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AN ADAPTIVE FEEDBACK APPROACH TO STRUCTURAL VIBRATION SUPPRESSION

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1. INTRODUCTION

Most applications of active structural vibration and acoustic control to date have involved the so-called feedforward control, which is depicted in Figure 1. In this approach, a measurement coherent with the disturbance is fed through an appropriate filter and the resulting signal is applied to the plant with the result that the outputs of certain error sensors, which measure the vibration, are reduced. In practice, large amounts of vibration suppression cannot be achieved without tuning the filter, and this process is usually automated by using a simple adaptive algorithm that monitors the error sensors and tunes the filter to optimize performance. This modification turns the open-loop system into a closed-loop feedback system, but if the tuning rate is slow enough, the system can profitably be viewed as an open-loop feedforward system.

The feedforward approach has provided significant levels of disturbance rejection in experiments. This is due to its two major strengths: it is adaptive and stability is not hard to maintain in many practical situations. Its major weakness is that it can be used only in situations where an independent (of the control input) measurement coherent with the disturbance can be obtained. In addition, the feedforward approach provides no protection against fast, transient disturbances, since it cannot adapt quickly enough.

In situations where the feedforward approach does not apply, one is led to consider some form of feedback control. Assuming the properties of the disturbance source are not well known, techniques that rely on a fixed model of the disturbance characteristics [1] are not appropriate. Although fixed-feedback techniques that do not rely on a disturbance model [2] will provide some disturbance attenuation, this paper considers adaptive feedback techniques in the hope that better performance will be achieved by adapting to the existing disturbance.

One proposed approach to adaptive feedback in acoustic control is to implement a "neutralization loop" so that the feedback connection between u and y is broken, resulting in a situation resembling the standard feedforward approach [3]. This is illustrated in Figure 2, which explicitly shows the four transfer functions between the two inputs, d and u, and the two outputs, e and y. Without the neutralization loop, the controller output u would feed back to its input y through G_{yu} with the potential of causing instability with a stable open-loop controller. The neutralization loop feeds the control signal u through a copy of G_{yu}, $\hat{G}_{yu}$, and subtracts the result from the actual output y prior to using the output in the controller. This effectively eliminates the only feedback path in the system, leaving only two forward paths: one from the disturbance directly to the error and the other from the disturbance to y, through the controller to u, and then from u to the error. Thus, if the controller and plant are stable, the overall system must be stable. Since the

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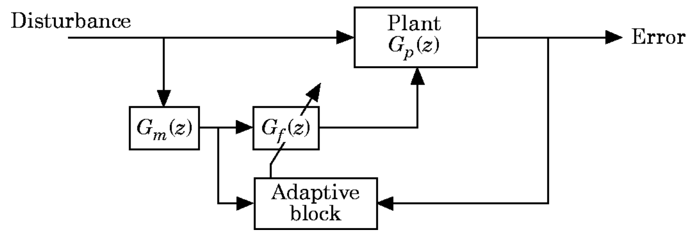

Figure 1. Feedforward control system.

neutralization loop will never provide perfect cancellation, stability will not be as easy to maintain with this configuration as with the standard feedforward configuration.

In this paper, an adaptive feedback technique due to Tay and Moore [4], which arose from a totally different theoretical setting, is applied to the structural disturbance rejection problem. This approach uses a fixed feedback to provide suppression of transient disturbances and an adaptive feedback to provide rejection of steady disturbances. It does not require an independent measurement of the disturbance. Thus, the approach eliminates two of the shortcomings of the feedforward approach, while maintaining the advantage of adaptivity. The neutralization loop technique is a special case of this adaptive feedback approach, and it appears that their ability to maintain stability should be equivalent.

Although this technique has been investigated via simulation on several second order plants, its usefulness for problems typical of structural and acoustic control has not been investigated. The contribution of the present paper is to investigate the usefulness of this technique for structural and acoustic systems that exhibit relatively high order, have almost all their poles close to the stability boundary, have non-minimum phase zeros and always include unmodelled dynamics in the high frequency range. The performance of the technique for harmonic, narrowband and broadband disturbances is explored via simulation, and the results are compared to the theoretically optimal results that can be obtained with feedback. The effect of system identification errors and unmodelled modes is investigated and a modification of the technique is introduced to reduce the sensitivity to unmodelled modes.

2. THE ADAPTIVE-Q FEEDBACK CONTROLLER

One approach to solving feedback control problems is to start by parameterizing the set of all stabilizing controllers. One parameterization, based on a nominal feedback

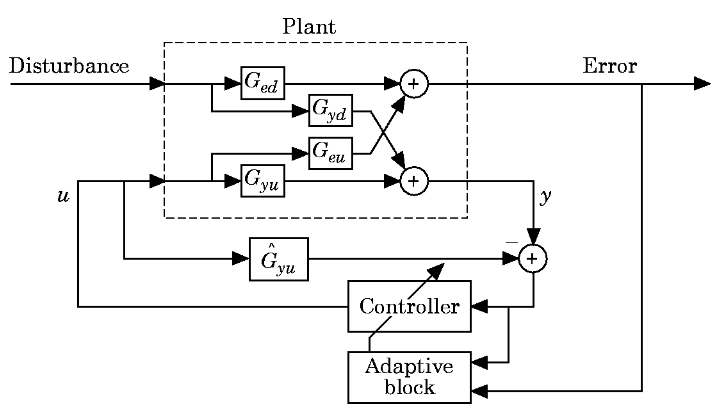

Figure 2. Adaptive feedback control using a neutralization loop.

