Journal of Sound and Vibration (1997) **205**(2), 235–242

LETTERS TO THE EDITOR

AN EXTENDED PROBABILITY EVALUATION METHOD FOR AN ACOUSTIC ENVIRONMENT BASED ON THE INTRODUCTION OF THREE FUNCTIONAL SYSTEM MODELS OF REGRESSION TYPE: THEORY AND APPLICATION TO A SOUND INSULATION SYSTEM

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(*Received 12 March 1996, and in final form 10 February 1997*)

1. INTRODUCTION

For evaluating minutely the probabilistic response of the actual environmental acoustic system excited by an arbitrary random input, it is important to predict a whole probability distribution form closely connected with the evaluation indexes L_x, L_{eq} and so on [1, 2]. However, the actual environmental acoustic system is too complicated to find the structural model of the dynamic type based on physical laws from a bottom-up viewpoint. In this note, a new type of evaluation method is proposed by introducing three functional models matched to a more precise prediction of the response output probability distribution from an object-oriented viewpoint. Originally, because of the positive variable of the sound intensity, the response output probability density function can be reasonably expressed theoretically in the general form of a statistical type of Laguerre expansion series [3]. Thus, the relationship between input and output is described by finding the regression relationship between the distribution parameters (including the expansion coefficients of this expression) and the stochastic input. These regression functions can be derived in terms of the orthogonal series expansion of the joint probability density function on input and output [5, 6]. Additionally, by considering the contamination of background noise in the observation of the output, three functional models are derived. By employing the statistical moments of the background noise known in advance under the assumption of stationarity of the background noise, their parameters are determined based on the well-known least squares error criteria [9].

2. THEORETICAL CONSIDERATION

2.1. *Three functional models of acoustic system contaminated by a background noise*

Let us consider the acoustic system contaminated by the background noise by letting x, y, v and z be the sound intensities of the input, the output, the background noise and the observation contaminated by v, respectively. Here, it is noted that the statistical moments of the stationary background noise v can be evaluated (in advance, in a special case in which the input is not added) under the assumption of stationarity of the background noise. Then, the probability density function (abbreviated to "p.d.f.") $p(y)$

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of output y, fluctuating only in the positive value region, can be reasonably expressed in the following general form of the statistical type of Laguerre expansion series [3]:

$$P(y) = P_\Gamma(y; m_y, s_y) \sum_{n=0}^{\infty} A_n L_n^{(m_y - 1)}(y/s_y), \tag{1}$$

with

$$m_y = \frac{\langle y \rangle^2}{\langle (y - \langle y \rangle)^2 \rangle}, \qquad s_y = \frac{\langle (y - \langle y \rangle)^2 \rangle}{\langle y \rangle} \qquad \text{and} \qquad A_n = \frac{\Gamma(m_y) n!}{\Gamma(m_y + n)} \left\langle L_n^{(m_y - 1)}\left(\frac{y}{s_y}\right)\right\rangle, \tag{2}$$

where x and v are originally independent of each other. Here, $\Gamma(m)$ denotes a gamma function and $\langle\ \rangle$ denotes the expectation of y. Furthermore, $p_\Gamma(\zeta; m, s)$ and $L_n^{(\alpha)}(x)$ denote, respectively, a gamma p.d.f. [4] and an associated Laguerre polynomial [5], defined as follows:

$$P_\Gamma(\zeta; m, s) = \frac{1}{\Gamma(m) s}\left(\frac{\zeta}{s}\right)^{m-1} \mathrm{e}^{-\zeta/s} \qquad \text{and} \qquad L_n^{(\alpha)}(x) = \sum_{j=0}^{n} \binom{n+\alpha}{n-j} \frac{(-x)^j}{j!}. \tag{3}$$

