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THE FORMULATION AND DYNAMIC ANALYSIS OF A MULTIPLE BELT SYSTEM

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In this paper a method is presented for analyzing the dynamic characteristics of driving systems consisting of multiple belts and pulleys. Firstly, an algorithm that derives the linear equations of motion for arbitrary multi-coupled belt systems is given. Secondly, the algorithm is used to develop a computer program that formulates equations of motion and calculates the transient responses of the belt system. The fundamental idea of the algorithm is as follows: complicated belt systems consisting of multiple belts and pulleys are regarded as combinations of simple belt systems consisting of a single belt and several pulleys. Therefore, the equations of motion for the belt systems can be derived by superposing the equations of motion for the simple belt systems. Thus, the responses of the arbitrary multi-coupled belt systems can be calculated. Finally, to verify the usefulness of this method, the simulation results are compared with experimental results.

1. INTRODUCTION

In a power train driving automotive accessories, a belt drive system supplies power to all of the accessories. The belt system consists of a single belt and several pulleys connected to the accessories, and supplies driving torque from an engine to the accessories. The vibration of the belt system has been studied in order to reduce torsional vibration and noise (see, for example, references [1, 2]). However, in machines such as a cash dispenser, a belt drive system conveys relatively light and thin goods such as paper money. In this system, the goods are put between belts and transferred at very high speed. This belt system is constructed by combining a large number of simple belt systems which consist of a single belt and several pulleys and have a large number of degrees of freedom. In a conventional cash dispenser, the belt system consists of about 40 simple belt systems. This belt system is usually driven by a three-phase induction motor and frequently starts and stops. Thus, this belt system must be able to accelerate and decelerate quickly. To improve the acceleration and deceleration of the belt system, the transient characteristics of the belt system at the start-up of the motor must be analyzed.

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In this paper a method is presented of systematically formulating a driving system consisting of multiple belts and pulleys. The method uses a computer to analyze the dynamic characteristics of the systems.

Firstly, an algorithm for formulating belt systems is given. This algorithm is used to make a computer program that can derive the linear equations of motion for arbitrary belt systems and solve these equations of motion. In order to calculate the transient responses exactly, the transient characteristics of an induction motor must be considered. These characteristics are also analyzed by modelling the motor as an equivalent two-phase induction motor. The transient torque of the motor is calculated with the presented program, and is used in calculating the responses of belt systems.

Secondly, to verify the usefulness of the presented program, the calculation results obtained with it are compared with experimental results of a test rig driven by an actual induction motor.

2. ALGORITHM FOR FORMULATING BELT SYSTEMS

The following assumptions are made for deriving the equations of motion for belt systems.

(1) A belt is replaced by a spring with a stiffness coefficient k_{ij} and a damping coefficient c_{ij}, as shown in Figure 1. The stiffness coefficient of a belt between a pulley i and a pulley j, k_{ij}, is expressed by (a list of nomenclature is given in the Appendix)

$$k_{ij} = AE/L_{ij}, \tag{1}$$

where A is the cross-sectional area of the belt, E is the elastic modulus of the belt, and L_{ij} is the distance between a pulley i and a pulley j.

(2) The mass and volume of the belt are ignored.

(3) There is no slip between the belt and pulley.

(4) The bending stiffness of the belt is negligible.

Complicated belt systems consisting of multiple belts and pulleys are regarded as combinations of single belt systems consisting of a single belt and several pulleys. Therefore, equations of motion for complicated belt systems can be derived by superposing equations of motion for single belt systems.

2.1. EQUATIONS OF MOTION FOR SINGLE BELT SYSTEMS

In this section, equations of motion are derived for a single belt system, which consists of a single belt and n pulleys, as shown in Figure 2. The pulleys of the single belt system are numbered from 1 to n arbitrarily. If pulley i is located next to pulleys h and j, as shown in Figure 2, the equation of motion for pulley i is

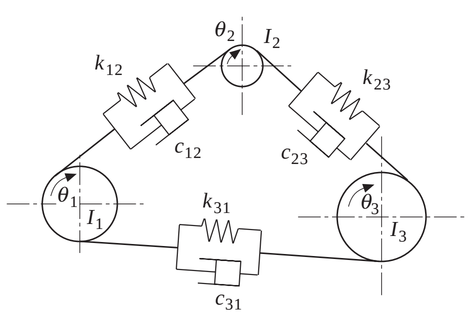

Figure 1. The belt model.