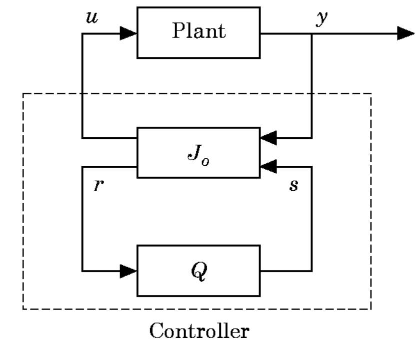

Figure 3. Parametrization of all stabilizing feedback controllers.

controller J_0, is shown in Figure 3. It can be shown [5] that as Q sweeps over the set of all stable transfer matrices of dimension $\dim(u) \times \dim(y)$, the controller consisting of J_0 and Q sweeps over all possible stabilizing controllers for the plant. Thus, the control design problem reduces to choosing the best stable transfer function Q. This form of the parameterization is useful for our purpose because it allows for a fixed controller to provide rejection for transient disturbances, which occur too fast to be reduced by an adaptive controller, and an adaptive part, Q, which one can tune to provide rejection for steady disturbances.

The adaptive disturbance–rejection scheme proposed in reference [4] is shown in Figure 4. One assumes that the plant can be modelled by a linear difference equation of the form

$$x_{k+1} = Ax_k + Bu_k + Ed_k, \qquad y_k = Cx_k, \qquad e_k = C_e x_k + D_e u_k,$$

where u_k is the control input, d_k is the disturbance input, y_k is the measured output and e_k is the signal to be minimized (and may be equal to y_k). It is assumed that e_k can be measured directly or constructed from signals that are measured or known. The controller is constructed using state estimate feedback and takes the form

$$\hat{x}_{k+1} = A\hat{x}_k + Bu_k + K(y_k - C\hat{x}_k), \qquad u_k = -F\hat{x}_k + s_k, \qquad r_k = y_k - C\hat{x}_k,$$

where $\hat{x}_k$ is the state estimate and r_k is the output estimation error. In the standard LQG framework, K would be the Kalman filter gain and F would be the LQR state feedback gain. The new input signal, s_k, is injected directly into the control input, u_k. Since the state

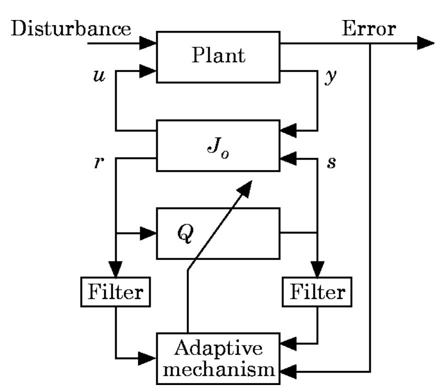

Figure 4. Adaptive disturbance-rejection feedback controller.

estimator has exact knowledge of the control input, s_k produces no estimation error in either the state or the output. Thus, the transfer function between s_k and r_k is identically equal to zero and the fixed feedback controller provides the neutralization loop.

What remains in the controller design is to find a way to adapt Q. The error results from the two inputs d and s, so that, using mixed notation,

$$e_k = T_{ed} d_k + T_{es} Q r_k, \tag{1}$$

where T_{ed} is the transfer function from d to e with $Q = 0$, and T_{es} is the transfer function from s to e. If Q is chosen to be an FIR filter, which alleviates any worry about keeping Q stable, this can be seen to be in the standard form of a problem to which the filtered-x LMS algorithm applies.

In addition, the problem of minimizing e can be reformulated as a system identification problem and the well developed algorithms from this field can be brought to bear on the problem of adapting Q. For example, in the case in which y and u, and hence r and s, are scalars, T_{es} and Q commute. Defining

$$i_k = -T_{es} r_k, \qquad z_k = T_{ed} d_k, \tag{2}$$

equation (1) can be written in the form

$$z_k = Q i_k + e_k,$$

which is the output error model used in system identification. In the remainder of this paper Q will be considered to be an FIR filter.

Experience with hybrid controllers, those that incorporate both fixed feedback and adaptive feedforward, has shown that the addition of a feedback loop to provide system damping results in faster convergence of the adaptive filter and lower filter orders for the same performance when compared with the undamped case [6, 7]. Thus, an added advantage of the J_0 feedback loop, as compared to a neutralization loop, may be shorter convergence time and lower required filter order.

