad \text{for all inner nodes,} \tag{8}$$

where, for example, $u_{(n)j-1,k}$ denotes the value of $u(x, y)$ at the point numbered by $(j - 1, k)$ after the nth round iteration, and δ_x, δ_y denote the mesh side lengths (Figure 1(b)). In fact, the Seidel iteration is used for equation (8) [9]. Finally, the values of the function $\tilde{u}_{(n+1)}(x, y)$ are obtained at the inner nodes.

(3) The third step is to obtain the new values of $w(x, y)$ at the inner nodes. By using the finite difference method, from (5) one has [2]

$$\tilde{w}_{(n+1)j,k} = \delta_x^2\delta_y^2/2(\delta_x^2 + \delta_y^2)\{(1/\delta_x^2)(w_{(n)j,k-1} + w_{(n)j,k+1})$$
$$+ (1/\delta_y^2)(w_{(n)j-1,k} + w_{(n)j+1,k}) - \tilde{u}_{(n+1)j,k}\} \qquad \text{for all inner nodes,} \tag{9}$$

Similarly, the Seidel iteration is used for (9) [9]. Finally, the values of the function $\tilde{w}_{(n+1)}(x, y)$ are obtained at the inner nodes.

(4) The fourth step is to adjust and to obtain the boundary values of $w(x, y)$ and $u(x, y)$ along the edge OA, in the clamped edge case, or in the traction free case. Clearly, in order

to perform the next computation stage, it is necessary to know the values of the functions $u(x, y)$ and $w(x, y)$ at the boundary nodes (Figure 1). From the condition (6) one sees that, the boundary values of $w(x, y)$ along the edges OB, AC, OA, BC and those of $u(x, y)$ along the edges OB, AC, BC are equal to zero. Those values are given beforehand and are not changed in the iterative process. The remaining problem is how condition (6h) can be satisfied.

In fact, the boundary value condition at any node $P(x_0, 0)$ on the edge OA can be satisfied in the following manner. First, at the vicinity of point P one assumes

$$w(x_0, y) = g(y/\delta_y)^2 + h(y/\delta_y)^4, \tag{10}$$

where δ_y denotes the mesh side length. Therefore, the condition (6h) is satisfied and the coefficient g and h can be evaluated by letting

$$w(x_0, \delta_y) = w_1 = g + h, \qquad w(x_0, 2\delta_y) = w_2 = 4g + 16h, \tag{11}$$

where w_1, w_2 denote the values of $w(x, y)$ at the points P_1, P_2, respectively (Figure 1). From equation (11) it follows

$$g = (16w_1 - w_2)/12, \qquad h = (w_2 - 4w_1)/12. \tag{12}$$

From (10) and (12) one has

$$\partial^2 w(x_0, y)/\partial y^2|_{y=0} = 2g/\delta_y^2 = (16w_1 - w_2)/6\delta_y^2 \tag{13}$$

and obtains the value of the function $u(x, y)$ at the point P (Figure 1) as

$$u|_P = (\partial^2 w/\partial x^2 + \partial^2 w/\partial y^2)|_P = \partial^2 w/\partial y^2|_P = (16w_1 - w_2)/6\delta_y^2. \tag{14}$$

The above derivation means that: (a) the boundary condition is satisfied implicitly by the suggested assumption shown by equation (10), (b) the values of $u(x, y)$ at the nodes on the edge OA is obtained and adjusted from the values of $w(x, y)$ at the inner nodes. Therefore, one can adjust and obtain the values of the function $u(x, y)$ along the edge OA by using equation (14).

In the second case, the edge OA is traction free. In this case, the boundary values of both the functions $w(x, y)$ and $u(x, y)$ should be adjusted. To this end, one assumes

$$w(x_0, y) = w_p + d_1(y/\delta_y) + d_2(y/\delta_y)^2 + d_3(y/\delta_y)^3, \qquad u(x_0, y) = u_p + (u_1 - u_p)(y/\delta_y), \tag{15, 16}$$

where w_p denotes the value of the function $w(x, y)$ at the point P, and u_p, u_1 denote the values of the function $u(x, y)$ at the points P and P_1, respectively (Figure 1).

