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EFFECTS OF GEOMETRY ON THE RESONANCE FREQUENCY OF HELMHOLTZ RESONATORS, PART II

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1. INTRODUCTION

In a previous study [1], equations were developed for the internal and external "end" corrections for use in calculating the resonance frequency of a Helmholtz resonator. They applied to a cavity that was a rectangular parallelepiped with an orifice that was either circular, rectangular or cross-shaped. The external end correction was that of a baffled piston. The internal end correction equations extended the work of Ingard [2] to a resonator of finite depth with an arbitrarily placed orifice. The resonance frequency was calculated for extremes of cavity dimensions and showed the wide range of frequencies that could be obtained for a cavity of constant volume and constant orifice area, unlike the Rayleigh equation [3], which predicts only one frequency. The validity of the new interior end correction was confirmed experimentally only with respect to cavity depth.

In this study, an equation was developed for the internal end correction of a resonator with a cylindrical cavity and an arbitrarily placed circular orifice on the circular end face. Experiments were performed on both types of resonators to establish the validity of the new corrections for both cavity shapes and asymmetric orifice positions. Comparison is made between the present theory and the experimental results of this study and measurements of others as well as with the resonance frequency equations developed by Rayleigh [3] and Nielsen [4].

2. RESONANCE EQUATIONS FOR A CYLINDRICAL RESONATOR

Applying the methods of Ingard [2] and the previous study [1] to the cylindrical cavity with arbitrarily placed circular orifice, yields the resonance frequency equation

$$\cot kd = \frac{A_c(l_o + \delta_i + \delta_e)}{A_o d}\, kd,$$

where k is the resonance wavenumber, A_c is the cross-sectional area and d the depth of the cylindrical cavity, A_o is the area and l_o the depth of the orifice, δ_i is the interior end correction and δ_e is the exterior end correction. The lowest frequency solution to that equation is relevant to the Helmholtz resonator.

The exterior end correction is that for a baffled circular piston radiator, while the interior end correction is now

$$\delta_i = \frac{d}{A_o A_c} \sum_{m=0}^{\infty}{}' \sum_{n=0}^{\infty}{}' \upsilon_{mn} \mathrm{I}_{mn}^2 \frac{\cot k_x d}{k_x d}, \qquad k_x^2 = k_{mn}^2 - k^2,$$

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where the primes indicate that the term 00 is excluded from the summation, and k_{mn} is the solution to the equation

$$\frac{\mathrm{d}}{\mathrm{d}r}\,\mathrm{J}_m(k_{mn}R) = 0,$$

which requires that the radial velocity be zero at the outer wall of radius R. The other terms are

$$\upsilon_{mn} = \frac{2}{\left[1 - \dfrac{m^2}{k_{mn}^2 R^2}\right]\mathrm{J}_m^2(k_{mn}R)}, \qquad \mathrm{I}_{mn} = 2\pi r_o^2 \mathrm{J}_m(k_{mn}a_o)\,\frac{\mathrm{J}_1(k_{mn}r_o)}{k_{mn}r_o},$$

where a_o is the radial position of the orifice center, and r_o is the radius of the orifice. These equations are similar to those presented by Ingard [2], except that his equations have been extended to include finite depth cavities and asymmetrically positioned orifices.

3. MEASUREMENTS OF RESONANCE FREQUENCY

Measurements were made with two resonators, one with a cylindrical cavity and one with a rectangular parallelepiped cavity. The orifice side of each resonator was set into a plane surface with a 90 cm radius to simulate the baffled piston condition on the exterior of the resonator. A Larson–Davis Model 2900 real time analyzer was used to make the measurements; this instrument has two microphone inputs and a signal generator that can create either broad band noise or a sinusoidal signal. The generator output was fed to a loudspeaker through a power amplifier. Signal levels were 30 dB above background sound levels in a reasonably sound absorptive room. One microphone was set into the resonator wall opposite the orifice to measure interior sound level and the other was placed 10 cm in front of the orifice to monitor the loudspeaker sound level.

The approximate resonance frequency was found first with broad band sound and then a sinusoidal sound was stepped, in one-tenth Hertz increments, around the approximate frequency and the interior sound level in the resonator was recorded. This resulted in a resonance frequency measurement accurate to one-tenth Hertz. Measurements were made at 23°C.

