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CRITICAL AND COINCIDENCE FREQUENCIES OF FLAT PANELS

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Critical and coincidence frequencies of panels are important in studying their behaviour under acoustic excitation. Most spacecraft structural panels are of honeycomb sandwich construction and many of them have face sheets made of composite material. The critical and coincidence frequencies of such panels are discussed here. Expressions for critical and coincidence frequencies of thick isotropic and thin as well as thick symmetric composite panels are derived. It is shown that the orthotropic behaviour and transverse shear flexibility of the panels affects the critical and coincidence frequencies. The critical frequency of a typical composite honeycomb sandwich panel obtained using the above expressions match well with experimental results.

1. INTRODUCTION

When an infinite plate is excited, the frequency at which the speed of the free bending wave becomes equal to the speed of acoustic wave in air is called the critical frequency [1]. The critical frequency is particularly important when one deals with sound radiation from structures. The sound radiation characteristics are highly dependent on whether the excitation frequency is above or below the critical frequency. Also, the radiation efficiency of a structure is very high near the critical frequency. Finite panels also show similar behaviour.

When a structure is excited acoustically, the frequency at which the speed of the forced bending wave in the structure and the speed of the free bending wave are equal is called the coincidence frequency [1]. Sound transmission is highest near the coincidence frequency. The sound transmission characteristics depend on whether the excitation frequency is above or below the coincidence frequency.

The vibration response of a panel to a reverberant acoustic field is highest around the critical frequency. Therefore, to obtain the response of a structure to acoustic excitation it is necessary to know its critical frequency accurately. The information on the critical frequency of the structure can be used in design if the response due to acoustics has to be reduced. For example, the structure can be designed in such a way that its critical frequency is beyond the range of frequencies in which the acoustic excitation is larger.

Thus, knowledge of critical and coincidence frequencies of a structure is essential to study the structural–acoustic interaction. It may also be noted that these two parameters are interrelated.

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The critical and coincidence frequencies of thin isotropic panels are discussed in detail in the literature [1–3]. Many aerospace components are of honeycomb sandwich construction. In such applications, transverse shear flexibility is expected to play an important role in their behaviour at high frequencies, and a thick plate theory will be more suitable. However, the critical and coincidence frequencies of thick panels are discussed in the literature [4–6] only to a very limited extent. In reference [4], transverse shear flexibility effect is included in the transmission loss expression, but an expression is not suggested for the critical frequency with transverse shear flexibility. Narayanan and Shanbag [5] have discussed coincidence frequencies of panels with transverse shear flexibility. However, the expression for the coincidence frequency is not given in a convenient and usable form. Hence, in practical situations in which the response of honeycomb panels to acoustic excitation is to be obtained using Statistical Energy Analysis (SEA), the geometric mean of two critical frequencies, one based on the total pure bending of the panel and the other based on pure bending of the face sheet, is used [6]. In the above calculations, the core shear flexibility is not included.

Most of the honeycomb sandwich panels encountered in aerospace structures have face sheets made of fibrous composite material. Information on critical and coincidence frequencies of such panels is necessary to gain a better understanding of their behaviour under acoustic excitation. No results have been reported yet on critical and coincidence frequencies of composite panels. A conventional procedure in such situations is to define two critical frequencies[2, 4, 7] in two principal material directions, using the two orthotropic flexural rigidity values. The geometric mean of the two frequencies is used as the critical frequency [7]. Guyader and Lesueur [8] have discussed the transmission loss of orthotropic panels but have not derived any expression for the critical frequencies.

Hence, there is an urgent need to obtain an expression for critical frequencies of panels considering the orthotropic behaviour and transverse shear flexibility. In this paper, the critical and coincidence frequencies of isotropic thick panels are discussed first. The above parameters are then derived for thin as well as thick orthotropic panels. The influence of transverse shear flexibility and the orthotropic behaviour on the above parameters is discussed. The expressions are validated by experiments.

2. ISOTROPIC THIN PLATE

In this section we briefly present the expressions for the critical and coincidence frequencies of isotropic plates. A basic procedure of obtaining these parameters is also highlighted. Thus, this section provides a prelude to the subsequent sections in which these expressions are derived for isotropic thick plates and orthotropic plates including transverse shear effects.

The free vibration of a thin plate is governed by the equation

$$\nabla^4 w + (\rho/D)(\partial^2 w/\partial t^2) = 0. \tag{1}$$

The plate has a flexural rigidity of D and a mass per unit area of ρ. The co-ordinate axes are shown in Figure 1. The panel is assumed to be in the X–Y plane. A list of symbols used is given in Appendix A.