Substituting $y = \delta_y$, $y = 2\delta_y$, $y = 3\delta_y$ into (15), one obtains

$$w_p + d_1 + d_2 + d_3 = w_1, \qquad w_p + 2d_1 + 4d_2 + 8d_3 = w_2, \qquad w_p + 3d_1 + 9d_2 + 27d_3 = w_3. \tag{17}$$

where w_1, w_2, w_3 denote the value of the function $w(x, y)$ at the inner node points P_1, P_2, P_3 respectively (Figure 1).

In addition, substituting (15) and (16) into the boundary condition (6i, j), it follows

$$2\alpha d_2 + u_p\delta_y^2 = 0, \qquad 6\beta d_3 + (u_1 - u_p)\delta_y^2 = 0. \tag{18}$$

Consider w_p, u_p, d_1, d_2, d_3 as unknowns, from five equations in (17) and (18), one obtains

$$w_p = 1/(\beta - 2\alpha)\{u_1\delta_y^2 + \alpha(-5w_1 + 4w_2 - w_3) + \beta(3w_1 - 3w_2 + w_3)\},$$
$$u_p = (\alpha/\delta_y^2)\{-2w_p + 5w_1 - 4w_2 + w_3\}. \tag{19}$$

The above derivation reveals that the boundary values w_p, u_p have a relation with the values w_1, w_2, w_3, u_1 which are the values of the function $w(x, y)$ and $u(x, y)$ at the inner nodes (Figure 1). Finally, one adjusts and obtains the new values of the functions $w(x, y)$ and $u(x, y)$ at the nodes along the edge OA by using equation (19).

(5) The fifth step is to obtain the final results after the $(n+1)$th iteration. Obviously, the investigated vibration mode can differ from each other by an arbitrary multiple. That is to say, in order to judge the approximation of the vibration mode, the mode should be normalized by some technique. Otherwise, comparison for different modes for the same eigenvalue is meaningless. To this end, one calculates

$$\int\int_{\mathrm{A}} [\tilde{w}_{(n+1)}(x, y)]^2 \,\mathrm{d}x\,\mathrm{d}y = Q^2, \tag{20}$$

where A denotes the rectangular region. Since the values of the function $\tilde{w}(x, y)$ at the nodes are known, the numerical integration can be easily deduced. In addition, one lets

$$w_{(n+1)}(x, y) = \tilde{w}_{(n+1)}(x, y)/Q, \qquad u_{(n+1)}(x, y) = \tilde{u}_{(n+1)}(x, y)/Q. \tag{21}$$

In this case, the function $w_{(n+1)}(x, y)$ satisfies

$$\int\int_{\mathrm{A}} [w_{(n+1)}(x, y)]^2 \,\mathrm{d}x\,\mathrm{d}y = 1. \tag{22}$$

From the above mentioned steps one sees that

$$\nabla^2\nabla^2\tilde{w}_{(n+1)}(x, y) = \nabla^2\tilde{u}_{(n+1)}(x, y) = (\rho h/D)\omega^2_{(n)}w_{(n)}(x, y). \tag{23}$$

In fact, equation (23) is satisfied in finite difference form. Substituting (21) into (23) yields

$$\nabla^2\nabla^2 w_{(n+1)}(x, y) = (\rho h/D)(1/Q)\omega^2_{(n)}w_{(n)}(x, y). \tag{24}$$

Simply considering two points: (a) the function $w_{(n)}(x, y)$ has been normalized in the previous computation stage (b) it is assumed that

$$w_{(n+1)}(x, y) \approx w_{(n)}(x, y) \tag{25}$$

Thus, the coefficient at the right side of (24) will be the new eigenvalue. Therefore, one has

$$\omega^2_{(n+1)} = (1/Q)\omega^2_{(n)} \tag{26}$$

(6) The final step is to evaluate the deviation in iteration. In this study, one assumes that if the following conditions:

$$|\omega^2_{(n+1)} - \omega^2_{(n)}| \leqslant \varepsilon_1, \qquad \max|w_{(n+1)}(x, y) - w_{(n)}(x, y)| \leqslant \varepsilon_2 \qquad \text{for all nodes} \tag{27, 28}$$

are satisfied, the fundamental vibration frequency and the vibration mode $w(x, y)$ are finally obtained. Otherwise, one needs to perform the next round of iterative computation.