Two parameters, m_y and s_y, and every expansion coefficient A_n depend on the fluctuation of input x and the stochastic relationship between input x and output y. If our concern is concentrated upon predicting a p.d.f. of the output y, especially based on the stochastic information of the input x, from an object-oriented viewpoint, it is desirable that the functional relationship between x and y should be described simultaneously in terms of the regression's styles: $\langle y|x\rangle$, $\langle (y - \langle y \rangle)^2 | x\rangle$ and $\langle L_k^{(m_y - 1)}(y/s_y)|x\rangle$ rather than based on only the physical rule of correspondence between x and y. Here, $\langle \cdot | x\rangle$ denotes the conditional expectation conditioned by x. These regression functions are defined in advance as follows:

$$\langle y|x\rangle = \int_0^\infty y p(y|x)\,\mathrm{d}y, \qquad \langle (y - \langle y \rangle)^2 | x\rangle = \int_0^\infty (y - \langle y \rangle)^2 p(y|x)\,\mathrm{d}y$$

and

$$\left\langle L_k^{(m_y - 1)}\left(\frac{y}{s_y}\right)\middle| x\right\rangle = \int_0^\infty L_k^{(m_y - 1)}\left(\frac{y}{s_y}\right) p(y|x)\,\mathrm{d}y, \tag{4}$$

where $p(y|x)$ is the conditional p.d.f. of y conditioned by x, and these regression functions are obviously of a non-linear type. For the purpose of finding general representations of these regression functions, let us use the method of orthogonal series expansion [5, 6] in close connection with the p.d.f. of y in equation (1), by letting $p(x)$ and $p(x, y)$ denote the p.d.f. of x and the joint p.d.f. of x and y. After expanding $p(x)$ and $p(x, y)$ into the orthonormal series expansion forms with weighting functions $p_\Gamma(x; m_x, s_x)$ and $p_\Gamma(x; m_x, s_x) p_\Gamma(y; m_y, s_y)$ as basic p.d.f.'s, respectively, and using the definition of the conditional probability $p(y|x) = p(x, y)/p(x)$ [7, 8], $p(y|x)$ can be expressed as follows [9]:

$$p(y|x) = \frac{p_\Gamma(y; m_y, s_y) \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} A_{mn} \varphi_m^{(1)}(x) \varphi_n^{(2)}(y)}{\sum_{m=0}^{\infty} A_{m0} \varphi_m^{(1)}(x)}, \tag{5}$$

where

$$\varphi_m^{(1)}(x) = \sqrt{\frac{\Gamma(m_x)m!}{\Gamma(m_x+m)}}\, L_m^{(m_x-1)}\left(\frac{x}{s_x}\right), \qquad \varphi_n^{(2)}(y) = \sqrt{\frac{\Gamma(m_y)n!}{\Gamma(m_y+n)}}\, L_n^{(m_y-1)}\left(\frac{y}{s_y}\right), \tag{6}$$

$$A_{m0} = \langle \varphi_m^{(1)}(x) \rangle \qquad \text{and} \qquad A_{mn} = \langle \varphi_m^{(1)}(x)\varphi_n^{(2)}(y) \rangle. \tag{7}$$

Here, $\varphi_m^{(1)}(x)$ and $\varphi_n^{(2)}(y)$ are the orthonormal polynomials satisfying the following relationships:

$$\int_0^\infty p_\Gamma(x; m_x, s_x)\varphi_m^{(1)}(x)\varphi_n^{(1)}(x)\,\mathrm{d}x = \delta_{mn}, \qquad \int_0^\infty p_\Gamma(y; m_y, s_y)\varphi_m^{(2)}(y)\varphi_n^{(2)}(y)\,\mathrm{d}y = \delta_{mn}, \tag{8}$$

where δ_{mn} denotes a Kronecker delta. Upon expanding y and $(y-\langle y \rangle)^2$ into the orthogonal series expansion forms in advance as follows:

$$y = \sum_{k=0}^{1} C_{1k}\varphi_k^{(2)}(y) \qquad \text{and} \qquad (y-\langle y \rangle)^2 = \sum_{k=0}^{2} C_{2k}\varphi_k^{(2)}(y), \tag{9}$$

the above regression functions can be expressed, respectively, as follows:

$$\langle y|x \rangle = \frac{\sum_{m=0}^{\infty} \alpha_m \varphi_m^{(1)}(x)}{\sum_{m=0}^{\infty} A_{m0}\varphi_m^{(1)}(x)}, \qquad \langle (y-\langle y \rangle)^2|x \rangle = \frac{\sum_{m=0}^{\infty} \beta_m \varphi_m^{(1)}(x)}{\sum_{m=0}^{\infty} A_{m0}\varphi_m^{(1)}(x)}$$

