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THE FORCED VIBRATION OF A THIN PLATE FLOATING ON AN INFINITE LIQUID

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In this paper a solution is presented for the harmonically forced vibration of an arbitrary thin plate floating on the surface of an infinite liquid. The full linear potential problem for the liquid is solved by the use of the appropriate Green's function. A variational equation which the plate–liquid system must satisfy is derived and a solution by the Rayleigh–Ritz method is presented. Examples of possible calculations are given for a square and a rectangular plate.

1. INTRODUCTION

The study of the vibration of thin plates floating on the surface of a liquid falls into two broad categories. In the first, a recent example of which is the work of Robinson and Palmer [1], the plate is assumed to float on a finite liquid, for example a tank, and the modes of vibration for the entire system are determined. No approximations about the liquid motion are made except that it is linear and incompressible. In the second approach the liquid is modelled as infinite and the changes in the frequency of vibration due to the presence of the liquid are investigated. Kwak and Kim [2] and Kwak [3] are recent work using this approach, although this line of investigation has a long history beginning with the work of Rayleigh and Lamb [2]. In such a model, strong approximations are made about the liquid motion and there is no wave generation by the vibrating plate.

It is well known that even for a simple geometry, such as a square thin plate with free edge conditions, analytic solutions for the thin plate modes of vibration are not possible. Therefore, the modes must be solved for numerically, the standard methods being derived from the variational equation for the thin plate (i.e., the Rayleigh–Ritz method or the finite element method). For this reason it seems natural to solve for the vibration of an arbitrary shaped plate floating on an infinite liquid by deriving a variational equation which the plate–liquid system must satisfy and this is the approach taken in this paper. Since a vibrating body on an infinite liquid will generate wave motion which will carry away energy it is not realistic to consider modal motion. Instead, it is natural to consider the plate motion as a function of a temporally harmonic forcing pressure. The forced vibration of a floating thin plate may be considered as a model of a large floating structure, for example a floating runway, subject to an external force, such as an aircraft landing.

The variational equation for an arbitrary thin plate subject to a harmonic forcing pressure is well known [4]. Of course, when the plate floats on a liquid the pressure due to the liquid motion is a complex function of the plate displacement. The solution presented in this paper is as follows. The relationship between the liquid motion and plate displacement is transformed into an integral equation by the use of the appropriate Green's

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function for an infinite liquid. This method is analogous to the boundary element method except that the Green's function is also chosen to satisfy the infinite boundary conditions so that the domain of integration is reduced to the plate. The integral equation relationship is then substituted into the variational equation to derive a variational equation for the plate–liquid system. This variational equation is then solved by the Rayleigh–Ritz method and some example calculations are given.

2. THE PROBLEM

We consider an arbitrary thin plate floating on the surface of infinite three-dimensional half-space of liquid as shown in Figure 1. This figure also shows the co-ordinate system used to solve the problem and the time independent boundary value problem for the liquid (equation (2)). The submergence of the plate is assumed to be negligible so that the boundary condition under the plate may be applied at the surface of the liquid. We restrict consideration to small amplitude response so that all the equations are linear. The spatial form of the forcing pressure on the plate will be arbitrary but we will restrict consideration to a harmonic time dependence. Since the problem is linear, this means that all the dependent variables must also be harmonic and the time derivatives are thus eliminated. Of course, solutions for complex time dependent pressures can be constructed using a spectral expansion of the forcing pressure as will be done subsequently. The linear potential theory for small amplitude wave motion [5] allows us to construct a linear operator relating velocity potential and displacement. This relationship is included in the variation equation for a thin plate [4] to give us a variation equation for the problem which we solve by the Rayleigh–Ritz method.