In fact, if all edges are simply supported, the fourth step is not necessary. In the case of $a = 1$ $\rho h/D = 1$ one chooses $\varepsilon_1 = 10^{-5}$, $\varepsilon_2 = 10^{-8}$. The iterative process is convergent in general. In addition, in the first round computation, the values of ω^2 and $u(x, y)$, $w(x, y)$ can be assumed freely. For example, ω^2 can be chosen to be an arbitrary positive value, $w(x, y)$ to be any positive values at inner nodes, $u(x, y)$ to be zero at inner nodes.

TABLE 1

The calculated fundamental frequency $f(b/a)$ for a rectangular bending plate with the simply supported edges (see Figure 1 and equation (29))

	b/a				
N	1	2	3	4	5
10	4·425	3·498	3·298	3·225	3·191
20	4·438	3·509	3·308	3·235	3·201
30	4·441	3·511	3·310	3·237	3·202
40	4·442	3·511	3·311	3·237	3·203
Exact	4·443	3·512	3·312	3·238	3·204

N: number of divisions used for each edge.

3. NUMERICAL EXAMPLES

Five examples are presented to verify the accuracy of the suggested method or to provide some new results.

Example 3.1.

In the first example, all edges of the rectangular plate with width a and height b were simply supported (Figure 1). In computation 10×10, 20×20, 30×30, 40×40 meshes were used to perform the iteration process. The final calculated results were expressed by

$$\omega^2 = (D/\rho h)(\lambda/a)^4, \qquad \lambda = f(b/a). \tag{29}$$

Meanwhile, the problem has an exact solution as follows [2, 3]

$$\lambda_{ext} = f(b/a) = \pi(1 + (b/a)^{-2})^{1/2} \tag{30}$$

The numerical results obtained for $f(b/a)$ are listed in Table 1. From which one sees that a convergent tendency can be seen when the mesh size decreases. In the case of a 40×40 mesh, deviation from the exact solution is negligible.

Example 3.2.

In the second example, two edges of the rectangular plate are clamped, while the other two edges are simply supported (Figure 1). For calculation, a 40×40 mesh was used to perform the iteration process. Three boundary conditions were assumed in the computation (a) type A, where the lower and upper edges are simply supported, and the left and right edges are clamped, (b) type B, where the lower and upper edges are clamped and the left and right edges are simply supported. (c) type C, where two adjacent edges

TABLE 2

The calculated fundamental frequency $g(b/a)$ for a rectangular bending plate with two simply supported edges and two clamped edges (see Figure 1 and equation (31))

	b/a				
Type	1	2	3	4	5
A	5·377 5·380†	4·876	4·791	4·762	4·749
B	5·377	3·697	3·269	3·262	3·215
C	5·198	4·213	4·045	3·990	3·965

†Obtained in reference [2].

TABLE 3

The calculated fundamental frequency $h(b/a)$ for a rectangular bending plate with various boundary conditions (see Figure 1 and equation (32))

	b/a				
Type	2·5	1·5	1·0	2/3	0·4
A	3·156	3·242	3·394	3·678	4·310
	3·183†	3·266†	3·418†	3·703†	4·336†
	3·187‡	3·267‡	3·418‡	3·703‡	4·336‡
B	3·099	3·090	3·080	3·069	3·058
	3·124†	3·114†	3·103†	3·092†	3·080†
C	1·223	2·046	3·080	4·635	7·748

† Obtained in reference [2].
‡ Obtained in reference [8].

are simply supported and other two adjacent edges are clamped. The final calculated results are expressed by

$$\omega^2 = (D/\rho h)(\lambda/a)^4, \qquad \lambda = g(b/a) \tag{31}$$

The numerical results obtained for $g(b/a)$ are listed in Table 2.

Example 3.3.