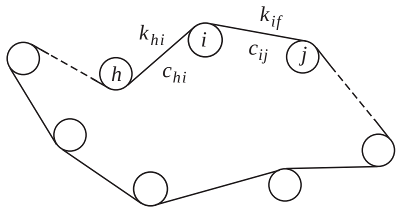

Figure 2. The single belt system.

$$I_i\ddot{\theta}_i - c_{hi}r_hr_i\dot{\theta}_h + (c_{hi} + c_{ij})r_i^2\dot{\theta}_i - c_{ij}r_ir_j\dot{\theta}_j - k_{hi}r_hr_i\theta_h + (k_{hi} + k_{ij})r_i^2\theta_i - k_{ij}r_ir_j\theta_j = T_i. \quad (2)$$

The equations of motion for the other pulleys can be derived in the same way as that for pulley i, and the equations of motion for all the pulleys are expressed in matrix form as follows:

$$\mathbf{I}\ddot{\boldsymbol{\theta}} + \mathbf{C}\dot{\boldsymbol{\theta}} + \mathbf{K}\boldsymbol{\theta} = \mathbf{T}, \quad (3)$$

where $\boldsymbol{\theta} = \{\theta_1, \theta_2, \ldots, \theta_n\}^{\mathrm{T}}$ and $\mathbf{T} = \{T_1, T_2, \ldots, T_n\}^{\mathrm{T}}$.

In equation (3), the equations of motion for the pulley i is in the ith column of the matrices $\mathbf{I}$, $\mathbf{C}$ and $\mathbf{K}$. Therefore, the matrix $\mathbf{I}$ is a diagonal matrix with the elements I_i ($i = 1, 2, \ldots, n$), and the matrix $\mathbf{K}$ is the matrix the elements of which in the hth, ith and jth rows are the coefficients of θ_h, θ_i and θ_j respectively. The matrix $\mathbf{C}$ is the same as the matrix $\mathbf{K}$.

If the pulleys are numbered according to location, the elements of the matrices $\mathbf{K}$ and $\mathbf{C}$ are arranged as shown in Figure 3.

2.2. EQUATIONS OF MOTION FOR BELT SYSTEMS

Equations of motion for belt systems can be derived by superposing the equations of motion for single belt systems. After this, in this paper, the term "loop" is used to refer to a single belt system which consists of a belt and several pulleys.

If two loops are connected by one pulley m, as shown in Figure 4, the equation of motion for the pulley m is

$$\begin{aligned} I_m\ddot{\theta}_m &- c_{lm}r_lr_m\dot{\theta}_l + (c_{lm} + c_{mn})r_m^2\dot{\theta}_m - c_{mn}r_mr_n\dot{\theta}_n - c_{l'm}r_{l'}r_m\dot{\theta}_{l'} \\ &+ (c_{l'm} + c_{mn'})r_m^2\dot{\theta}_m - c_{mn'}r_mr_{n'}\dot{\theta}_{n'} - k_{lm}r_lr_m\theta_l + (k_{lm} + k_{mn})r_m^2\theta_m \\ &- k_{mn}r_mr_n\theta_n - k_{l'm}r_{l'}r_m\theta_{l'} + (k_{l'm} + k_{mn'})r_m^2\theta_m - k_{mn'}r_mr_{n'}\theta_{n'} = T_m. \end{aligned} \quad (4)$$

The equations of motion for the other pulleys are obtained in the same manner, and the equations of motion for all pulleys can be expressed in matrix form in the same way as equation (3).

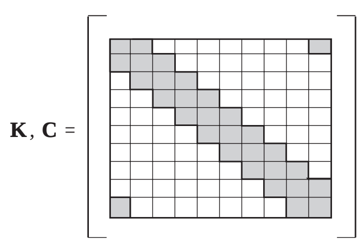

Figure 3. The form of matrices $\mathbf{K}$ and $\mathbf{C}$.

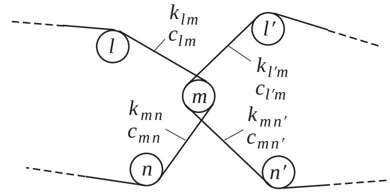

Figure 4. Two loops connected by one pulley, m.

In equation (4), the coefficients of θ_m and $\dot{\theta}_m$ are the summation of the coefficients of θ_m and $\dot{\theta}_m$ in the equation of motion which is derived separately with respect to each loop. Therefore, the matrices $\mathbf{K}$ and $\mathbf{C}$ of the equation of motion for a belt system which is connected by one pulley in each loop can be obtained by superposing the matrices $\mathbf{K}$ and $\mathbf{C}$ of the equation of motion for each loop.