The solution for the infinite plate may be written as

$$w = \mathrm{A}e^{j(\omega t - k_x x - k_y y)}, \tag{2}$$

where k_x and k_y are the wavenumber components in the X and Y directions. They are related by the expression $k_x^2 + k_y^2 = k^2$, where k is the wavenumber. The wavenumber at a frequency ω is defined as $k = \omega/c$, where c is the speed of wave propagation. Hence, the wavenumber is a measure of the number of waves in unit length. For a thin plate, substituting equation (2) in equation (1), $k_x^4 + 2k_x^2k_y^2 + k_y^4 = \rho\omega^2/D$, and hence $k^4 = \rho\omega^2/D$. From the above equations, the speed of a free bending wave, c_b, for a thin plate can be obtained as

$$c_b^4 = \omega^2 D/\rho. \tag{3}$$

At the critical frequency, by definition, $c_b = c$. Hence, the critical frequency, ω_{cr} in rad/s for a thin plate, is given by

$$\omega_{cr}^2 = c^4\rho/D. \tag{4}$$

When a sound wave strikes the panel at an angle of incidence θ (Figure 1), it produces a trace wave in the plate, called a forced wave. The speed of this forced bending wave is $(c/\sin\theta)$ [1, 3]. Coincidence occurs when the speed of the forced bending wave matches the speed of the free bending wave. Hence the coincidence frequency ω_{co} can be shown to be

$$\omega_{co}^2 = (c^4\rho/D)\sin^4\theta. \tag{5}$$

Using equation (4), the expression for coincidence frequency can be simplified as

$$\omega_{co}^2 = \omega_{cr}^2/\sin^4\theta. \tag{6}$$

Equation (6) suggests that the coincidence frequency is directly related to the critical frequency.

3. ISOTROPIC THICK PLATE

The free vibration of a thick plate is governed by the equation [9, 10]

$$\nabla^4 w + (\rho/D)(\partial^2 w/\partial t^2) - (\rho/N)(\partial^2\{\nabla^2 w\}/\partial t^2) = 0, \tag{7}$$

where N is the shear rigidity of the plate. The above equation is based on Mindlin's theory, and is very accurate in representing the shear deformation in honeycomb sandwich panels. For a honeycomb sandwich panel, having t and h as the thickness of the face sheets and core, respectively, the shear rigidity is $Gh\{1 + (t/h)\}^2$ where G is the shear modulus of the core. The solution for the infinite plate is the same as equation (2). Substituting equation (2) in the differential equation (7) and using a similar procedure as in the case of a thin plate, the expression for the speed of free bending wave can be shown to be

$$c_b^2 = 2N\{\rho + (\rho^2 + \{4\rho N^2/[\omega^2 D]\})^{1/2}\}^{-1}. \tag{8}$$

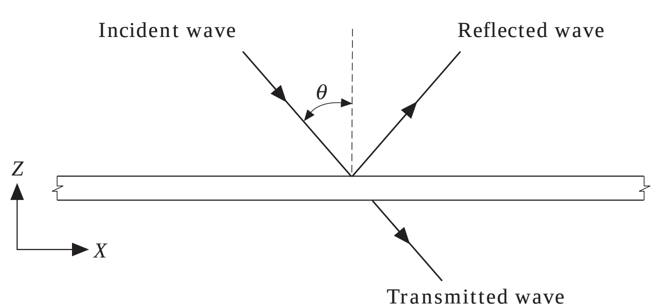

Figure 1. The definition of the co-ordinate axes.

The critical frequency can then be obtained as

$$\omega_{cr}^2 = (c^4\rho/D)/\{1 - (c^2\rho/N)\}. \tag{9}$$

The effect of transverse shear flexibility on the critical frequency can easily be understood from Figure 2. In the above figure, the critical frequency is normalized with respect to $c^4\rho/D$, the critical frequency of a thin plate having the same flexural rigidity. From the results, it is clear that the critical frequency increases with the transverse shear flexibility of the panel. For a thin plate, i.e., when N is very large, $\omega_{cr}^2 = c^4\rho/D$, which is the same as equation (4).

It should be noted that if $c^2\rho/N \geqslant 1$, the critical frequency does not exist; i.e., at no frequency is the speed of free bending wave speed equal to the speed of sound in air. In such cases, the speed of the free bending wave is less than the speed of sound in air.

In a similar way as was done for a thin plate, the coincidence frequency of the thick plate can be shown to be

$$\omega_{co}^2 = (c^4\rho/D)/(1 - \{c^2\rho/N\}/\sin^2\theta)/\sin^4\theta. \tag{10}$$

Coincidence occurs only if $c^2\rho/N < \sin^2\theta$. It can be seen that the coincidence frequency cannot be obtained as $\omega_{co}^2 = \omega_{cr}^2/\sin^4\theta$. For thin panels (large values of N), equation (10) converges to equation (5).