and

$$\left\langle L_k^{(m_y-1)}\left(\frac{y}{s_y}\right)\middle|\, x \right\rangle = \frac{\sum_{m=0}^{\infty} \gamma_{mk}\varphi_m^{(1)}(x)}{\sum_{m=0}^{\infty} A_{m0}\varphi_m^{(1)}(x)}, \tag{10}$$

where

$$\alpha_m = \sum_{n=0}^{1} C_{1n}A_{mn}, \qquad \beta_m = \sum_{n=0}^{2} C_{2n}A_{mn} \qquad \text{and} \qquad \gamma_{mk} = \sqrt{\Gamma(m_y+k)/\Gamma(m_y)k!}\,A_{mk}, \tag{11}$$

after substituting equations (5) and (9) into equation (4) and using equation (8). Since the observed quantity is not y but z contaminated by the background noise v, it is necessary to find a way to identify the above regression parameters in equation (10) only by using observed z values and the previously known statistical moments of v. On the basis of the additivity of the sound intensity quantity and the addition theorem for the associated Laguerre polynomial, the following relationships can be found:

$$z = y + v, \tag{12}$$

$$(z-\langle z \rangle)^2 = (y-\langle y \rangle)^2 + 2(y-\langle y \rangle)(v-\langle v \rangle) + (v-\langle v \rangle)^2, \tag{13}$$

and

$$L_n^{(m_y)}\left(\frac{z}{s_y}\right) = L_n^{(m_y)}\left(\frac{y+v}{s_y}\right) = \sum_{k=0}^{n} L_k^{(m_y-1)}\left(\frac{y}{s_y}\right) L_{n-k}^{(0)}\left(\frac{v}{s_y}\right). \tag{14}$$

After replacing y, $(y - \langle y \rangle)$, $(y - \langle y \rangle)^2$ and $L_k^{(m_y-1)}(y/s_y)$ by $\langle y|x \rangle$, $\langle y - \langle y \rangle | x \rangle$, $\langle (y - \langle y \rangle)^2 | x \rangle$ and $\langle L_k^{(m_y-1)}(y/s_y)|x \rangle$, the three functional system models for the resultant observation z can be introduced, corresponding to equations (12), (13) and (14) respectively, from the top-down viewpoint of object-oriented type as follows:

$$z = \frac{\sum_{m=0}^{M} \alpha_m \varphi_m^{(1)}(x)}{\sum_{m=0}^{M} A_{m0} \varphi_m^{(1)}(x)} + v + \varepsilon_1, \tag{15}$$

$$(z - \langle z \rangle)^2 = \frac{\sum_{m=0}^{M} \beta_m \varphi_m^{(1)}(x)}{\sum_{m=0}^{M} A_{m0} \varphi_m^{(1)}(x)} + (v - \langle v \rangle)^2 + 2\langle y - \langle y \rangle | x \rangle (v - \langle v \rangle) + \varepsilon_2 \tag{16}$$

and

$$L_n^{(m_y)}\left(\frac{z}{s_y}\right) = \sum_{k=0}^{n} \frac{\sum_{m=0}^{M} \gamma_{mk} \varphi_m^{(1)}(x)}{\sum_{m=0}^{\infty} A_{m0} \varphi_m^{(1)}(x)} L_{n-k}^{(0)}\left(\frac{v}{s_y}\right) + \varepsilon_3, \tag{17}$$

especially after taking only the first finite number M in the original infinite number of expansion expression (10). Here, ε_1, ε_2 and ε_3 are accidental errors due to truncating infinite terms of theoretical regression functions. In the standard method, it is usual that only equation (12) is employed and then y is expressed by a linear regression model or a convolution model based on some physical laws only from a bottom-up viewpoint. In the practical case when predicting a response probability distribution by using only this artificially simplified physical model, some filtering effect inevitably appears, especially at both ends of the fluctuating amplitude. On the other hand, if the above proposed three functional models supported by the mean, variance and higher order moments of the specific Laguerre polynomial type are employed simultaneously, one can predict in minute detail the whole probability distribution form of the output fluctuation.