If two loops are connected by the pulley l and the pulley m, as shown in Figure 5, the equation of motion for the pulley m is

$$
\begin{aligned}
I_m\ddot{\theta}_m - c_{lm}r_lr_m\dot{\theta}_l + (c_{lm} + c_{mn})r_m^2\dot{\theta}_m - c_{mn}r_mr_n\dot{\theta}_n - c'_{lm}r_lr_m\dot{\theta}_l \\
+ (c'_{lm} + c_{mn'})r_m^2\dot{\theta}_m - c_{mn'}r_mr_{n'}\dot{\theta}_{n'} - k_{lm}r_lr_m\theta_l + (k_{lm} + k_{mn})r_m^2\theta_m \\
- k_{mn}r_mr_n\theta_n - k'_{lm}r_lr_m\theta_l + (k'_{lm} + k_{mn'})r_m^2\theta_m - k_{mn'}r_mr_{n'}\theta_{n'} = T_m.
\end{aligned}
\tag{5}
$$

In equation (5), the coefficients of θ_m and $\dot{\theta}_m$ are the summation of the coefficients of θ_m and $\dot{\theta}_m$ in the equations of motion which are derived separately with respect to each loop, and the coefficients of θ_l and $\dot{\theta}_l$ are obtained in the same way. Therefore, the matrices $\mathbf{K}$ and $\mathbf{C}$ of the equation of motion for the belt system which is connected by two pulleys in each loop can be obtained by superposing the matrices $\mathbf{K}$ and $\mathbf{C}$ of the equation of motion for each loop.

If two loops are connected by more than three pulleys, the matrices $\mathbf{K}$ and $\mathbf{C}$ of the equation of motion for belt systems can be obtained in the same manner.

For the reasons mentioned above, by using the method for assembling a stiffness matrix in the finite element method (see, for example, references [3]), the matrices $\mathbf{K}$ and $\mathbf{C}$ of the equation of motion for belt systems consisting of multiple loops can be obtained by superposing the matrices $\mathbf{K}$ and $\mathbf{C}$ of the equation of motion for each loop with respect to the common pulleys which belong in more than two loops.

For example, consider the belt system shown in Figure 6. This belt system consists of two loops, L_1 and L_2, which are connected by the common pulley 3. The stiffness matrices $\mathbf{K}_{L_1}$ and $\mathbf{K}_{L_2}$ of the equation of motion for the loops L_1 and L_2 are as follows:

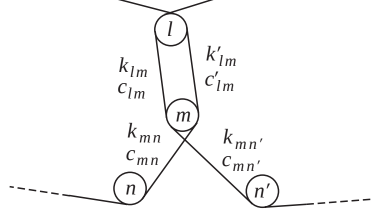

Figure 5. Two loops connected by two pulleys, l and m.

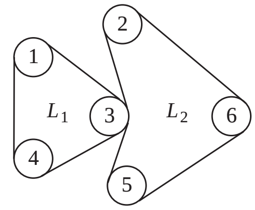

Figure 6. A belt system consisting of two loops.

$$\mathbf{K}_{L_1} = \begin{bmatrix} (k_{41}+k_{13})r_1^2 & -k_{13}r_1r_3 & -k_{41}r_3r_4 \\ -k_{13}r_1r_3 & (k_{13}+k_{34})r_3^2 & -k_{34}r_3r_4 \\ -k_{41}r_4r_1 & -k_{34}r_3r_4 & (k_{34}+k_{41})r_4^2 \end{bmatrix}, \tag{6}$$

$$\mathbf{K}_{L_2} = \begin{bmatrix} (k_{62}+k_{23})r_2^2 & -k_{23}r_2r_3 & 0 & -k_{62}r_6r_2 \\ -k_{23}r_2r_3 & (k_{23}+k_{35})r_3^2 & -k_{35}r_3r_5 & 0 \\ 0 & -k_{35}r_3r_5 & (k_{35}+k_{56})r_5^2 & -k_{56}r_5r_6 \\ -k_{62}r_6r_2 & 0 & -k_{34}r_3r_4 & (k_{56}+k_{62})r_6^2 \end{bmatrix}. \tag{7}$$

The equation of motion for the belt system is

$$\mathbf{I}\ddot{\boldsymbol{\theta}} + \mathbf{C}\dot{\boldsymbol{\theta}} + \mathbf{K}\boldsymbol{\theta} = \mathbf{T}, \tag{8}$$

where $\boldsymbol{\theta} = \{\theta_1, \theta_2, \ldots, \theta_6\}^{\mathrm{T}}$, $\mathbf{I} = \mathrm{diag}[I_1, I_2, \ldots, I_6]$ and $\mathbf{T} = \{T_1, T_2, \ldots, T_6\}^{\mathrm{T}}$. The stiffness matrix of the belt system is