4. THIN COMPOSITE PANEL

The critical and coincidence frequencies of symmetric cross-ply laminates, neglecting transverse shear effects, are discussed here. The equation for free vibration (thin plate theory) [11] is

$$D_{11}(\partial^4 w/\partial x^4) + 2(D_{12} + 2D_{66})(\partial^4 w/\partial x^2\partial y^2) + D_{22}(\partial^4 w/\partial y^4) + \rho(\partial^2 w/\partial t^2) = 0. \tag{11}$$

The solution for an infinite plate is

$$w = A\,e^{j(\omega t - k_x x - k_y y)}. \tag{12}$$

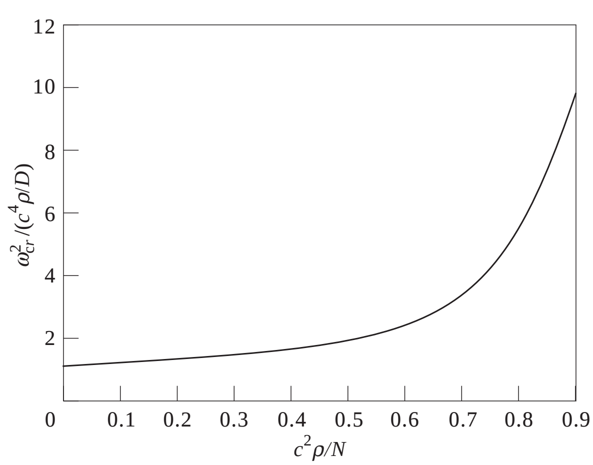

Figure 2. The effect of transverse shear flexibility on the critical frequency of an isotropic plate.

From equations (11) and (12),

$$D_{11}k_x^4 + 2(D_{12} + 2D_{66})k_x^2k_y^2 + D_{22}k_y^4 = \rho\omega^2. \tag{13}$$

By definition, $k_x^2 + k_y^2 = k^2$.

From equation (13), it can be shown that k is a function of ω and k_x or k_y. Hence, at a particular frequency one can have different values for wavenumbers and hence the wave speed, depending on the values of k_x or k_y. In contrast, an isotropic plate has a particular value of wavenumber as well as wave speed at a particular frequency.

In wavenumber plane (k_x along the X-axis and k_y along the Y-axis), the constant ω curve for an isotropic plate is an arc of a circle. The distance from the origin to a point on the curve is the wavenumber (Figure 3) and is a constant for an isotropic plate. For an orthotropic plate, the wavenumber varies with k_x or k_y.

Using a polar co-ordinate system (Figure 3), $k_x = r\cos\theta$, $k_y = r\sin\theta$ and $k^2 = r^2$. For an isotropic plate, r is a constant for a particular frequency, but for an orthotropic plate r is a function of θ (this θ is different from the angle of incidence).

The value of r at a particular frequency as a function of θ can be obtained from equation (13) as

$$r^4 = \rho\omega^2/(D_{11}\cos^4\theta + D_{22}\sin^4\theta + 2(D_{12} + 2D_{66})\sin^2\theta\cos^2\theta). \tag{14}$$

Equation (14) can be written as

$$r^4 = (\rho\omega^2/\sqrt{D_{11}D_{22}})/(\sqrt{D_{11}/D_{22}}\cos^4\theta + \sqrt{D_{22}/D_{11}}\sin^4\theta + 2\alpha\sin^2\theta\cos^2\theta), \tag{15}$$

where $\alpha = (D_{12} + 2D_{66})/\sqrt{D_{11}D_{22}}$. α is a parameter that represents the orthotropic properties of the plate. For an isotropic plate, $\alpha = 1$.

A simplification of equation (15) can be carried out when $D_{11} = D_{22} = D$. This happens in many practical situations. In such a case, the value of r is given by

$$r^4 = (\rho\omega^2/D)/(1 - [(1-\alpha)/2]\sin^2(2\theta)). \tag{16}$$

Here, $\alpha = (D_{12} + 2D_{66})/D$. For an isotropic plate, $r^4 = \rho\omega^2/D$.

From equation (16), it is clear that the wavenumber at a particular frequency depends on θ and the orthotropic parameter α. The above dependence is shown in Figure 4. Here the wavenumber is normalized with respect to the wavenumber for an isotropic plate

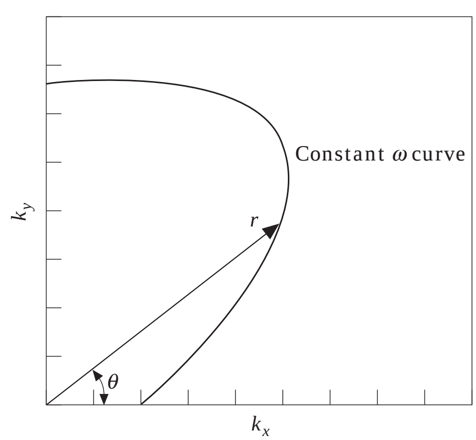

Figure 3. The wavenumber plane.

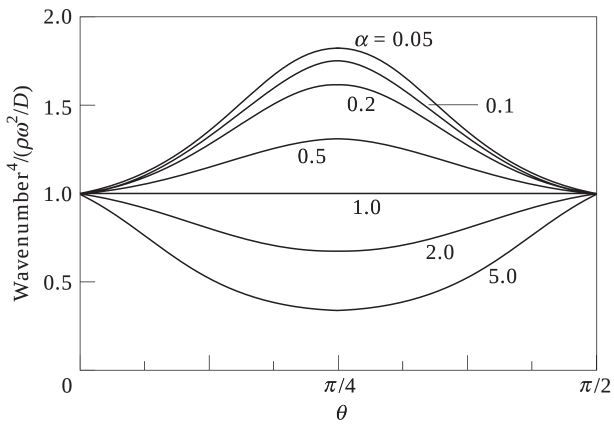

Figure 4. Variation of the wavenumber of a composite panel with the polar co-ordinate θ of the wavenumber plane.

having the same flexural rigidity. The above results can be classified into three cases depending on the value of α.