2.2. *Identification of functional models and prediction of the response probability distribution*

The unknown parameters α_k $(k = 0, 1, \ldots, M)$, β_k $(k = 0, 1, \ldots, M)$ and γ_{mn} $(m = 0, 1, \ldots, M$ and $n = 1, 2, \ldots, M)$ in equation (10) can be determined by the well-known least squares error critieria [10] (i.e., $\langle \varepsilon_i^2 \rangle \rightarrow \min$ $(i = 1, 2, 3)$). Since $\langle y \rangle = \langle \langle y|x \rangle \rangle_x$ and $\langle (y - \langle y \rangle)^2 \rangle = \langle \langle (y - \langle y \rangle)^2 | x \rangle \rangle_x$ can be evaluated once after $\langle y|x \rangle$ and $\langle (y - \langle y \rangle)^2 | x \rangle$ are identified, two distribution parameters, m_y and s_y, can easily be estimated from equation (2). Here, $\langle\ \rangle_x$ denotes an expectation operation about x. For instance, the parameters α_m in equation (15) can be estimated based on minimizing the expectation of ε_1^2 as follows:

$$\langle \varepsilon_1^2 \rangle = \left\langle \left(z - \frac{\sum_{m=0}^{M} \alpha_m \varphi_m^{(1)}(x)}{\sum_{m=0}^{M} A_{m0} \varphi_m^{(1)}(x)} - v \right)^2 \right\rangle_{x,z,v}, \tag{18}$$

where $\langle\ \rangle_{x,z,v}$ denotes the expectation about x, z and v. More concretely, through the minimization operation: $\partial\langle \varepsilon_1^2\rangle/\partial\alpha_k = 0$ $(k = 0, 1, \dots, M)$, the following simultaneous equations are obtained:

$$\sum_{m=0}^{M} \left\langle \frac{\varphi_k^{(1)}(x)\varphi_m^{(1)}(x)}{\left(\sum_{m=0}^{M} A_{m0}\varphi_m^{(1)}(x)\right)^2} \right\rangle \alpha_m = \left\langle \frac{\varphi_k^{(1)}(x)}{\sum_{m=0}^{M} A_{m0}\varphi_m^{(1)}(x)} (z - \langle v \rangle) \right\rangle_{x,z} \quad (k = 0, 1, \dots, M). \tag{19}$$

In the derivation of equation (19), the natural condition of the statistical independency between x and v is employed. Similarly, after minimizing the expectation of the squared error in equation (16) (i.e., $\langle \varepsilon_2^2\rangle \to \min$), the unknown parameters β_k in equation (16) can be estimated by solving the following equation:

$$\sum_{m=0}^{M} \left\langle \frac{\varphi_k^{(1)}(x)\varphi_m^{(1)}(x)}{\left(\sum_{m=0}^{M} A_{m0}\varphi_m^{(1)}(x)\right)^2} \right\rangle \beta_m = \left\langle \frac{\varphi_k^{(1)}(x)}{\sum_{m=0}^{M} A_{m0}\varphi_m^{(1)}(x)} \{(z - \langle z \rangle)^2 - \langle (v - \langle v \rangle)^2 \rangle\} \right\rangle_{x,z}$$

$$(k = 0, 1, 2, \dots, M). \tag{20}$$

Here, in the derivation of equation (20), the statistical independency property between the input and the background noise has been employed. To estimate the γ_{mk}, it is necessary to estimate m_y and s_y in advance, responding to the identification input. That is, by solving equations (19) and (20), α_m and β_m must be estimated. Then, by executing the simple expectations $\langle\langle y|x\rangle\rangle_x$ and $\langle\langle (y - \langle y \rangle)^2|x\rangle\rangle_x$ according to the input data used in the above estimation procedure of α_m and β_m, by use of equation (10) and employing equation (2), m_y and s_y can be estimated. After the above preparation, let us estimate the parameters γ_{mk} in equation (17) by using the similar least squares error criterion (i.e., $\langle \varepsilon_3^2\rangle \to \min$). That is, we finally have the following equations:

$$\sum_{m=0}^{M} \left\langle \frac{\varphi_i^{(1)}(x)\varphi_m^{(1)}(x)}{\left(\sum_{m=0}^{M} A_{m0}\varphi_m^{(1)}(x)\right)^2} \right\rangle \gamma_{mn}$$