$$\mathbf{K} = \begin{bmatrix} (k_{41}+k_{13})r_1^2 & 0 & -k_{13}r_1r_3 & -k_{41}r_3r_4 & 0 & 0 \\ 0 & (k_{62}+k_{23})r_2^2 & -k_{23}r_2r_3 & 0 & 0 & -k_{62}r_6r_2 \\ -k_{13}r_1r_3 & -k_{23}r_2r_3 & (k_{13}+k_{34}+k_{23}+k_{35})r_3^2 & -k_{34}r_3r_4 & -k_{35}r_3r_5 & 0 \\ -k_{41}r_4r_1 & 0 & -k_{34}r_3r_4 & (k_{34}+k_{41})r_4^2 & 0 & 0 \\ 0 & 0 & -k_{35}r_3r_5 & 0 & (k_{35}+k_{56})r_5^2 & -k_{56}r_5r_6 \\ 0 & -k_{62}r_6r_2 & 0 & 0 & -k_{34}r_3r_4 & (k_{56}+k_{62})r_6^2 \end{bmatrix}, \tag{9}$$

where the element with respect to the common pulley 3, (3, 3), is the summation of the elements of the stiffness matrix in each loop. The location of the elements of the stiffness matrices $\mathbf{K}_{L_1}$ and $\mathbf{K}_{L_2}$ in the stiffness matrix of the belt system are shown in Figure 7.

As another example, consider the belt system shown in Figure 8. This belt system consists of two loops, L_1 and L_2, which are connected by the common pulleys 2 and 3. The equation of motion for the belt system is

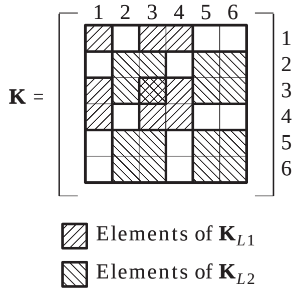

Figure 7. The locations of elements of the stiffness matrices of each loop in the stiffness matrix of the belt system.

$$\mathbf{I}\ddot{\boldsymbol{\theta}} + \mathbf{C}\dot{\boldsymbol{\theta}} + \mathbf{K}\boldsymbol{\theta} = \mathbf{T}, \tag{10}$$

where $\boldsymbol{\theta} = \{\theta_1, \theta_2, \ldots, \theta_5\}^{\mathrm{T}}$, $\mathbf{I} = \mathrm{diag}[I_1, I_2, \ldots, I_5]$ and $\mathbf{T} = \{T_1, T_2, \ldots, T_5\}^{\mathrm{T}}$. The stiffness matrix of the belt system is

$$\mathbf{K} = \begin{bmatrix} (k_{51}+k_{12})r_1^2 & -k_{12}r_1r_2 & 0 & 0 & -k_{51}r_5r_1 \\ -k_{12}r_1r_2 & (k_{12}+k_{23}+k_{42}+k'_{23})r_2^2 & (-k_{23}-k'_{23})r_3^2 & -k_{42}r_4r_2 & 0 \\ 0 & (-k_{23}-k'_{23})r_2^2 & (k_{23}+k_{35}+k'_{23}+k_{34})r_3^2 & -k_{34}r_3r_4 & -k_{35}r_3r_5 \\ 0 & -k_{42}r_4r_2 & -k_{34}r_3r_4 & (k_{34}+k_{42})r_4^2 & 0 \\ -k_{51}r_5r_1 & 0 & -k_{35}r_3r_5 & 0 & (k_{35}+k_{51})r_5^2 \end{bmatrix}, \tag{11}$$

where the elements with respect to the common pulleys 2 and 3, (2, 2), (2, 3), (3, 2) and (3,3), are the summation of the elements of the stiffness matrix in each loop. The locations of the elements of the stiffness matrices of each loop in the stiffness matrix of the belt system are shown in Figure 9.

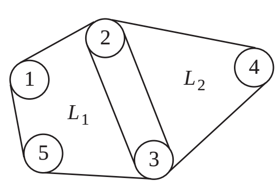

Figure 8. A belt system consisting of two loops connected by two pulleys.

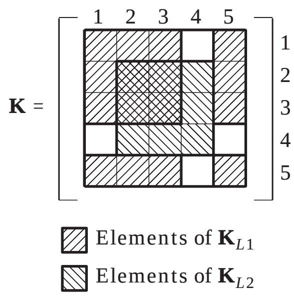

Figure 9. The locations of elements of the stiffness matrices of each loop in the stiffness matrix of the belt system.

3. DETERMINING WHETHER OPPOSING BELT MOVEMENT OCCURS

In a belt system consisting of multiple loops, if a pulley belongs to more than two loops, and the belt of each loop rotates in the opposite direction at the pulley, the belt system will not operate. Thus, before analyzing the dynamic characteristics of a belt system, it is necessary to determine whether opposing belt movement has occurred.

3.1. CLASSIFICATION OF BELT SYSTEMS

Belt systems are classified into three types, as shown in Figure 10: (1) the series type—the loops are connected in series; (2) the branch type—a series of loops branches at a loop; (3) the circulation type—the loops are connected in a ring.