(a) $\alpha = 1$. In this case, the plate is isotropic and the wavenumber at a particular frequency is independent of θ.

(b) $\alpha < 1$. In this case, the wavenumber is always greater than the wavenumber of the equivalent isotropic plate, and is a maximum when $\theta = \pi/4$. The maximum value of r^4 is given by $r^4_{max} = (\rho\omega^2/D)/(\{1 + \alpha\}/2)$. If the value of α is low, r^4_{max} will be large. The maximum value of r^4_{max} is $2\rho\omega^2/D$ when $\alpha = 0$.

(c) $\alpha > 1$. In this case, the wavenumber is always less than the wavenumber of the equivalent isotropic plate and is a minimum when $\theta = \pi/4$. The minimum value of r^4 is given by $r^4_{min} = (\rho\omega^2/D)/(\{1 + \alpha\}/2)$.

Since for many composite panels (an example will be discussed later) the value of α is very low (as low as 0·08), a large variation of wavenumber with θ is expected and the wavenumber is always greater than the wavenumber of an equivalent isotropic plate. At a particular frequency, k^4 can vary from $\rho\omega^2/D$ to $2\rho\omega^2/D$ depending on the value of k_x or k_y.

Since the occurrence of k_x or k_y cannot be defined deterministically, for further discussions on wavenumber a probabilistic framework is used. Equation (16) can be written as

$$r^4\{1 - [(1 - \alpha)/2] \sin^2 (2\theta)\} - (\rho\omega^2/D) = 0. \tag{17}$$

Taking the expectation of equation (17),

$$r^4 \int_0^{\pi/2} \{1 - [(1 - \alpha)/2] \sin^2 (2\theta)\} p(\theta) \, \mathrm{d}\theta - (\rho\omega^2/D) = 0, \tag{18}$$

where $p(\theta)$ is the probability density function for the occurrence of θ (i.e., k_x or k_y). While carrying out the expectation operation, it should be considered that θ is the only independent variable and that r is a dependent variable.

An uniform probability density function can be assumed for the occurrence of θ and is an appropriate assumption for SEA applications. Hence,

$$p(\theta) = (2/\pi)\{U(\theta - 0) - U(\theta - [\pi/2]), \tag{19}$$

where U is the unit step function. Using equation (19) for the probability density function for the occurrence of θ and performing the integral given in equation (18), the expected value of r^4 can be derived as

$$r^4[(3+\alpha)/4] - (\rho\omega^2/D) = 0. \quad (20)$$

From equation (20), the expected wavenumber for a thin orthotropic plate at a particular frequency is

$$k^4 = (\rho\omega^2/D)/\{(3+\alpha)/4\}. \quad (21)$$

For an isotropic plate, the above equation reduces to $k^4 = \rho\omega^2/D$.

4.1. BENDING WAVE SPEED

Having obtained the expression for the wavenumber, the speed for the free bending wave in an orthotropic thin plate can be obtained as

$$c_b^4 = (\omega^2 D/\rho)\{(3+\alpha)/4\}. \quad (22)$$

For an isotropic plate, the above equation converges to $c_b^4 = \omega^2 D/\rho$, which is the same as equation (3).

4.2. CRITICAL FREQUENCY

Using a similar procedure as used for thin isotropic plate the critical frequency of a thin orthotropic plate is

$$\omega_{cr}^2 = (c^4\rho/D)/\{(3+\alpha)/4\}. \quad (23)$$

For an isotropic plate the above equation reduces to $\omega_{cr}^2 = c^4\rho/D$, which is the same as equation (4). The dependence of the critical frequency on the orthotropic characteristics of the panel are shown in Figure 5. The critical frequency is normalized with respect to the critical frequency of the equivalent isotropic plate (having the same D). It can be seen that orthotropic behaviour of the panel significantly affect the critical frequency. If $\alpha < 1$, which happens in most of the cases, the critical frequency is higher than that of the equivalent isotropic plate. The increase in critical frequency depends on the value of α; the lower the value of α the higher the increase. If $\alpha > 1$, the critical frequency is lower than that of the equivalent isotropic plate.

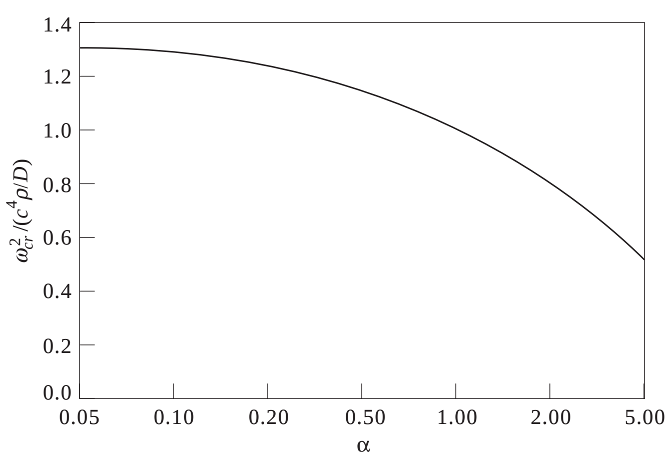

Figure 5. The effect of orthotropic behaviour on the critical frequency.