3. EQUATIONS OF MOTION FOR THE LIQUID

The linearized boundary value problem for the velocity potential Φ for the liquid assuming irrotational and inviscid flow is as follows:

$$\nabla^2\Phi = 0, \qquad -\infty < z < 0,$$

$$\frac{\partial \Phi}{\partial z} = 0, \qquad z \to -\infty,$$

$$\frac{\partial \Phi}{\partial z} = \frac{\partial W}{\partial t}, \qquad z = 0, \quad \mathbf{P} \in \Delta,$$

$$\rho\left(g\frac{\partial \Phi}{\partial z} + \frac{\partial^2 \Phi}{\partial t^2}\right) = -\frac{\partial p}{\partial t}, \qquad z = 0. \tag{1}$$

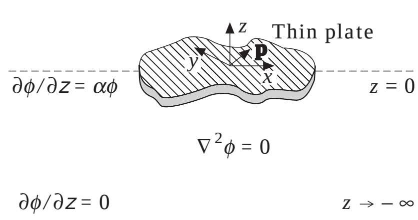

Figure 1. A schematic diagram of a thin plate floating on an incompressible infinite liquid.

In equation (1), W is the displacement of the plate, $\mathbf{P}$ is a point on the liquid surface, p is the pressure on the liquid surface, which is assumed to be constant except beneath the plate, ρ is the density of the liquid, g is the acceleration due to gravity and Δ is the region of the liquid surface occupied by the plate. We also require appropriate conditions to be met as $|\mathbf{P}| \to \infty$ and at $t = 0$.

Equation (1) is now non-dimensionalized using

$$\bar{W} = \frac{W}{a}, \qquad \bar{\mathbf{P}} = \frac{\mathbf{P}}{a}, \qquad \bar{z} = \frac{z}{a}, \qquad \bar{t} = t\sqrt{\frac{g}{a}}, \qquad \bar{\Phi} = \frac{\Phi}{a\sqrt{ag}},$$

where a is some length parameter associated with the plate. We will consider only a periodic forcing pressure with non-dimensional radian frequency $\sqrt{\alpha}$, so we may write $\bar{\Phi}(\bar{x}, \bar{z}, \bar{t}) = \bar{\phi}(\bar{x}, \bar{z})\, \mathrm{e}^{-\mathrm{i}\bar{t}\sqrt{\alpha}}$, and likewise for W. The boundary value problem (1) becomes

$$\nabla^2\bar{\phi} = 0, \qquad -\infty < \bar{z} < 0,$$

$$\frac{\partial\bar{\phi}}{\partial\bar{z}} = 0, \qquad \bar{z} \to -\infty,$$

$$\frac{\partial\bar{\phi}}{\partial\bar{z}} = -\mathrm{i}\sqrt{\alpha}\bar{W}, \qquad \bar{z} = 0, \quad \bar{\mathbf{P}} \in \bar{\Delta},$$

$$\frac{\partial\bar{\phi}}{\partial\bar{z}} - \alpha\bar{\phi} = 0, \qquad \bar{z} = 0, \quad \bar{\mathbf{P}} \notin \bar{\Delta}. \tag{2}$$

From now on, we shall omit the overbars for clarity. The boundary condition as $|\mathbf{P}| \to \infty$ is the Sommerfield radiation condition [5]

$$\sqrt{|\mathbf{P}|}\left(\frac{\partial}{\partial|\mathbf{P}|} - \mathrm{i}\alpha\right)\phi = 0, \qquad \text{as } |\mathbf{P}| \to \infty. \tag{3}$$

The standard solution method to the infinite linear wave problem is to transform the boundary value problem into an integral equation by the use of the appropriate Green's function and then to seek a solution of the integral equation [6, 7]. Performing just such a transformation, equations (2) and (3) become

$$\phi(\mathbf{P}) = \iint_{\Delta} G_\alpha(\mathbf{P}; \mathbf{Q})(\alpha\phi(\mathbf{Q}) + \mathrm{i}\sqrt{\alpha}W(\mathbf{Q}))\,\mathrm{d}S_{\mathrm{Q}}, \tag{4}$$

where G_α is the Green function, given by [8]

$$G_\alpha(\mathbf{P}; \mathbf{Q}) = \frac{1}{2\pi|\mathbf{P} - \mathbf{Q}|} + \frac{1}{4\pi}(\mathrm{i}2\pi\alpha \mathrm{J}_0(\alpha|\mathbf{P} - \mathbf{Q}|) - \pi\alpha\{\mathbf{H}_0(\alpha|\mathbf{P} - \mathbf{Q}|) + \mathrm{Y}_0(\alpha|\mathbf{P} - \mathbf{Q}|)\}), \tag{5}$$