3.2. HOW TO DETERMINE WHETHER OPPOSING BELT MOVEMENT HAS OCCURRED

The direction of rotation of a pulley is decided by the direction of velocity of a belt. For the combination of two pulleys and a belt shown in Figure 11(a), the two pulleys rotate in the same direction, but, for the combination shown in Figure 11(b), the two pulleys rotate in opposite directions. The velocity of a belt at a pulley i is

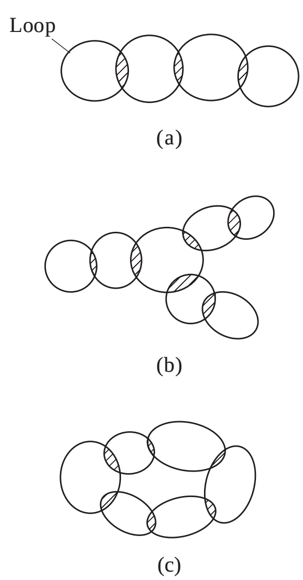

Figure 10. A classification of belt systems: (a) series type; (b) branch type; (c) circulation type.

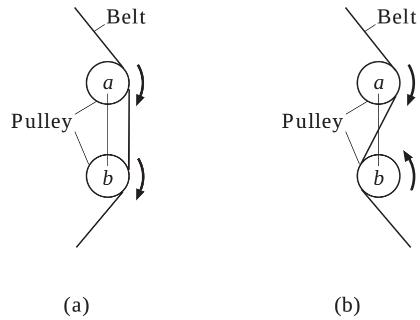

Figure 11. The directions of rotation of the pulleys: (a) same direction; (b) opposite direction.

$$v_i = r_i\dot{\theta}_i. \tag{12}$$

For the combination shown in Figure 11(a), the relationship of the velocity of the belt at pulley a and pulley b is expressed as

$$v_b = v_a. \tag{13}$$

For the combination shown in Figure 11(b), the relationship of the velocity of the belt at pulley a and pulley b is expressed as

$$v_b = -v_a. \tag{14}$$

Therefore, if a belt system consists of n pulleys, the relationship between the velocity of the belt at pulley i and the velocity of the belt at pulley j, when pulley i is located next to pulley j $(i, j = 1, \ldots, n)$, is expressed by equations (13) and (14). Unless all the equations are satisfied, opposing belt movement will occur.

For the belt system in Figure 12, for exmaple, the equations of the velocities of the belts are obtained as

$$\left.\begin{array}{ll} v_1 = v_2, \quad v_2 = v_7, \quad v_7 = v_1 & \text{for loop } L_1 \\ v_3 = v_2 & \text{for loop } L_2 \\ v_3 = -v_4, \quad v_4 = v_5, \quad v_5 = v_6, \quad v_6 = -v_3 & \text{for loop } L_3 \\ v_6 = v_7 & \text{for loop } L_4 \end{array}\right\}. \tag{15}$$

Because these equations are not satisfied, opposing belt movement will occur.

4. TRANSIENT OUTPUT TORQUE OF INDUCTION MOTORS

Belt systems are usually driven by a three-phase induction motor with a squirrel cage rotor. Therefore, to analyze the transient responses of belt systems, the transient output torque of the induction motor must be examined.

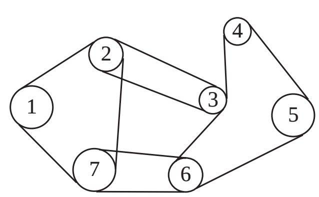

Figure 12. An example of a belt system with opposing belt movement.

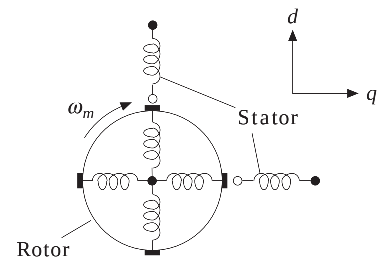

Figure 13. A model of two-phase induction motor.

As is well known, a three-phase induction motor can be transformed to an equivalent two-phase induction motor (see, for example, reference [4]). Therefore, in this paper, the transient output torque of an induction motor is examined by using the two-phase induction motor model shown in Figure 13. Moreover, by using a d–q transformation, a rotor coil can be transformed to an equivalent stator coil (see, for example, references [5, 6]). Therefore, a three-phase induction motor is transformed to an equivalent two-phase induction motor whose two windings in the rotor and stator are fixed in the d- and q-axes.