$$= \left\langle \frac{\varphi_i^{(1)}(x)}{\sum_{m=0}^{M} A_{m0}\varphi_m^{(1)}(x)} \left\{ L_n^{(m_y)}\left(\frac{z}{s_y}\right) - \sum_{k=0}^{n-1} \frac{\sum_{m=0}^{M} \gamma_{mk}\varphi_m^{(1)}(x)}{\sum_{m=0}^{M} A_{m0}\varphi_m^{(1)}(x)} \left\langle L_{n-k}^{(0)}\left(\frac{v}{s_y}\right)\right\rangle \right\} \right\rangle_{x,z}$$

$$(i = 0, 1, 2, \dots, M \text{ and } n = 1, \dots, M). \tag{21}$$

Here, in the derivation of equation (21), the statistical independency property between the input and the background noise has also been employed. By solving equation (21), γ_{mk} can be estimated.

Thus, by executing the expectations $\langle\langle y|x\rangle\rangle_x$, $\langle\langle (y-\langle y\rangle)^2|x\rangle\rangle_x$ and $\langle\langle L_k^{(m_y-1)}(y/s_y)|x\rangle\rangle_x$ according to an arbitrary random input x by employing equation (10) with the above estimated α_m, β_m and γ_{mk}, m_y, s_y and A_n in equation (2) can be predicted. Accordingly, from equation (1), the objective output response p.d.f. $p(y)$ of y to an arbitrary random input can be explicitly predicted.

3. EXPERIMENTAL CONSIDERATIONS

To confirm the effectiveness of the proposed method, the present experiments have been carried out for the sound insulation system [11, 12] shown in Figure 1. A sound insulation wall was set up between the transmission room and the reception room. Specifically, a sound-bridge type double wall with two 1.2 mm thick aluminium plates was employed. Owing to the complicated mechanism of this sound insulation system, the structural model based on physical laws cannot be found theoretically. Road traffic noise was radiated in the transmission room from a loudspeaker, and the response level fluctuation was recorded synchronously by sound level meters (with a fast mode and an A weighting network) in each room. This road traffic noise fluctuating non-stationarily was recorded in a city in advance. Each sound level was sampled every one second and data of size 1000 was obtained. Then, the observation was artificially composed of the above output data and the white noise generated by a noise generator in an anechoic room (positively adjusted 3 dB lower in the mean than the above output sound level in advance) for the purpose of comparing the proposed theoretical curve with experimental values. After transforming the sound level data to the sound intensity data, the proposed theoretical method was

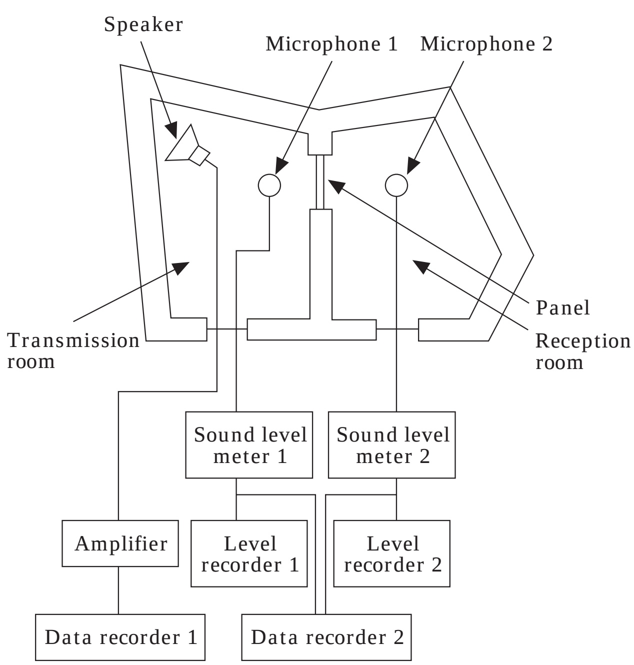

Figure 1. A block diagram for measuring the stochastic transmission characteristics of the sound insulation system when excited by an actual random noise.