In the absence of equation (23), the practice is to define two critical frequencies, $\omega_{cr,x}^2 = c^4\rho/D_{11}$ and $\omega_{cr,y}^2 = c^4\rho/D_{22}$ for orthotropic plates. These values correspond to the bending wave speed when $\theta = 0$ and $\theta = \pi/2$. The geometric mean of these two critical frequencies is taken as the critical frequency of the plate. In such cases the effects of D_{12} and D_{66} are not considered.

In many practical cases, $D_{11} = D_{22} = D$, and in the conventional way $\omega_{cr}^2 = c^4\rho/D$. By comparing equation (23) it is clear that the actual critical frequency is influenced by the parameter α. Since α is usually very small for composite panels, because $D_{12} + 2D_{66}$ is very small compared to D, the conventional equations can under estimate the critical frequency by 1·155 (1·33 on ω_{cr}^2).

4.3. COINCIDENCE FREQUENCY

In a similar manner as was done for an isotropic plate, the coincidence frequency of a thin orthotropic plate can be derived as

$$\omega_{co}^2 = (c^4\rho/D)/\{(3+\alpha)/4\}/\sin^4\theta. \tag{24}$$

In this case, $\omega_{co}^2 = \omega_{cr}^2/\sin^4\theta$.

5. THICK COMPOSITE PANELS

The critical and coincidence frequencies of symmetric cross-ply laminates, considering transverse shear effects, are discussed here. Many spacecraft structural elements follow this construction. In this case the average shear angle is taken as the rotation of the transverse plane due to shear (Mindlin's theory). This formulation is very accurate in representing the transverse shear deformation in honeycomb sandwich panels.

The free vibration of such panels is governed by the equation [12]

$$D_{11}(\partial^4 w/\partial x^4) + 2(D_{12}+2D_{66})(\partial^4 w/\partial x^2\partial y^2) + D_{22}(\partial^4 w/\partial y^4) + \rho(\partial^2 w/\partial t^2)$$
$$-(\rho/N)[\partial^2\{\mathrm{D}_{11}(\partial^2 w/\partial x^2) + D_{22}(\partial^2 w/\partial y^2)\}/\partial t^2 = 0 \tag{25}$$

Using a procedure similar to that used for thin orthotropic plates,

$$D_{11}k_x^4 + 2(D_{12}+2D_{66})k_x^2k_y^2 + D_{22}k_y^4 - (\rho\omega^2/N)(D_{11}k_x^2 + D_{22}k_y^2) - \rho\omega^2 = 0. \tag{26}$$

In polar co-ordinates, equation (26) becomes

$$r^4\{D_{11}\cos^4\theta + 2(D_{12}+2D_{66})\sin^2\theta\cos^2\theta + D_{22}\sin^4\theta\}$$
$$-(\rho\omega^2/N\{D_{11}\cos^2\theta + D_{22}\sin^2\theta\}r^2 + \rho\omega^2 = 0. \tag{27}$$

If $D_{11} = D_{22} = D$, equation (27) can be simplified as

$$r^4\{1 - [(1-\alpha)/2]\sin^2(2\theta)\} - r^2(\rho\omega^2/N) - (\rho\omega^2/D) = 0. \tag{28}$$

As defined earlier, $\alpha = (D_{12} + 2D_{66})/D$.

On taking an expectation of equation (28),

$$r^4\,E[1 - \{(1-\alpha)/2\}\sin^2(2\theta)] - (\rho\omega^2/N)r^2 - (\rho\omega^2/D) = 0. \tag{29}$$

For an uniform probability density function for θ, equation (29) can be reduced to

$$\{(3+\alpha)/4\}r^4 - (\rho\omega^2/N)r^2 - (\rho\omega^2/D) = 0. \tag{30}$$

Hence, the expected wavenumber at a particular frequency is given by

$$r^2 = (1/2N)\{\rho\omega^2 + \omega(\rho^2\omega^2 + [4\rho N^2/D][(3+\alpha)/4])^{1/2}\}/\{(3+\alpha)/4\}. \tag{31}$$

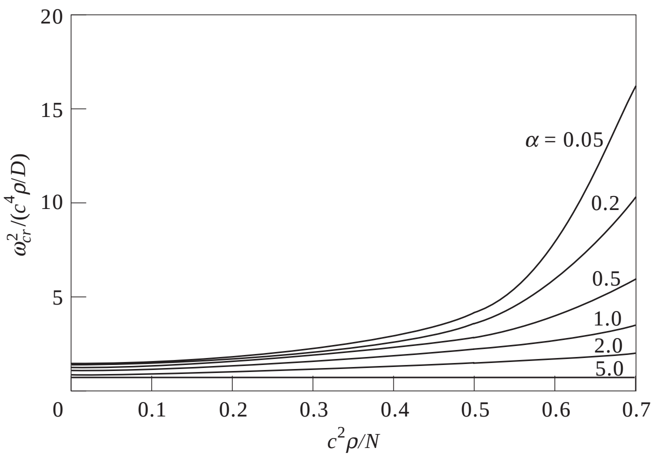

Figure 6. The effect of shear flexibility on the critical frequency of a composite panel.