In the third example for the rectangular plate (Figure 1), $v = 0{\cdot}3$ is taken and a 30×30 mesh was used in computation. Three boundary conditions were assumed as follows (a) type A, the lower edge is free, and others are simply supported. (b) type B, the lower and upper edges are free and the left and right edges are simply supported. (c) type C, the lower and upper edges are simply supported and the left and right edges are free. The final calculated results are expressed by

$$\omega^2 = (D/\rho h)(\lambda/a)^4, \qquad \lambda = h(b/a) \tag{32}$$

The obtained numerical results for $h(b/a)$ are listed in Table 3, from which one can see that the deviation of the results obtained in different ways is very small.

Example 3.4.

In the fourth example of the rectangular plate (Figure 1), $v = 0{\cdot}3$ is taken a 20×20 mesh was used in computation. Five boundary conditions were assumed as follows (a) type A:

TABLE 4

The calculated fundamental frequency $p(b/a)$ for a rectangular bending plate with various boundary conditions (see Figure 1 and equation (33))

	b/a				
Type	2·5	1·5	1·0	2/3	0·4
A	3·380	3·772	4·438	5·658	8·450
(exact)	3·384	3·776	4·443	5·664	8·459
B	3·379	3·769	4·431	5·640	8·386
C	3·364	3·722	4·311	5·295	6·792
D	3·292	3·532	3·889	4·415	5·265
E	3·144	3·230	3·382	3·666	4·297

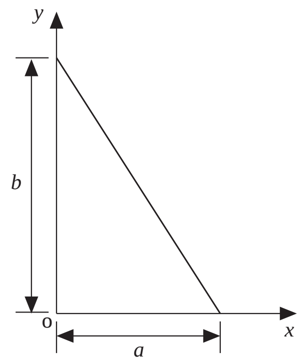

Figure 2. A triangular bending plate.

all edges are simply supported, (b) type B: 1/4 of the lower edge is free and others are simply supported, (c) type C: 2/4 of the lower edge is free and others are simply supported, (d) type D: 3/4 of the lower edge is free and others are simply supported, (e) type E: the lower edge is free and others are simply supported. The final calculated results are expressed by

$$\omega^2 = (D/\rho h)(\lambda/a)^4, \qquad \lambda = p(b/a) \tag{33}$$

The numerical results obtained for $p(b/a)$ are listed in Table 4, from which one can see that compared with the free edge case, the natural frequency is higher in the simply supported edge case.

Example 3.5.

In the fifth example, a triangular bending plate is considered for evaluating the fundamental vibration frequency (Figure 2). The inclined edge is assumed to be simply supported. For the other two edges, the following four types of boundary condition are assumed: (a) type A: both edges are simply supported, (b) type B: both edges are clamped, (c) type C: the horizontal edge is simply supported and the vertical edge is clamped, (d) type D: the horizontal edge is clamped and the vertical edge is simply supported. In computation, 40 divisions were used for each edge to perform the iteration process. The final calculated results are expressed by

$$\omega^2 = (D/\rho h)(\lambda/a)^4, \qquad \lambda = q(b/a) \tag{34}$$

The numerical results obtained for $q(b/a)$ are listed in Table 5.

TABLE 5

The calculated fundamental frequency $q(b/a)$ for a triangle bending plate with various boundary conditions (see Figure 2 and equation (34))

	b/a									
Type	1	2	3	4	5	6	7	8	9	10
A	7·019	5·264	4·670	4·363	4·173	4·041	3·945	3·870	3·810	3·761
B	8·550	6·413	5·691	5·320	5·091	4·934	4·819	4·730	4·659	4·601
C	7·770	6·024	5·433	5·129	4·940	4·810	4·714	4·640	4·581	4·533
D	7·770	5·633	4·912	4·542	4·314	4·157	4·042	3·953	3·883	3·825

4. REMARKS

In the literature, the natural vibration frequency has been mainly investigated in two different ways. The first was to use the Rayleigh–Ritz principle. The second way was to investigate the frequency by using the partial differential equation. The suggested approach belongs to the latter. However, in this paper the partial differential equation is approximated by the finite difference equation. Generally speaking, the suggested approach provides a relatively simple way to study the fundamental vibration frequency. For example, by using a rather simple program with 250 source lines, very accurate results have been obtained.

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