3. APPLICATION TO A STRUCTURAL PLANT

To assess the performance of this technique for structural vibration suppression, a modal model of the form

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} x_p \\ x_v \end{bmatrix} = \begin{bmatrix} 0 & I \\ -\Omega^2 & -2Z\Omega \end{bmatrix}\begin{bmatrix} x_p \\ x_v \end{bmatrix} + \begin{bmatrix} 0 \\ \Phi_u \end{bmatrix} u + \begin{bmatrix} 0 \\ \Phi_d \end{bmatrix} d \triangleq Ax + Bu + Ed, \tag{3a}$$

$$y = [0 \quad \Phi_y^{\mathrm{T}}] \begin{bmatrix} x_p \\ x_v \end{bmatrix} \triangleq Cx, \tag{3b}$$

is considered, where the inputs are forces and the measurements are velocities. The values of the parameters are taken from experimental data for the simply supported plate discussed in reference [8] and are given in Table 1. For simulation and control design purposes, it is necessary to determine a discrete time equivalent model of the above system. Consider first the simulation model. Although the control input will be held constant between sample times, the disturbance input will not, since it is being generated by a continuous time process. To form an accurate discrete time model, the disturbance will be assumed to be generated by white, Gaussian noise that is passed through a shaping filter. The equations of this filter are

TABLE 1

Parameters for the modal model

diag ($\Omega/2\pi$)	diag (Z)	$\mathbf{\Phi}_u$	$\mathbf{\Phi}_d$	$\mathbf{\Phi}_y$
49·45	0·0077	0·7033	0·7033	0·4001
108·96	0·0117	0·3664	−0·3062	−0·2333
130·25	0·0083	0·1091	0·1353	0·8233
188·53	0·0027	0·2185	−0·0554	−0·1619
203·25	0·0027	0·5573	0·5246	0·3468
265·62	0·0024	0·6715	0·6389	−0·6213
285·78	0·0012	0·1343	0·1319	0·5346
326·08	0·0013	−0·4100	0·3292	−0·2604
338·30	0·0022	−0·5521	0·6070	0·4473

$$\dot{z} = A_d z + B_d v, \qquad d = C_d z, \tag{4}$$

where v is unit intensity white noise. The discrete time equivalent of the overall system is given by

$$\xi_{k+1} = A_a \xi_k + B_a u_k + E_a w_k, \qquad y_k = C_a \xi_k + n_k, \tag{5a, b}$$

where $\xi = [x_p^{\mathrm{T}} \;\; x_v^{\mathrm{T}} \;\; z^{\mathrm{T}}]^{\mathrm{T}}$ is the augmented state, w_k is a unit-intensity discrete time white noise process, and to which the measurement noise term n_k has been added.

To determine the optimal suppression obtainable by a feedback controller, consider the problem of minimizing $E[e_k^{\mathrm{T}} e_k]$, where E is the expected value operator and $e_k = C_e x_k + D_e u_k$. This is in the form of a Linear-quadratic Gaussian (LQG) control problem. Typically, C_e and D_e are chosen to be of the form

$$C_e = \begin{bmatrix} Q^{1/2} \\ 0 \end{bmatrix}, \qquad D_e = \begin{bmatrix} 0 \\ R^{1/2} \end{bmatrix}$$

and then the cost function takes the standard form

$$E[e_k^{\mathrm{T}} e_k] = E[x_k^{\mathrm{T}} Q x_k + u_k^{\mathrm{T}} R u_k].$$

In many cases, the control effort is not significant and does not need to be penalized. In these cases, as R is decreased a point is reached where the performance of the controller ceases to improve. This level of performance is the optimal suppression obtainable by feedback, since the controller is using all of the *a priori* information about the disturbance and is essentially unrestricted as to the amount of control effort that can be applied.

Although the discrete time system (5) is suitable for simulation, it cannot be used for control system design because it contains information that will be unknown in an actual application. The way in which the disturbance enters the plant, given by the matrix E, and the dynamics of the noise, given by the shaping filter (4), will not be known and are subject to change. Thus, the controller should not depend on these values. For control design, the model

$$\dot{x} = Ax + Bu + \begin{bmatrix} 0 \\ I \end{bmatrix} v, \qquad y = Cx + n, \tag{6a, b}$$

will be used, where A, B, and C are defined in equation (3) and the disturbance is modelled as white noise acting independently on each mode. This model requires no spectral information and no knowledge of how the disturbance enters the plant, but provides some information about the disturbance for design of the state estimator. If an adaptive controller can be made to work with this minimal amount of disturbance characterization, it will truly be independent of *a priori* information. The actual design uses a discretized version of equation (6).

Due to modelling inaccuracies and unmodelled modes, the transfer function from s to r will not be zero in practice. It is now possible for a stable Q to drive the system unstable, and this is indeed what will happen with an unmodelled mode. A simple fix is to prefilter r, prior to sending it through Q, so that there is little energy in the inaccurate frequency region. For example, in the case of unmodelled high frequency dynamics, one will low pass filter r. This will re-establish the near zero transfer function from s to r, eliminating instability due to a stable Q.