where J_0 and Y_0 are, respectively, Bessel functions of the first and second kind of order zero, and $\mathbf{H}_0$ is the Struve function of order zero [9]. We write equation (4) in operator notation as

$$\phi = \mathbf{G}_\alpha(\alpha\phi + \mathrm{i}\sqrt{\alpha}W),$$

where $\mathbf{G}_\alpha$ represents the linear integral operator defined in equation (4). This operator is self-adjoint in the Hilbert Space defined by integration over the region Δ. This notation will also be used in the numerical solution, where ϕ and W are now finite-dimensional vectors and $\mathbf{G}_\alpha$ is a matrix. We solve equation (4) by constructing the operator $\mathbf{H}_\alpha^{-1}$, defined by

$$\mathbf{H}_\alpha^{-1} = (1 - \alpha \mathbf{G}_\alpha)^{-1}. \tag{6}$$

Solution of equation (4) depends on the construction of $\mathbf{H}_\alpha^{-1}$, and this operator is calculated numerically by the constant panel method [3, 5]. In this method, Δ is divided into n subregions, over which the plate displacement and velocity potential may be considered to be constant. Therefore, W and ϕ may be represented by vectors in an n-dimensional space. Likewise, the operator $\mathbf{G}_\alpha$ becomes an $n \times n$ matrix, the elements of which are determined by the integration of G_α over the appropriate subregion. Obviously, once the matrix $\mathbf{G}_\alpha$ has been calculated, the matrix $\mathbf{H}_\alpha^{-1}$ may be calculated trivially.

4. THE VARIATIONAL EQUATION FOR THE THIN PLATE

The principle of virtual work applied to the free plate gives us the following variational equation [4],

$$\delta \iint_\Delta \tfrac{1}{2} D(W_{xx}^2 + W_{yy}^2 + 2\nu W_{xx} W_{yy} + 2(1-\nu) W_{xy}^2)\, \mathrm{d}S + \iint_\Delta \rho_r h \frac{\partial^2 W}{\partial t^2} \delta W \, \mathrm{d}S = \iint_\Delta (p_w + p_f) \delta W \, \mathrm{d}S, \tag{7}$$

where W is the plate displacement as before, ρ_r is the plate density, h is the plate thickness, D is the modulus of rigidity, p_w is the pressure due to the liquid and p_f is the forcing pressure. The pressure p_w is given by the linearized Bernoulli's equation at the liquid surface,

$$p_w = -\rho g W - \rho \, \partial \Phi / \partial t, \tag{8}$$

and the harmonic forcing pressure is given by

$$p_f = p_0(\mathbf{P}) \, \mathrm{e}^{-\mathrm{i}t\sqrt{g\alpha/a}}. \tag{9}$$

Non-dimensionalizing and restricting consideration to a single frequency as before and, substituting equations (8) and (9) into equation (7), we obtain

$$\delta \iint_\Delta \left[\tfrac{1}{2} \beta (W_{xx}^2 + W_{yy}^2 + 2\nu W_{xx} W_{yy} + 2(1-\nu) W_{xy}^2) + \frac{1 - \alpha\gamma}{2} W^2 \right] \mathrm{d}S - \delta \iint_\Delta p_0 W \, \mathrm{d}S = \iint_\Delta \mathrm{i}\sqrt{\alpha} \phi \delta W \, \mathrm{d}S, \tag{10}$$

where

$$\beta = D / g\rho a^4 \qquad \text{and} \qquad \gamma = \rho_r h / \rho a.$$

Again, the overbars have been removed to avoid clutter. Using $\mathbf{H}_\alpha^{-1}$ we solve equation (4) as

$$\phi = \mathrm{i}\sqrt{\alpha}\mathbf{H}_\alpha^{-1}\mathbf{G}_\alpha W. \tag{11}$$