For thin plates, N is very large and hence $r^4 = (\rho\omega^2/D)/[(3+\alpha)/4]$, which is the same as equation (21).

5.1. BENDING WAVE SPEED

Having obtained the expression for wavenumber, the speed of the bending wave can be derived as

$$c_b^2 = \{(3+\alpha)/4\}2N\{\rho + (\rho^2 + [4\rho N^2/(\omega^2 D)][(3+\alpha)/4])^{1/2}\}^{-1}. \tag{32}$$

The above equation can be shown to converge to the equation for isotropic panel for $\alpha = 1$ and to the equation for a thin orthotropic panel when $N \to \infty$.

5.2. CRITICAL FREQUENCY

In a manner similar to that done above, the critical frequency of a symmetric composite panel can be shown to be

$$\omega_{cr}^2 = (c^4\rho/D)/\{[(3+\alpha)/4] - [c^2\rho/N]\}. \tag{33}$$

The above equation may be compared with the critical frequency of a thin isotropic plate given by equation (4), that for a thick isotropic plate given by equation (9) and that for a thin orthotropic plate given by equation (23). With suitable mathematical operations, equation (33) can be shown to converge to the above equations.

The dependence of the critical frequencies on the transverse shear flexibility and orthotropic material properties is shown in Figure 6. It can be seen from the above results that the critical frequencies of isotropic as well as composite panels increase with transverse shear flexibility. Orthotropic behaviour increases the critical frequency further (if $\alpha < 1$).

The critical frequency does not exist if $(c^2\rho/N) \geqslant ([3+\alpha]/4)$. A similar situation also exists for isotropic thick plates. It may be noted that thin plates (isotropic or orthotropic) always have critical frequencies, but thick plates have critical frequencies under certain conditions only.

5.3. COINCIDENCE FREQUENCY

When the plate is subjected to an acoustic field at an angle of incidence θ, the coincidence frequency can be obtained from equation (33) by using an approach similar to that used earlier. The coincidence frequency can be shown to be

$$\omega_{co}^2 = (c^4\rho/D)/\{[(3+\alpha)/4] - [(c^2\rho/N)/\sin^2\theta]\}/\sin^4\theta. \tag{34}$$

In this case the coincidence frequency cannot be obtained as $\omega_{co}^2 = \omega_{cr}^2/\sin^4\theta$. Coincidence occurs only if $c^2\rho/N < [\{3 + \alpha\}/4]\sin^2\theta$.

It is interesting to compare equation (34) with the coincidence frequencies for thin isotropic plates, thick isotropic plates and thin orthotropic plates, which are given by equations (5), (10) and (24) respectively.

It can be seen that $\omega_{co}^2 = \omega_{cr}^2/\sin^4\theta$ is valid only for thin (isotropic or orthotropic) plates and is not valid for thick plates. Thin plates always have coincidence frequencies, but thick plates have coincidence frequencies only under certain conditions.

6. THE CRITICAL FREQUENCY OF A TYPICAL COMPOSITE PANEL

In the absence of the equations derived here for the critical frequencies of composite panels, one would be using the corresponding equations for isotropic plates. It is interesting to compare these values for a practical case. For this, a honeycomb sandwich panel, which is typical of a spacecraft panel, is taken as an example.

6.1. DETAILS OF THE PANEL

The properties of the panel are as follows: dimensions, $2{\cdot}15 \times 1{\cdot}80$ m; area, $3{\cdot}87$ m^2; mass, 10·918 kg; face sheet thickness, 0·2 mm; face sheet material, two layers of (0/90) CFRP; core material, aluminium honeycomb; core thickness, 18 mm; core density, 16 kg/m^3; core shear modulus, $8{\cdot}158 \times 10^7$ N/m^2.

Each CFRP layer has the following properties: Young's modulus along fibre direction, 30×10^{10} N/m^2; Young's modulus along transverse direction, $0{\cdot}607 \times 10^{10}$ N/m^2; major Poisson ratio, 0·346; shear modulus, $0{\cdot}5 \times 10^{10}$ N/m^2.

The following structural properties of the panel can be derived from the properties of the individual layer [11]. $D_{11} = 5135{\cdot}27$ N m, $D_{22} = 5028{\cdot}02$ N m, $D_{12} = 69{\cdot}74$ N m, $D_{66} = 165{\cdot}63$ N m, and $N = 15{\cdot}01 \times 10^5$ N/m.

6.2. CRITICAL FREQUENCIES

Critical frequencies are obtained for the four cases discussed above. The speed of sound in air is taken as 343·0 m/s.