4. SIMULATION RESULTS

The continuous time plant described by equations (3) with the parameters given in Table 1 is used as the test system. The frequency response of this system is shown in Figure 5. To simulate and control the system, a discretized version of equations (3) is used and the sampling frequency is 2000 Hz. The nominal controller, J_0, is designed with the LQG methodology using a process noise covariance of 1 and a measurement covariance of 1×10^{-4}. In all cases, the state weighting is $Q = C_{nom}^{\mathrm{T}} C_{nom}$, where C_{nom} is the output matrix of the nominal model, i.e., the model on which the control system design is based. The value of the control weighting will vary with the cases considered. The error is taken to be $e_k = C_{act} x_k$, where C_{act} is the output matrix of the actual plant. A standard RLS algorithm is used for all adaptations and the starting covariance P is initialized as the identity matrix multiplied by a suitable scalar.

In the first case, a harmonic disturbance is considered. It is assumed in this case that the disturbance is held constant between sampling instants so that a simple discretization of equations (3) can be used as the discrete time plant. For simplicity, only the first five modes of the continuous time plant were used and it was assumed that the nominal plant and the actual plant were identical. The adaptive feedback consisted of a three term FIR

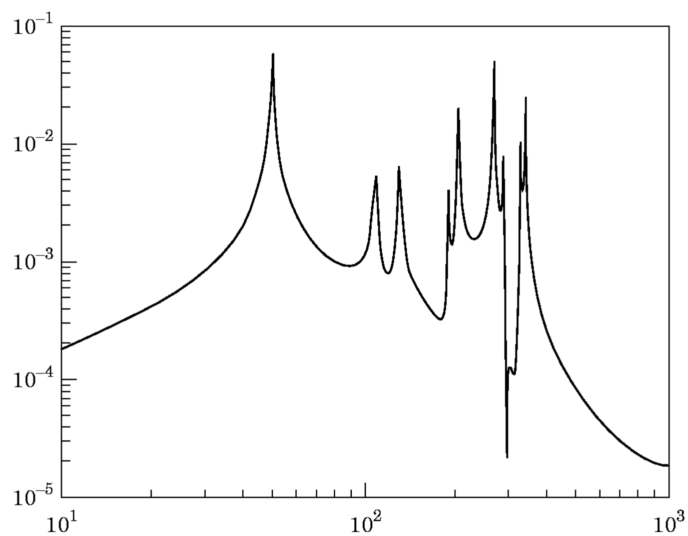

Figure 5. Frequency response of the test system: nine mode plant.

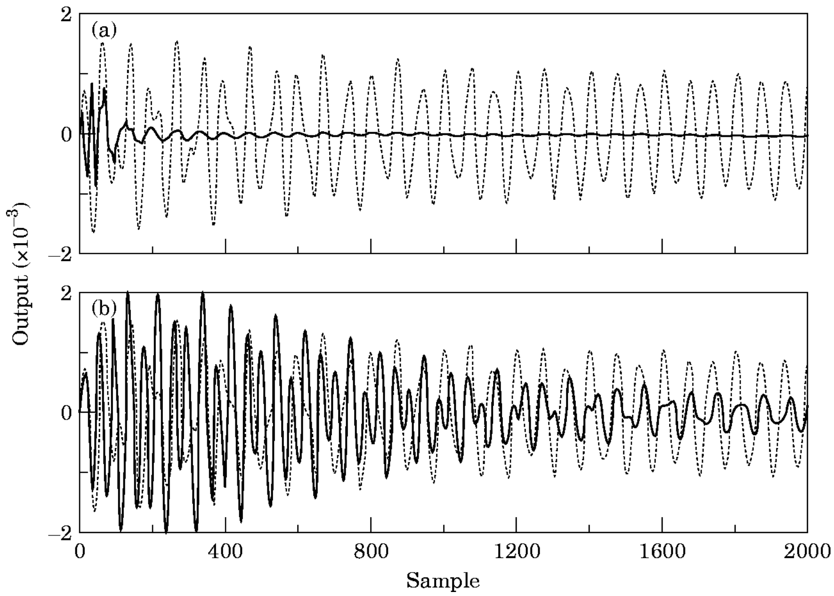

Figure 6. Simulation results for a harmonic disturbance. Dotted lines show the open loop response and solid lines show the closed loop adaptive response. (a) Control weighting is 10^{-6} with P initialized at 10^{14} and (b) control weighting is 10^{-3} with P initialized at 10^{12}.

filter. The disturbance is at 30 Hz and the simulation results are shown in Figure 6. In Figure 6(a), J_0 was designed with a control weighting of 1×10^{-6} and the RLS routine was initialized with $P = 1 \times 10^{14}$. Clearly, the harmonic is greatly suppressed with respect to the open loop response (shown dotted). It should also be noted that the response of the closed loop system with no adaptation ($s = 0$) has almost the same magnitude of response as the open loop system. Thus, the suppression is coming from the adaptive feedback and not from the fixed feedback.