In the equivalent two-phase induction motor, the relationship between voltages and currents is expressed as

$$\mathbf{V} = \mathbf{R}\mathbf{i} + \mathbf{L}\dot{\mathbf{i}} + \frac{p}{2}\omega_m \mathbf{G}\mathbf{i}, \tag{16}$$

where

$$\mathbf{R} = \begin{bmatrix} R_1 & 0 & 0 & 0 \\ 0 & R_1 & 0 & 0 \\ 0 & 0 & R_2 & 0 \\ 0 & 0 & 0 & R_2 \end{bmatrix}, \quad \mathbf{L} = \begin{bmatrix} L_1 & 0 & L_m & 0 \\ 0 & L_1 & 0 & L_m \\ L_m & 0 & L_2 & 0 \\ 0 & L_m & 0 & L_2 \end{bmatrix},$$

$$\mathbf{G} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & L_m & 0 & L_2 \\ -L_m & 0 & -L_2 & 0 \end{bmatrix}$$

$$\mathbf{i} = \{i_{1d}, i_{1q}, i_{2d}, i_{2q}\}^{\mathrm{T}}, \qquad \mathbf{V} = \{V_{1d}, V_{1q}, V_{2d}, V_{2q}\}^{\mathrm{T}},$$

$$L_1 = L_m + L_1', \qquad L_2 = L_m + L_2'.$$

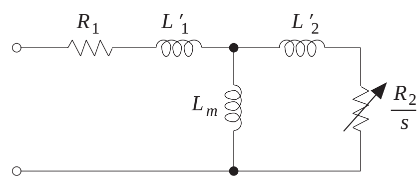

Figure 14. The equivalent circuit of the induction motor.

The constants R_1, R_2, L_1', L_2' and L_m are obtained from the equivalent circuit of an induction motor in a steady state (ω_m is constant). The equivalent circuit is shown in Figure 14.

By multiplying by $\mathbf{i}^{\mathrm{T}}$, equation (16) becomes

$$\mathbf{i}^{\mathrm{T}}\mathbf{V} = \mathbf{i}^{\mathrm{T}}\mathbf{R}\mathbf{i} + \mathbf{i}^{\mathrm{T}}\mathbf{L}\dot{\mathbf{i}} + \frac{p}{2}\,\omega_m \mathbf{i}^{\mathrm{T}}\mathbf{G}\mathbf{i}. \tag{17}$$

In equation (17), $\mathbf{i}^{\mathrm{T}}\mathbf{V}$ is the electric power, which is supplied from an outer power source; the first term is power dissipated in resistance, the second term is power stored as magnetic energy and the third term is energy, which is transformed to mechanical energy. Therefore, the output torque is expressed as

$$T_e = \left(\frac{p}{2}\,\omega_m \mathbf{i}^{\mathrm{T}}\mathbf{G}\mathbf{i}\right)\bigg/\omega_m = \frac{p}{2}\,L_m(i_{1q}i_{2d} - i_{1d}i_{2q}). \tag{18}$$

By integrating equation (16), i_{1q}, i_{2d}, i_{1d} and i_{2q} are obtained. Therefore, the output torque can be calculated. For a three-phase induction motor, in order to apply equations (16) and (18), the three-phase voltages must be transformed to two-phase voltages by using the following equation:

$$\begin{Bmatrix} V_{1d} \\ V_{1q} \end{Bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{Bmatrix} V_a \\ V_b \\ V_c \end{Bmatrix}, \tag{19}$$

where V_a, V_b and V_c are three-phase sinusoidal voltages, and because of the squirrel cage rotor, $V_{2d} = V_{2q} = 0$.

5. THE PRESENTED COMPUTER PROGRAM

By using the algorithm mentioned above, a computer program that calculates the transient responses of an arbitrary belt system can be developed. The program is divided into two parts, the formulation of the belt system and the calculation of transient responses. In formulating the belt system, the input parameters of the program are the number of loops in the belt system, the number of pulleys in each loop, the radius and momentum of inertia of each pulley, the stiffness and damping coefficients of each belt, and the load at each pulley. By defining the pulleys which connect each loop, the program firstly determines whether opposing belt movement has occurred, and secondly derives the equation of motion for the belt system. In calculating transient responses, the program solves the formulated equation of motion by using the four order Runge–Kutta method. At every time interval, the output motor torque is calculated by using equations (16) and (18). Then, the responses of the system—that is, the angular displacement and angular velocity of each pulley and the belts deflections, and so on—are calculated. The angular velocity of the motor, which is equal to that of the pulley connected with the motor, is used in calculating the motor torque in the next time interval. The flowchart of the program is shown in Figure 15.