First, experiments were performed with a rectangular cavity and a square orifice. Three boxes were fabricated of Plexiglas, 0·95 cm (3/8 in) thick, each with internal dimensions of 10·2 cm (4 in) by 15·2 cm (6 in) by 20·5 cm (12 in). Four orifices were placed on a different face of each box; this is shown diagramatically in Figure 1. The four orifices were positioned as follows; (1) in the center of the face (halfway down both sides); (2) in the corner (at the extreme of both sides); (3) halfway down the first side and at the extreme of the second side; and (4) halfway down the second side and at the extreme of the first side. Each face had a different area so each set of orifices looked into a cavity with a different width, breadth and depth (direction normal to the face with the orifice). The orifices were 2·54 cm (1 in) square; but only one was open for each experiment. Since the volume of the cavity and the orifice area was identical in each of the tests, the Rayleigh equation must predict only one resonance frequency for all tests, while the Nielson equation predicts three, since it accounts for cavity depth. One of the equations (6) of reference [1], which calculates I_{mn} for a rectangular cavity with a rectangular orifice, explictly includes all the cavity dimensions and so predicts 12 resonance frequencies. Comparison of experiment with theory is reported in Table 1. The measurements

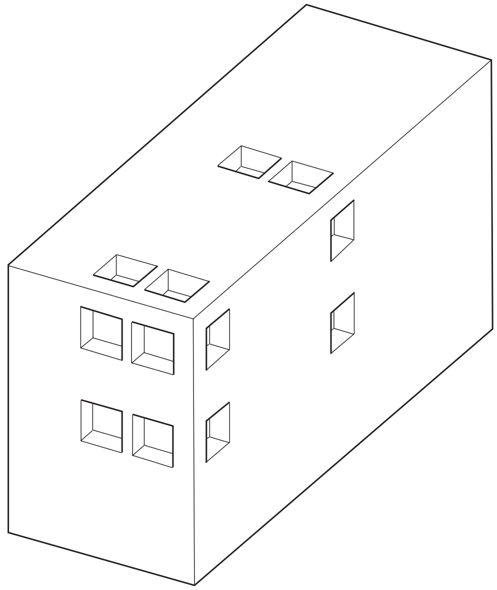

Figure 1. The geometry of the experimental resonator. There were 12 orifice positions on an asymmetric rectangular resonator with constant volume and orifice area.

clearly show the 12 distinct frequencies, but are several percent higher than those calculated.

Then, experiments were performed with a cylindrical cavity of radius 5·08 cm (2 in) and depth 59·7 cm (23·5 in) with a circular orifice of radius 1·27 cm (0·5 in). The orifice center,

TABLE 1

The calculated and measured resonance frequencies for a resonator with a rectangular cavity with several square orifices of identical area at 12 *different positions on the cavity faces. The results show that orifice position has an important influence on the resonance frequency despite the fact that the orifice area and cavity volume are the same for each case. The measured frequencies are several percent higher than those calculated with the present theory. The frequencies calculated from the Rayleigh* [3] *equation or the Nielsen* [4] *equation do not agree closely*

Cavity			Orifice center		Resonance Frequency				
Width	Breadth	Depth	Width	Breadth	Measured	Theory	Nielson	Rayleigh	% low
10·2	15·2	30·5	1·27	1·27	99·5	97·0	104·0	110·4	2·5
			1·27	7·62	106·9	104·2	104·0	110·4	2·5
			5·08	1·27	104·7	102·8	104·0	110·4	1·8
			5·08	7·62	111·3	108·8	104·0	110·4	2·2
10·2	30·5	15·2	1·27	1·27	100·7	97·2	108·7	110·4	3·5
			1·27	15·2	110·1	107·1	108·7	110·4	2·7
			5·08	1·27	107·6	103·0	108·7	110·4	4·2
			5·08	15·2	115·5	112·0	108·7	110·4	3·0
15·2	30·5	10·2	1·27	1·27	98·5	97·2	109·7	110·4	1·3
			1·27	15·2	110·1	107·1	109·7	110·4	2·7
			7·62	1·27	107·5	104·4	109·7	110·4	2·9
			7·62	15·2	115·8	113·0	109·7	110·4	2·4