Figure 6(b) shows the results for J_0 designed with a control weighting of 1×10^{-3} and the adaptive routine initialized with $P = 1 \times 10^{12}$. The performance is much poorer than in the first case. Although the harmonic is being suppressed, the convergence rate is much slower than in the first case. If P is increased to 10^{14} the results are even worse. The reason for this is that the fixed feedback provides much less damping in this case and the rapid adaptations excite the first mode of the structure, which is close in frequency to the disturbance. This added sinusoid in the output evidently makes it more difficult for the adaptive feedback to converge.

The second case considered is that of a narrowband disturbance. Again, a 5-mode model of the plant is used and the adaptive feedback is a 3-term FIR filter. The disturbance is centered at 60 Hz and Figure 7 shows the frequency response of the shaping filter (4). The transfer function of the filter is $120\pi s/(s^2 + 24\pi s + 14400\pi^2)$. Figure 8(a) shows the open loop response of the system to the disturbance and Figure 8(b) shows the response of the system with the fixed feedback (control penalty = 1e-5) and no adaptation. The fixed feedback provides some disturbance rejection and although this could be increased in simulation, in practice there will be a limit to the reduction achievable while maintaining stability. The additional rejection provided by the adaptive feedback is shown in Figure 9(a). In addition, the optimal rejection achievable by feedback, as discussed in section 2, is shown in Figure 9(b). It can be seen that the response with the three-term adaptive loop is within a factor of 2 or 3 of the optimal possible response.

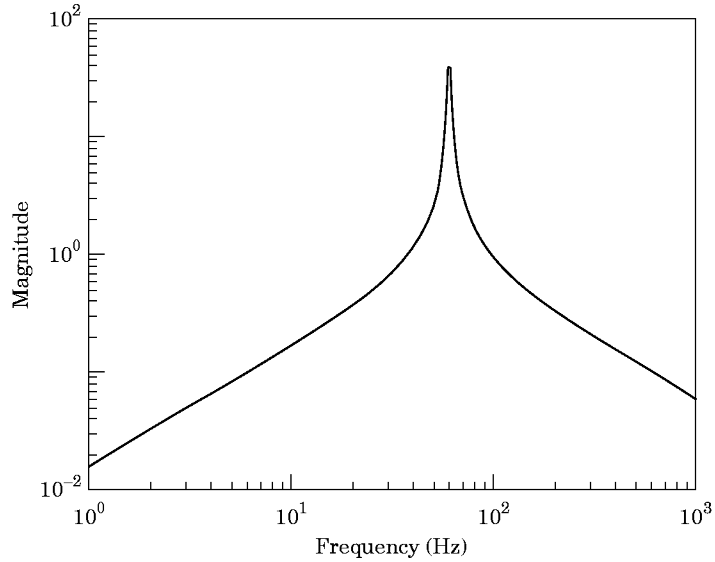

Figure 7. Frequency response of narrowband shaping filter.

The third case involves a broadband disturbance. The disturbance has a bandwidth of 100 Hz and a center frequency of 80 Hz. The frequency response of the shaping filter is shown in Figure 10. The open loop response of the five mode model of the system and the response with the fixed feedback controller (control penalty = 1e-3) are shown in Figure 11(a) and (b). The fixed feedback provides very little suppression in this case. The response of the adaptive feedback controller with a 12-term FIR filter is shown in Figure 12(a) and the optimal performance of a feedback controller is shown in Figure 12(b). Although the suppression provided by the adaptive controller is not dramatic in this

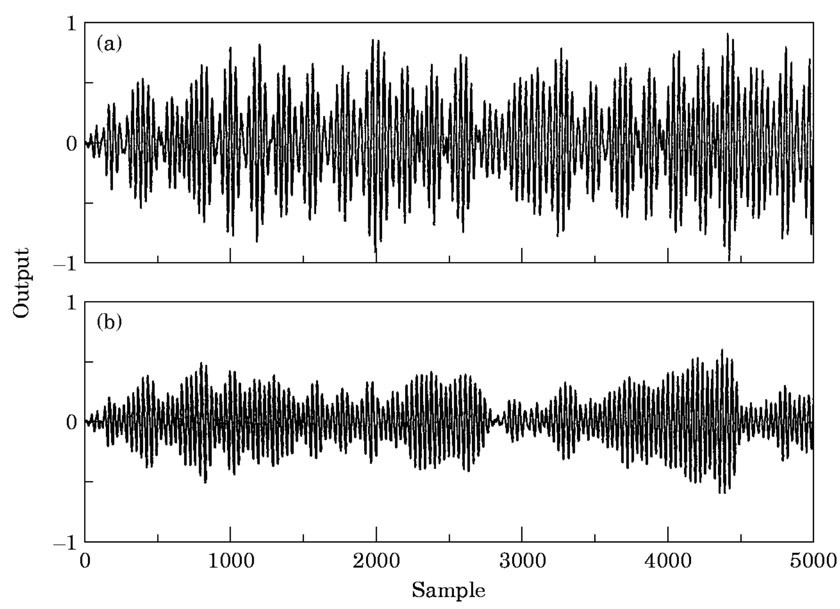

Figure 8. (a) Open loop response to narrowband disturbance and (b) closed loop response without adaptation to narrowband disturbance.