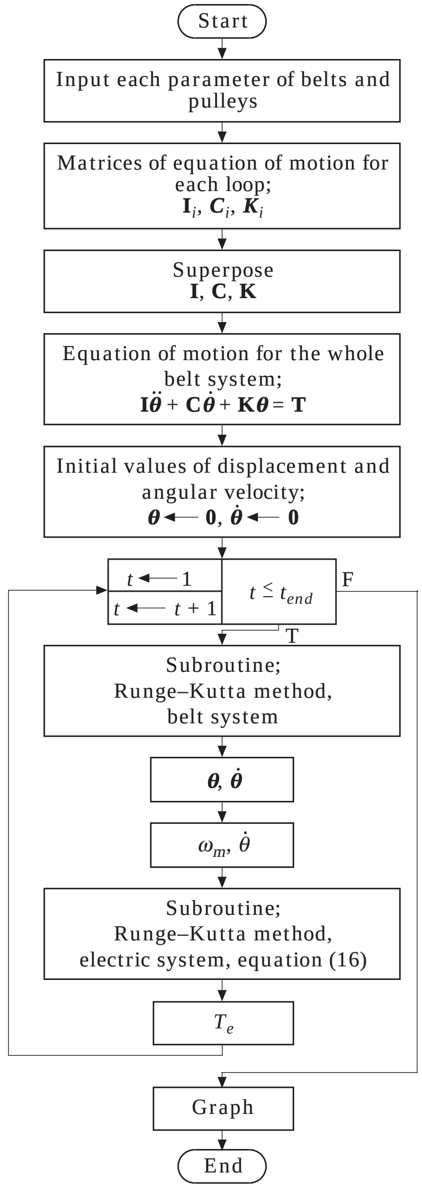

Figure 15. A flowchart of the computer program.

6. A COMPARISON OF EXPERIMENTAL AND CALCULATED RESULTS

6.1. THE TEST RIG

The test rig is shown in Figure 16. It consists of a belt system with four loops and a motor. The belts are the flat belts widely used in cash dispensers. The pulleys are numbered, and the pulley with a 10 mm radius has a moment of inertia of $1{\cdot}34 \times 10^{-5}$ Kg m^2. The moment of inertia of the pulley is computed by using the density of the material and the dimensions of the pulley. The motor is a three-phase induction motor, the moment of inertia of which is $7{\cdot}33 \times 10^{-4}$ Kg m^2. The output torque is measured by a torque transducer that is attached between the motor and pulley 10 by a sufficiently rigid shaft. The moment of inertia of the torque transducer is $1{\cdot}20 \times 10^{-5}$ Kg m^2. The stiffness of each

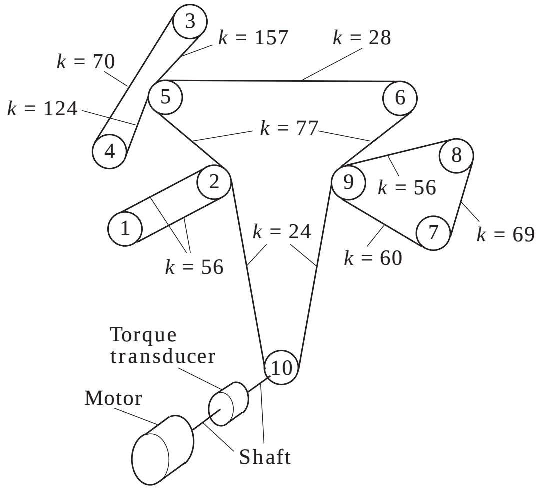

Figure 16. The test rig.

belt, k (KN/m), is shown in Figure 16, and the damping coefficients of the belts are $1{\cdot}0 \times 10^{-6}$ Ns/m. The stiffness and damping coefficients of the belts are determined by the measurement of displacement against the load and damping ratio.

6.2. THE EXPERIMENTAL AND CALCULATED RESULTS AND DISCUSSION

The experimental and calculated results of the time responses of the motor output torque, the rotational frequency of pulley 10, and the rotational frequency of pulley 6, respectively, are shown in Figures 17, 18 and 19. In the calculation, a small step size for Runge–Kutta analysis is chosen so that its influence on the calculation results does not need to be considered.

The angular velocity of the rotor of the induction motor ω_m—that is, the rotational frequency of pulley 10—affects the motor output torque as shown in equations (16) and (18). Therefore, the motor output torque vibrates in connection with the rotational frequency of pulley 10, and the amplitude of the motor output torque becomes larger as the fluctuation of the rotational frequency of pulley 10 becomes larger. The motor output

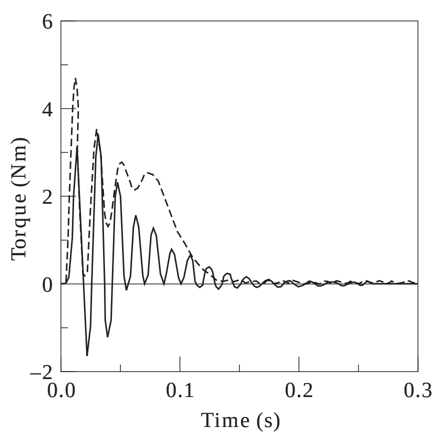

Figure 17. The responses of the motor output torque. ——, Experimental; – – – –, simulation.