The critical frequency of the panel is 595 Hz, taking into account the orthotropic behaviour of the face sheets and the core shear flexibility. Here, D_{11} is approximately equal to D_{22} and their geometric mean is used as D. If core shear flexibility is not considered, the estimated critical frequency is 503 Hz. The critical frequency obtained by assuming the face sheet to be isotropic but considering the core shear flexibility is 500 Hz. For this, the Young's modulus of the face sheet is assumed to be $13{\cdot}5 \times 10^{10}$ N/m^2, so that the flexural rigidity of the isotropic panel is the same as the geometric mean of D_{11} and D_{22} of the composite panel. The Poisson ratio is taken as 0·346, which is the major Poisson ratio of the CFRP face sheet. If the core shear flexibility is also neglected, the estimated frequency is 441 Hz.

From the above results, it is clear that both the orthotropic behaviour of the face sheet and the core shear flexibility should be considered to obtain the critical frequency. In the absence of the expression derived here, the critical frequency would be estimated as 441 Hz instead of 595 Hz.

6.3. EXPERIMENTAL RESULTS

To validate the expression derived, the critical frequency of the above panel is obtained experimentally. Here, the critical frequency is obtained in an indirect way. In this method the total loss factor, which is the sum of dissipation loss factor and radiation loss factor, of the panel is obtained. Generally, the dissipation loss factor decreases with frequency. On the other hand, the radiation loss factor is very low below the critical frequency of the panel and is very large near the critical frequency. Hence, the total loss factor suddenly increases to a large value near the critical frequency. This behaviour is used here to obtain the critical frequency of the panel.

To obtain the total loss factor, the panel is mechanically excited using shaker systems. As per the energy balance, the average power input to the panel is equated to the power dissipated. The power dissipated, π_{diss}, at a frequency ω, is given by

$$\pi_{diss} = \eta \omega M \langle v^2 \rangle_x, \tag{35}$$

where η is the loss factor and M is the total mass of the panel. The panel is assumed to be of uniform mass distribution. Any local mass concentration should be neglected. In equation (35), $\langle v^2 \rangle_x$ is the spatial average value of the mean square value of the velocity of the panel. From the above energy balance, Clarkson [13] has shown that the loss factor of the panel having the modal density $n(f)$ at a frequency f is

$$\eta = F^2(t) n(f) / \{8\pi f M^2 \langle v^2 \rangle_x\}, \tag{36}$$

where $F^2(t)$ is the mean square value of the exciting force.

The panel used for the experiment is the same as the one described earlier. The panel was mounted to a fixture at six locations, called hold down points. The fixture in turn was mounted on a seismic mass. The test set-up is shown in Figure 7.

In the present experiment, the panel was excited at one point. To obtain the dissipated power, the acceleration responses of the panel were measured at three locations. The above locations and the driving point are shown in Figure 8. Responses are measured using very light accelerometers, having a mass less than 3 g, to avoid the error due to accelerometer mass. To obtain the power dissipated, the response at the driving point is not taken into

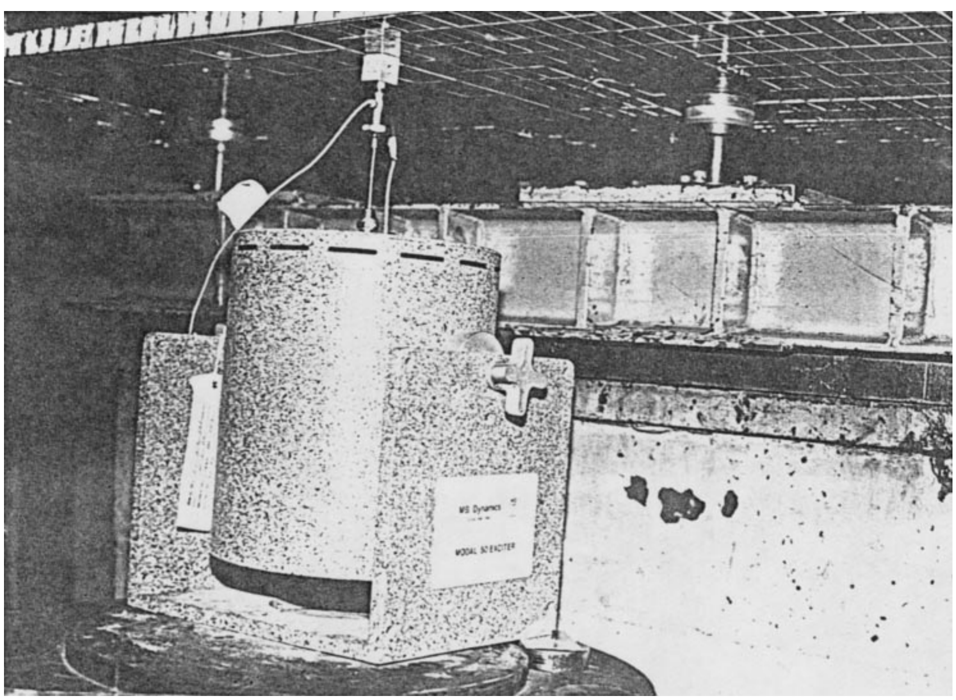

Figure 7. A view of the test set-up.