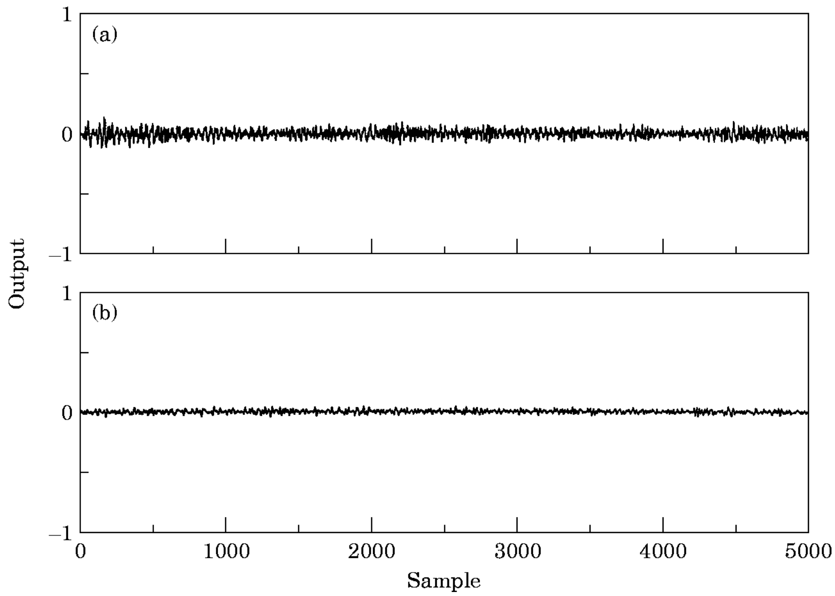

Figure 9. (a) Closed loop response with adaptation to narrowband disturbance and (b) optimal closed loop response to narrowband disturbance.

example, it is within a factor of 2 or so of the optimal suppression. The suppression could be improved by changing the control actuator location, or adding an additional actuator, but the point here is not to generate an example that shows dramatic rejection, but rather to show that the adaptive mechanism approaches relatively closely to the optimal. Thus, this control scheme appears to be capable of obtaining suppression near the optimal obtainable with feedback for a given system configuration.

One question that needs to be addressed is the ability of the controller to perform well when the system model used for control design is inaccurate. To get some idea of the

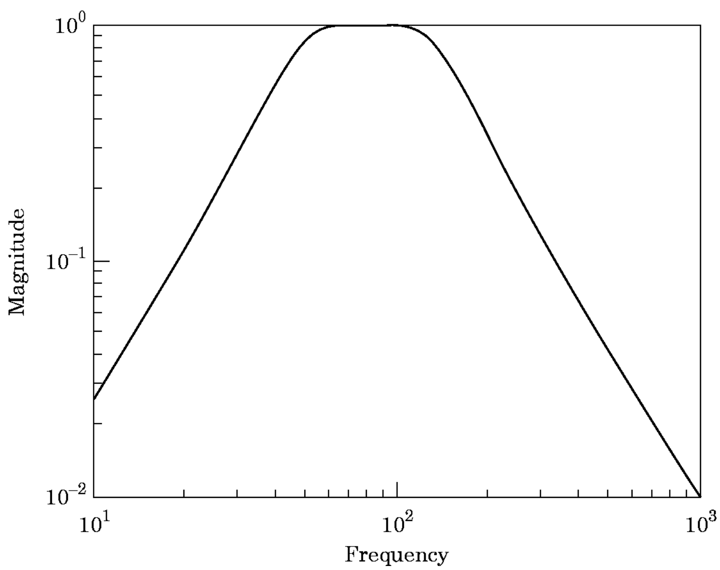

Figure 10. Frequency response of broadband shaping filter.