TABLE 2

The calculated and measured resonance frequencies for a resonator with a cylindrical cavity with a circular orifice the center of which is at several radial positions with respect to the cavity center. The results show that orifice position has an important influence on the resonance frequency despite the fact that the orifice area and cavity volume are the same for each case. Again, the measured frequencies are several percent higher than those calculated with the present theory. The frequencies calculated from the Rayleigh [3] *equation or the Nielsen* [4] *equation do not agree closely*

Cavity		Orifice center	Resonance frequency				
Radius	Depth	Radius	Measured	Theory	Nielson	Rayleigh	% low
5·08	59·7	3·81	86·4	85·3	85·1	101·4	1·2
		2·54	89·1	87·7	85·1	101·4	1·6
		1·27	90·1	88·4	85·1	101·4	1·9
		0	90·6	88·6	85·1	101·4	2·2

a_o, was placed at four radial locations. Comparison of experiment with theory is reported in Table 2. Again the measurements clearly show four distinct frequencies, but are several percent higher than those calculated.

Comparison of the theory was made with resonance frequencies measured by Panton and Miller [5] and by Selamet, Dickey and Novak [6], and are shown in Table 3. The calculated frequencies are quite close to those of reference [5], but are several percent low with respect to those of reference [6].

Comparison of the theory was made with an empirical equation for the interior end

TABLE 3

The calculated and measured resonance frequencies for a resonator with a cylindrical cavity and with a centered circular orifice, as measured by others. The calculations show close agreement with those of reference [5] *and are slightly high with respect to those of reference* [6]

	Cavity		Orifice			Resonance frequency		
Reference	Radius	Depth	Radius	Depth	Center radius	Measured	Theory	% low
5	1·91	10·82	0·254	0·318	0	252	252·0	0
5	1·91	18·90	0·381	0·318	0	234	235·2	−0·5
5	1·91	27·46	0·508	0·318	0	211	211·2	−0·1
5	1·27	6·10	0·254	0·318	0	498	503·7	−1·1
5	1·27	10·64	0·381	0·318	0	472	464·3	1·6
5	1·27	10·64	0·508	0·318	0	538	535·2	0·5
6	7·66	24·42	2·02	8·5	0	89	84·6	4·9
6	6·37	35·28	2·02	8·5	0	87	82·9	4·7
6	5·08	55·55	2·02	8·5	0	81	77·7	4·1
6	4·47	71·65	2·02	8·5	0	75	72·7	3·1
6	3·86	96·01	2·02	8·5	0	66	64·6	2·1
6	3·10	148·5	2·02	8·5	0	50	49·6	0·8

correction of cylindrical cavities with centered, circular orifices, reported by Ingard [2] to be in very good agreement with experiment. The equation was

$$\delta_i = 0{\cdot}48\sqrt{A_o}(1 - 1{\cdot}25\xi), \qquad \xi = r_o/R.$$

Ingard stated that this equation had an upper limit of usefulness of $\xi = 0{\cdot}4$. The comparison with present theory is shown in Figure 2. His equation is within two percent of the present theory from $\xi = 0{\cdot}22$ to $\xi = 0{\cdot}52$. Since his experiments probably did not include wide, shallow cavities, it is likely that the divergence at low values was not detected.

4. CONCLUSIONS

These results, over a wide range of resonator geometry, support the validity of the *form* of the equations for both the rectangular and cylindrical cavity resonator, but, as pointed out in the previous study [1], the equations cannot be exact since they depend on a uniform velocity profile in the orifice, ignore viscous and thermal effects, and assume no flexure of the cavity walls. The deviations caused by this inexactness, however, turn out to be small for small amplitude oscillations.

One major result of this study is that it confirms the important influence of orifice asymmetry on the resonance frequency for both rectangular and cylindrical cavities and, by inference, other shaped cavities. The interior end correction is composed of the evanescent cross modes which are determined by the geometry of the cavity. When the orifice is circular, and centered on either a cubical cavity or a nearly cubical cylindrical cavity (ratio of A_c to d the same in both cases) of identical volume, the resonance frequencies are nearly identical. In Figure 3 is shown the influence of an off-center orifice for each cavity type, as a percentage of its maximum position and centered frequency. For the cylindrical resonator with a circular orifice, the maximum position is $r_{max} = R - r_o$. For the resonator with rectangular cavity and a circular orifice, the maximum position is