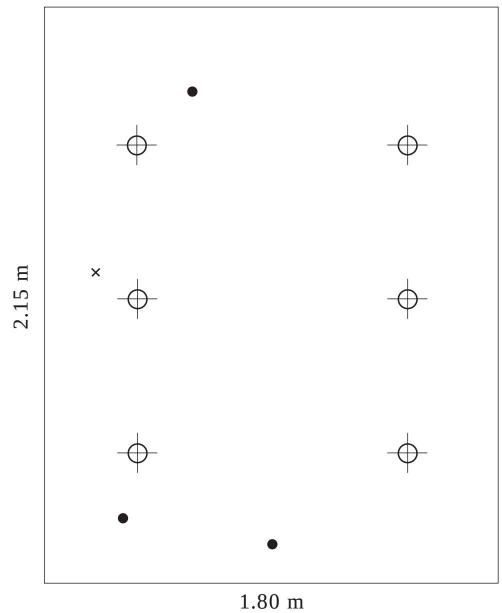

Figure 8. The accelerometer (●) and driving point (×) locations.

account [14]. The panel is excited using broadband random excitation with 162·76 Hz resolution. This very low value of resolution was selected to avoid frequency averaging [15].

From measured values of the driving force and response accelerations, the loss factor is calculated using equation (36). For the above calculation, modal density is obtained using the expression derived for composite honeycomb sandwich panels [12]. It is also to be noted that even in the lowest frequency band there are nine modes, which are sufficient for statistical accuracy. The total loss factor thus obtained is shown in Figure 9.

The results clearly show that the critical frequency of the panel lies in the 651·04 Hz band. The estimated critical frequency of the panel is 595 Hz. The experimental results match very well with the estimated critical frequency. The reliability of the results could be improved by using many driving point positions. The present results with one driving

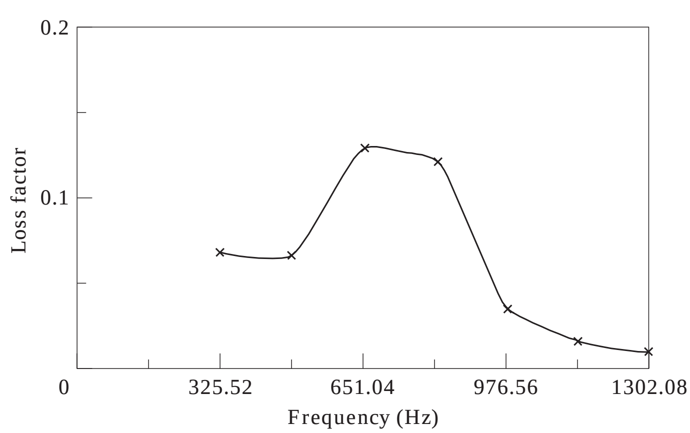

Figure 9. The measured total loss factor of the panel.

point are quite adequate since the driving point is located at random, not at any symmetric point, and there are more than nine modes in all the frequency bands [16]. In the absence of the expressions derived here, i.e., neglecting core shear flexibility and orthotropic behaviour, the critical frequency would have been estimated to be at 441 Hz, which can cause significant error in the estimated responses of these panels to acoustic excitation.

7. CONCLUSIONS

Expressions for critical and coincidence frequencies for thick isotropic panels and thin as well as thick composite panels are derived. For composite panels these frequencies are to be obtained in a statistical framework. It is seen that all thin panels have critical and coincidence frequencies, but thick panels have critical and coincidence frequencies under certain conditions only. For thin panels, coincidence frequencies are proportional to critical frequencies, but for thick panels they are not related by a simple function. The critical frequency increases with the transverse shear flexibility of the panel. The orthotropic nature of the panel has a significant influence on the critical frequency. The critical frequency of a typical composite honeycomb sandwich panel is obtained experimentally. The experimental results match very well with the critical frequency predicted using the present expression. If the transverse shear flexibility and the orthotropic nature of the panel are not considered there can be considerable error in the estimated critical frequencies.

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APPENDIX A: LIST OF SYMBOLS

c	speed of sound in air	ω_{co}	coincidence frequency, in rad/s
c_b	speed of the bending wave		
D_{11}, D_{22}, D_{12}, D_{66}	flexural rigidity values of the laminate	ω_{cr}	critical frequency, in rad/s
k_x, k_y, k	wavenumbers	ρ	mass per unit area of the panel
$p(\theta)$	probability of occurrence of θ	θ	angle of incidence
		t	thickness of face sheet
N	shear rigidity of the panel	h	thickness of the core
r, θ	polar co-ordinates	G	shear modulus of core
r_{max}	maximum value of r	$F^2(t)$	mean square value of force
r_{min}	minimum value of r	$n(f)$	modal density at frequency f
U	unit step function		
w	displacement normal to the panel	M	mass of the panel
		η	loss factor
α	parameter representing orthotropic properties of the panel	π_{diss}	mean power dissipated
		ϑ^2	mean square value of velocity
ω	frequency, in rad/s	$\langle\ \rangle$	ensemble average