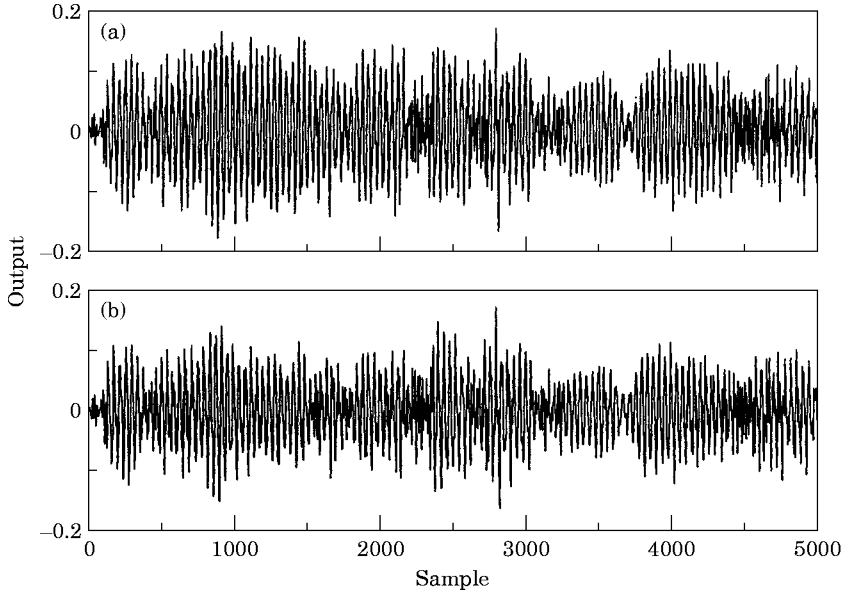

Figure 11. (a) Open loop response to broadband disturbance and (b) closed loop response without adaptation to broadband disturbance.

amount of uncertainty that could be tolerated, the parameters of the continuous time system (3) were randomly varied by up to $\pm 10\%$ to generate the control design model. The resulting adaptive controller drove the system unstable. When the frequencies were varied by up to $\pm 2\%$ and the other parameters, including damping, by up to $\pm 10\%$, the resulting controller was stable and achieved performance similar to that for the above cases. Thus, the controller can tolerate some uncertainty, but is most sensitive to frequency

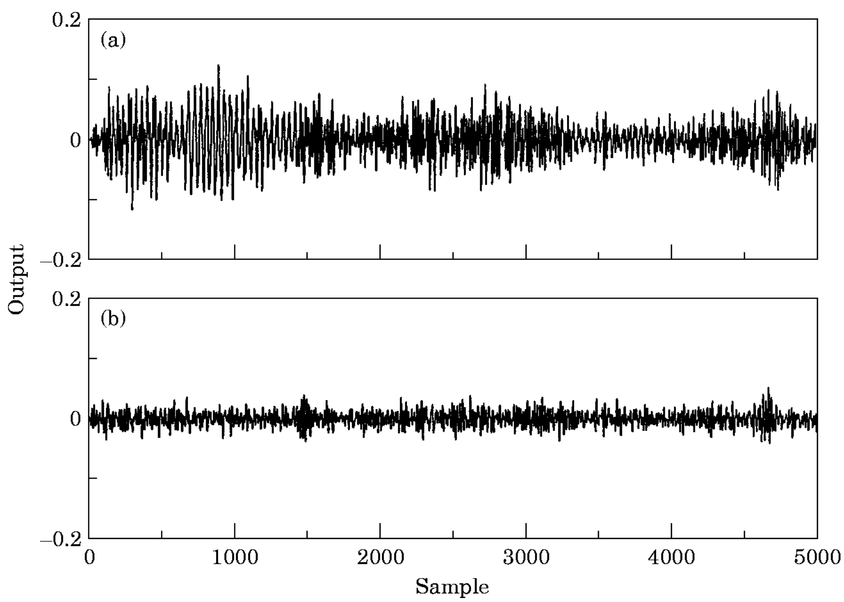

Figure 12. (a) Closed loop response with adaptation to broadband disturbance and (b) optimal closed loop response to broadband disturbance.

uncertainty. Presumably this sensitivity is due to the completely (180°) inaccurate gradient information provided to the adaptive controller in the neighborhood of the perturbed frequencies.

In practical situations it is relatively easy to measure the modal frequencies accurately. But if these frequencies can change significantly during the operation of the controller, the proposed control scheme is not adequate. In this case it is probably necessary to consider a fully adaptive controller that adapts to both the plant and the disturbance.

Another situation to consider is the presence of unmodelled dynamics, so that the control design model does not include some of the system dynamics. If the actual system one considers is modelled with nine modes, but the control design model only includes five modes, the resulting controller will be unstable in the broadband case discussed above. There are two ways in which the controller can go unstable. First, since the transfer function from s to r will no longer be even close to zero in the region of unmodelled dynamics, even a stable Q can create an unstable system. Second, if there is significant noise power in the unmodelled region to which the adaptation mechanism attempts to respond, the incorrect gradient information, due to the inaccurate filter T_{es} in equation (2), can cause the filter coefficients to diverge.

The first mechanism can be taken care of by filtering r as discussed in section 3. In the case being considered, a fourth order Butterworth filter with a breakpoint of 100 Hz may be used to reduce the effect of the unmodelled modes on the transfer function from s to r. The second mechanism is most easily taken care of by making sure that the model is accurate in the region where the noise power is significant. For simulation purposes, the disturbance will be changed to have a center frequency of 60 Hz and a bandwidth of 30 Hz. In practice, the system must be modelled well in the regions where there are disturbances if this controller is to reject them. An alternative may be to filter the error signal so that the adaptive mechanism does not respond to disturbances in poorly modelled regions.