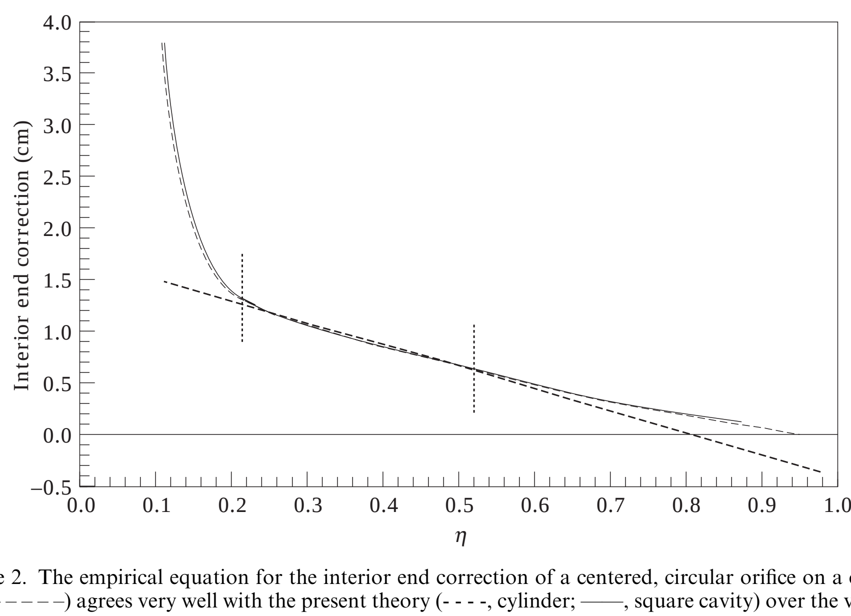

Figure 2. The empirical equation for the interior end correction of a centered, circular orifice on a cylindrical cavity (– – – – –) agrees very well with the present theory (- - - -, cylinder; ——, square cavity) over the valid range of the equation: η is the ratio of the orifice radius to the cylinder radius.

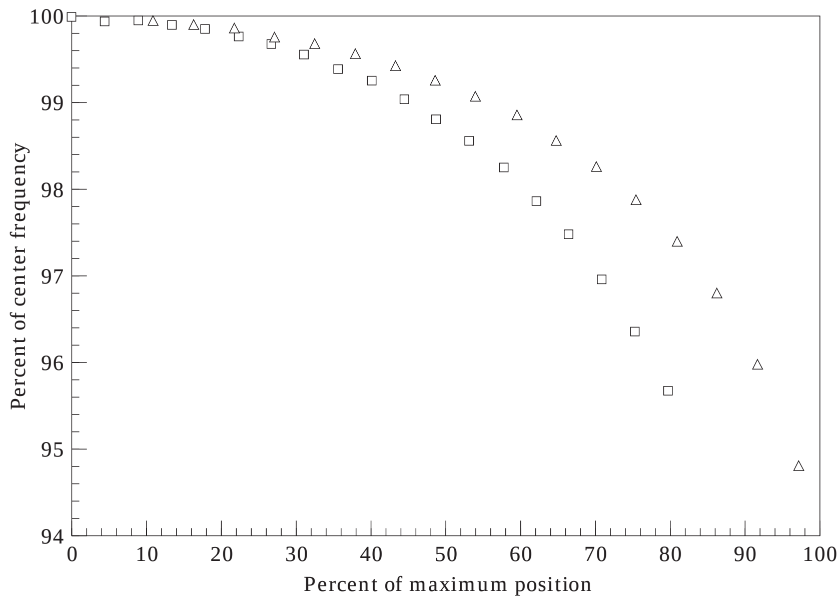

Figure 3. The percentage change of resonance frequency caused by a symmetrically placed circular orifice for rectangular (△) and cylindrical (□) cavities. The reduction is significant when the orifice is positioned far off center.

$r_{max} = \sqrt{(a - r_o)^2 + (b - r_o)^2}$, where a and b are the half-widths of the cavity face containing the orifice.

The motive for this study was to develop a set of resonators with a range of resonance frequencies using uniformly sized hexcell-type cavities sandwiched between two plates. The use of a variable orifice position expands the bandwidth of the set another five percent beyond that obtainable by orifice diameter changes.

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