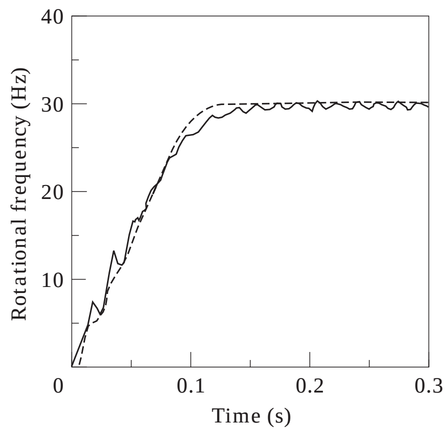

Figure 18. The rotational frequency of pulley 10. Key as Figure 17.

torque has peaks at the maximum derivative of the rotational frequency with respect to time. When the derivative of the rotational frequency is negative, the output torque becomes negative. The fluctuation of the rotational frequency of pulley 10 during the acceleration in the experimental results is larger than that in the calculated results, so the fluctuation of the output torque in the experimental results becomes larger than that in the calculated results. On the other hand, as shown in Figure 19, the fluctuation of the rotational frequency of pulley 6 during the acceleration in the experimental results is smaller than that in the calculated results. In theoretical analysis, the slip between the belts and the pulleys is not considered. Therefore, the rotational frequency of pulley 10 during the acceleration is almost equal to that of pulley 6, and the frequency of the vibration of pulleys 6 and 10 in the steady state rotational frequency (30 Hz) is about 85 Hz. However, in the experimental results, the rotational frequency of pulley 6 during the acceleration is not equal to that of pulley 10, and the maximum difference of the rotational frequency is about 6 Hz. The frequency of the vibration of pulley 6 in the steady state rotational frequency is not equal to that of pulley 10. In the steady state responses of pulley 6, the vibration at a frequency of about 85 Hz occurs periodically with the frequency at about 30 Hz.

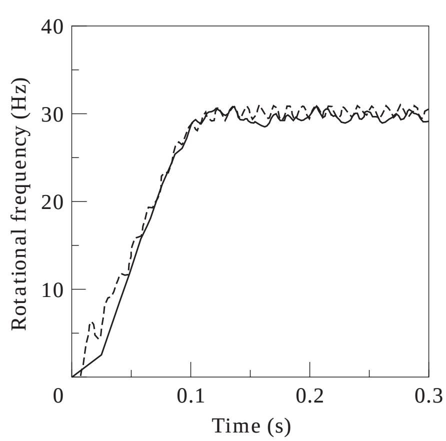

Figure 19. The rotational frequency of pulley 6. Key as Figure 17.

This makes it clear that the slip between the belts and the pulleys always occurs, and that the magnitude of the slip during the acceleration is larger than that in the steady state rotational frequency. Therefore, strictly speaking, the vibration of the belt system must be considered as a non-linear vibration system with friction damping. However, the time from the start-up of the motor to the steady state rotational frequency, and the frequency of the vibration of the output motor torque directly after the start-up in the experimental results, agree very well with those in the calculated results.

7. CONCLUSIONS

The main conclusions are as follows.

(1) Complicated belt systems consisting of multiple belts and pulleys can be regarded as a combination of single belt systems consisting of a single belt and several pulleys, and equations of motion for complicated belt systems can be derived by superposing equations of motion for single belt systems.

(2) This algorithm can be used to obtain a computer program that can derive equations of motion for arbitrary belt systems driven by a three-phase induction motor, and that can calculate the transient responses of the belt systems.

(3) Comparing the experimental and calculated results shows that the vibration of the belt system must be considered as a non-linear vibration system with friction damping. However, the time from the start-up of the motor to the steady state rotational frequency and the frequency of the vibration of the output motor torque directly after the start-up can be simulated by using the presented program.

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APPENDIX: NOMENCLATURE

c_{ij}	damping coefficient of belt between pulley i and pulley j
I_i	moment of inertia of pulley i
k_{ij}	stiffness coefficient of belt between pulley i and pulley j
r_i	radius of pulley i
T_i	torque applied at pulley i
v_i	velocity of belt at pulley i
θ_i	rotational angle of pulley i
R_1, R_2	primary and secondary resistance, respectively
L_1, L_2	primary and secondary self-inductance, respectively
L_1', L_2'	primary and secondary leakage inductance, respectively

L_m	mutual inductance
ω_m	angular velocity of rotor of induction motor
s	slip of induction motor
p	number of poles of induction motor
V_{1d}, V_{2d}	primary and secondary voltages in d-axis, respectively
V_{1q}, V_{2q}	primary and secondary voltages in q-axis, respectively
i_{1d}, i_{2d}	primary and secondary currents in d-axis, respectively
i_{1q}, i_{2q}	primary and secondary currents in q-axis, respectively

Superscript

T	transpose of matrix.