Figures 13(a) and (b), and 14(a) and (b), show the result of a simulation in which the control model contains inaccurate information about the first five modes of the system,

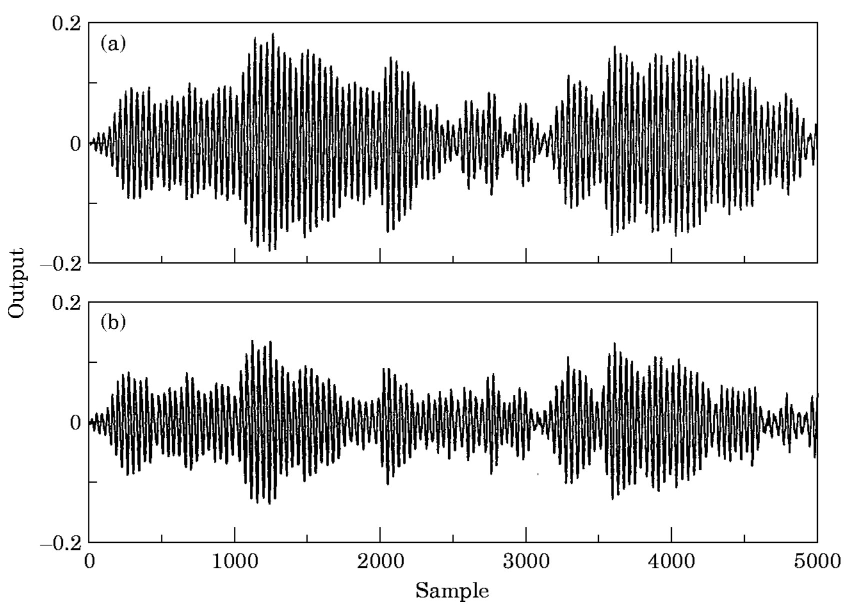

Figure 13. Results using an inaccurate control design model. (a) Open loop response to disturbance and (b) closed loop response without adaptation to disturbance.

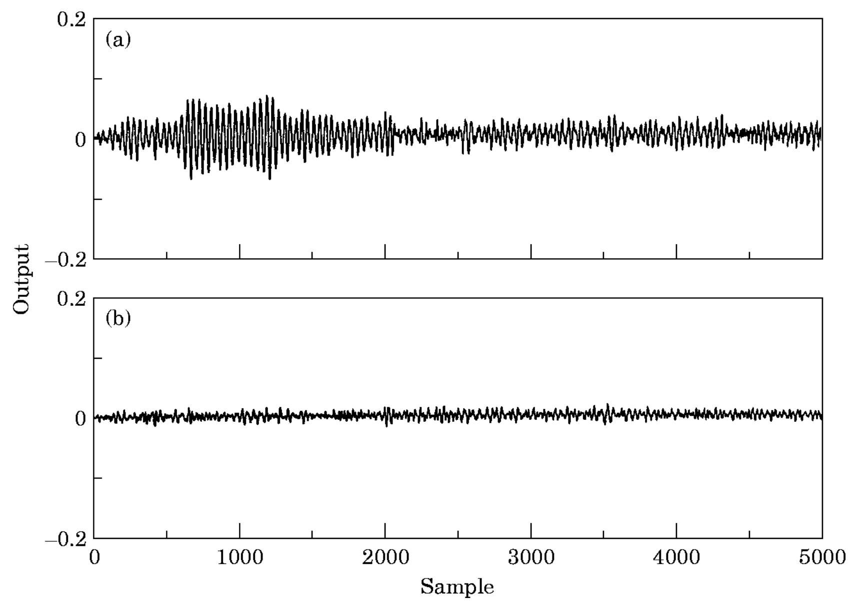

Figure 14. Results using an inaccurate control design model. (a) closed loop response with adaptation to disturbance and (b) optimal closed loop response to disturbance.

due to the random parameter perturbations discussed above, and no information about the remaining four modes of the system. The nominal controller is designed with a control penalty of 1e-3 and does not destabilize the actual plant. A 12 term MA filter is used for the adaptive feedback and the adaptations are initialized at $P = 10^9 * I$. Again, it can be seen that the controller provides significant suppression which is within a factor of 2 or so of the optimal.

5. CONCLUSION

This paper has demonstrated the potential of an adaptive feedback technique to suppress disturbances in structures. The proposed controller requires no more system modelling than standard filtered-x LMS approaches and does not require an independent measurement coherent with the disturbance source. In addition, the proposed controller provides rejection of transient disturbances and speeds up the convergence of the adaptive algorithm, features not available with straightforward neutralization loop techniques.

Finally, this controller should significantly outperform fixed feedback controllers in practice. It does not require specific knowledge of the disturbance spectrum or how the disturbance enters the plant. With adaptation of simple, low order filters, this technique appears capable of performance within a factor of 2 or so of the optimal performance achievable with feedback.

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