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# LETTERS TO THE EDITOR

## THE MOTION OF A DAMPED *n*-DEGREE-OF FREEDOM SYSTEM WITH A SINGLE NATURAL "FREQUENCY"

E. V. Wilms
*Department of Civil and Geological Engineering, University of Manitoba, Winnipeg, Manitoba, Canada*

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Consider the system shown in Figure 1. Each body has unit mass, and is connected to each of the other bodies by a spring of constant unity, and by a dashpot of constant $c$. Motion takes place in the horizontal direction. The displacements of the bodies are $x_1$, $x_2$ and $x_3$.

The forces acting on body 1 are shown in Figure 2. The equation of motion of body 1 is

$$\ddot{x}_1 = x_2 - x_1 + x_3 - x_1 + c(\dot{x}_2 - \dot{x}_1) + c(\dot{x}_3 - \dot{x}_1). \tag{1}$$

Similarly, the equations of motion of bodies 2 and 3 are

$$\ddot{x}_2 = x_3 - x_2 + x_1 - x_2 + c(\dot{x}_3 - \dot{x}_2) + c(\dot{x}_1 - \dot{x}_2), \tag{2}$$

$$\ddot{x}_3 = x_1 - x_3 + x_2 - x_3 + c(\dot{x}_1 - \dot{x}_3) + c(\dot{x}_2 - \dot{x}_3). \tag{3}$$

Adding equations (1)–(3),

$$\ddot{x}_1 + \ddot{x}_2 + \ddot{x}_3 = 0, \tag{4}$$

Integrating,

$$\dot{x}_1 + \dot{x}_2 + \dot{x}_3 = 0. \tag{5}$$

Integrating again,

$$x_1 + x_2 + x_3 = 0. \tag{6}$$

Setting the constants of integration equal to zero eliminates an overall rigid body mode. Equation (5) may also be obtained from conservation of momentum. Substituting equations (5) and (6) into equation (1) yields

$$\ddot{x}_1 + 3c\dot{x}_1 + 3x_1 = 0. \tag{7}$$

If we have $n$ bodies rather than three, equations (5)–(7) become

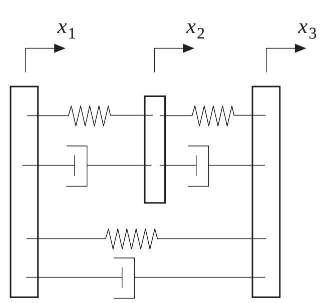


Figure 1. The system.

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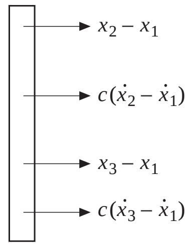


Figure 2. The forces acting on body one.

$$\sum_{i=1}^{n} x_i = 0, \qquad \sum_{i=1}^{n} \dot{x}_i = 0, \tag{8}$$

and

$$\ddot{x}_i + nc\dot{x}_i + nx_i = 0, \qquad i = 1, \ldots, n. \tag{9}$$

The solutions of equations (9) are

$$x_i = \mathrm{e}^{-(nc/2)t}\{b_i/q + a_i nc/2q\} \sin qt + \mathrm{e}^{-(nc/2)t} a_i \cos qt, \qquad i = 1, \ldots, n. \tag{10}$$

In equations (10), $x_i(0) = a_i$, $\dot{x}_i(0) = b_i$ and $q = (n - n^2c^2/4)^{1/2}$.

There are a total of $n$ constants $a_i$. Only $n - 1$ are independent. The last one is obtained from equation (8), evaluated at time $t = 0$. A similar argument holds for the constants $b_i$.

Equation (10) indicates that motion occurs with a single natural "frequency",

$$q = (n - n^2c^2/4)^{1/2}.$$