Substituting equation (11) for ϕ into the right side of equation (10), we obtain

$$\begin{aligned}\iint_\Delta \mathrm{i}\sqrt{\alpha}\phi\delta W\,\mathrm{d}S &= -\alpha\iint_\Delta (\mathbf{H}_\alpha^{-1}\mathbf{G}_\alpha W)\delta W\,\mathrm{d}S \\ &= -\frac{\alpha}{2}\,\delta\iint_\Delta (\mathbf{H}_\alpha^{-1}\mathbf{G}_\alpha W)W\,\mathrm{d}S. \end{aligned}\tag{12}$$

The last step in which the variation is taken outside the integral and the factor of a $\frac{1}{2}$ is included follows from the fact that the operators $\mathbf{H}_\alpha^{-1}$ and $\mathbf{G}_\alpha$ are self-adjoint and commuting. Substituting equation (12) into equation (10), we derive the following variational equation for the problem:

$$\begin{aligned}\delta\iint_\Delta \left\{\tfrac{1}{2}\beta(W_{xx}^2 + W_{yy}^2 + 2\nu W_{xx}W_{yy} + 2(1-\nu)W_{xy}^2) + \frac{1-\alpha\gamma}{2}\,W^2\right\}\mathrm{d}S \\ + \frac{\alpha}{2}\,\delta\iint_\Delta (\mathbf{H}_\alpha^{-1}\mathbf{G}_\alpha W)W\,\mathrm{d}S = \delta\iint_\Delta p_0 W\,\mathrm{d}S. \end{aligned}\tag{13}$$

5. SOLUTION OF THE VARIATIONAL EQUATION BY THE RAYLEIGH–RITZ METHOD

The advantage of having transformed the equations of motion to a variational equation is that a numerical solution is straightforward. One of the simplest numerical techniques, and one which is used extensively to analyse thin plate vibrations, is the Rayleigh–Ritz method. In this method the displacement is expanded as

$$W = \sum_{i=0}^{I}\sum_{j=0}^{J} C_{ij}W_{ij} \tag{14}$$

and substituted into the variational equation which is then solved by seeking a minimum with respect to the C_{ij} coefficients. Therefore substituting the expansion for W (equation (14)) into equation (13), deriving with respect to the coefficient C_{pq}, and equating to zero, we derive the following simultaneous equations for coefficients C_{ij}:

$$\begin{aligned}\sum_{i=0}^{I}\sum_{j=0}^{J} C_{ij}\left\{\frac{\beta}{2}f(i,j;p,q) + \frac{1-\alpha\gamma}{2}\,g(i,j;p,q) + \frac{\alpha}{2}\,(h(i,j;p,q) + h(p,q;i,j))\right\} \\ = \iint_\Delta p_0 W_{pq}\,\mathrm{d}S, \quad p = 0,1,\ldots,I, \quad q = 0,1,\ldots,J, \end{aligned}\tag{15}$$

where

$$f(i, j; p, q) = \iint_{\Delta} \left\{ 2 \frac{\partial^2 W_{ij}}{\partial x^2} \frac{\partial^2 W_{pq}}{\partial x^2} + 2 \frac{\partial^2 W_{ij}}{\partial y^2} \frac{\partial^2 W_{pq}}{\partial y^2} \right.$$

$$\left. + 2\nu \left(\frac{\partial^2 W_{ij}}{\partial x^2} \frac{\partial^2 W_{pq}}{\partial y^2} + \frac{\partial^2 W_{ij}}{\partial y^2} \frac{\partial^2 W_{pq}}{\partial x^2} \right) + 4(1-\nu) \frac{\partial^2 W_{ij}}{\partial x \partial y} \frac{\partial^2 W_{pq}}{\partial x \partial y} \right\} \mathrm{d}S_P,$$

$$g(i, j; p, q) = \iint_{\Delta} 2 W_{ij} W_{pq} \, \mathrm{d}S$$

and

$$h(i, j; p, q) = \iint_{\Delta} W_{ij} (\mathbf{H}_{\alpha}^{-1} \mathbf{G}_{\alpha} W_{pq}) \, \mathrm{d}S.$$

The matrices in equation (15) are independent of the parameters β and γ. This means that once the matrices f, g and h have been calculated, it is trivial to solve the problem for any value of the non-dimensional stiffness β or the non-dimensional mass γ.