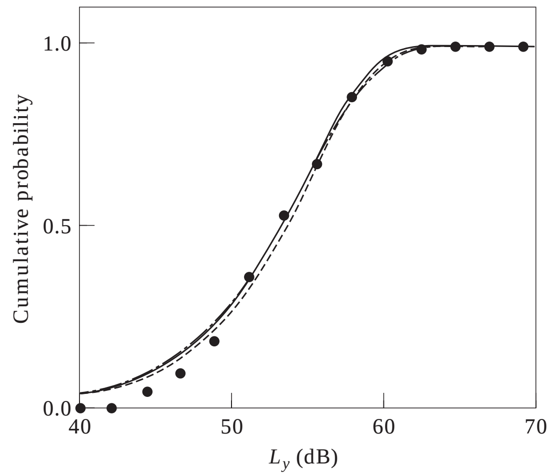

Figure 2. A comparison between the theoretically predicted cumulative probability distributions for the case $M = 2$ in equations (19), (20) and (21) and the experimentally sampled values of the sound level without the background noise in a transmission room excited by road traffic noise. Experimentally sampled values are marked by dots (●) and theoretical curves are shown with the degree of approximation (——, first approximation; – – –, second approximation; — - —, third approximation).

applied. Then, the parameters of the functional models were determined by using the first 800 pairs of input and observation values, and the p.d.f. of the response sound fluctuation without a background noise was predicted corresponding to the other 200 input level data. Again, these response sound intensity data were transformed to the sound pressure level data and their corresponding curves were drawn on a dB scale. The response cumulative probability distributions for two cases with $M = 2$ and $M = 3$ are shown in Figures 2 and 3, respectively. Both figures show good agreement between the theoretically predicted curves and experimentally sampled points, except for the lower part of the probability distribution curves.

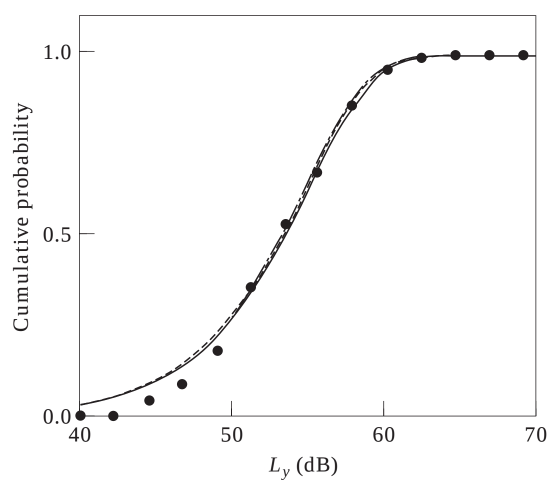

Figure 3. A comparison between the theoretically predicted cumulative probability distributions for the case $M = 3$ in equations (19), (20) and (21) and the experimentally sampled values of the sound level without the background noise in a transmission room excited by road traffic noise. Experimentally sampled values are marked by dots (●) and theoretical curves are shown with the degree of approximation (——, first approximation; – – –, second and third approximations; — - —, fourth approximation).

4. CONCLUSIONS

To predict more precisely the probability distribution function form of the response fluctuation for an environmental acoustic system with an arbitrary sound input under the existence of background noise, a new stochastic method has been proposed by introducing three functional models on the sound intensity scale matched to the problem-oriented viewpoint of, especially, evaluating the probabilistic response. First, the framework of response probability distribution expression has been established in a unified form of a statistical Laguerre series expansion type. Then, the relationship between input and output has been described by extended regression functions to input for the lower and higher order moments of a specific type directly connected to this expansion form of the response p.d.f. These extended regression functions have been derived by use of the orthogonal series expansion of the joint p.d.f. of the input and output. After these regression functions have been directly related to the observation data contaminated by the background noise through the proposed functional models, all of the parameters of the regression function curves have been estimated based on the well-known least squares error criteria under the assumption of stationarity of the background noise. Finally, the proposed method has also been confirmed experimentally by applying it to the actual sound insulation system.

Such an estimation method, reflecting a top-down viewpoint (supported by the high order moments) on the basis of a bottom-up viewpoint (supported by the lower order moments, such as mean and variance) for functionally predicting the output response p.d.f. seems as yet to be at an early stage of study, and so there remain many future problems, such as: to apply this method to many other engineering fields; and to find a more practical estimation method through any approximation of this method and to find many other types of methods of unification between this functional estimation method from the top-down viewpoint owing to the variety of p.d.f. forms and the usual estimation method from the bottom-up viewpoint owing to the physical mechanism.

ACKNOWLEDGMENT

We would like to express our cordial thanks to Mr T. Nakano and Mr M. Masuno for their assistance.

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