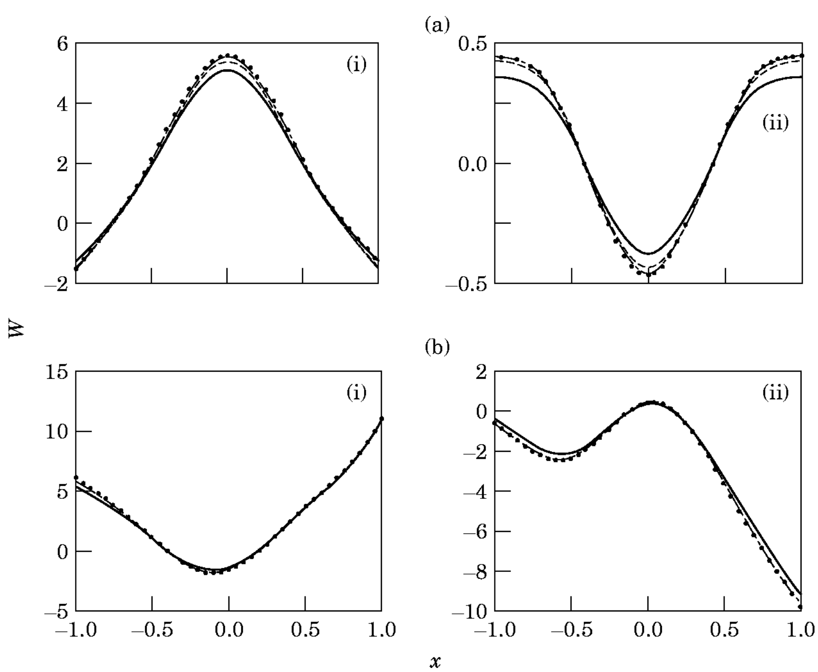

Figure 2. The real (a, i and b, i) and imaginary (a, ii and b, ii) parts of the displacement function (W) along the x-axis for a 2×2 square thin plate subject to a pressure $p_0 = \delta(\mathbf{P} - \mathbf{P}_0)$, where $\mathbf{P}_0 = (0, 0)$ (a, i and a, ii) and $\mathbf{P}_0 = (0, 1)$ (b, i and b, ii). $I = J = 8$ and $n = 100$ (——), $n = 225$ (- - - -), $n = 400$ (– – – – –) and $n = 625$ (. . . . . .). $\alpha = \pi$, $\beta = 0{\cdot}005$ and $\gamma = 0{\cdot}01$.

6. SOLUTION EXAMPLES

Given that, beyond the parameters α, β and γ, we may vary the plate geometry and the pressure p_0, it is obviously not possible to present anything approaching a comprehensive survey of results. Instead, we give a few examples of the sort of calculations which are possible. The choice of the basis functions $W_{ij}(\mathbf{P})$ can have significant effects on the solution accuracy but, as is standard in thin plate analysis, we choose here simple polynomials; i.e.,

$$W_{ij}(\mathbf{P}) = x^i y^j.$$

The main reason for this choice is the simplicity of the resulting equations. Because we are solving a variational equation subject to the so called natural boundary conditions (i.e., the free plate boundary conditions), we do not need to impose any restrictions on the basis functions except completeness [10]. There are two parameters which govern the numerical solution, the number of basis functions used in the expansion of W (I and J) and the number of panels used to discretize the plate (n). In Figures 2 and 3 are shown plots of some simple convergence tests for a 2×2 square plate with $\alpha = \pi$, $\beta = 0{\cdot}005$ and $\gamma = 0{\cdot}01$. In all cases, p_0 is chosen to be a delta function at a point $\mathbf{P}_0$; i.e., $p_0 = \delta(\mathbf{P} - \mathbf{P}_0)$ with $\mathbf{P}_0 = (0, 0)$ (a, i and a, ii) and $\mathbf{P}_0 = (0, 1)$ (b, i and b, ii). We plot the real (a, i and b, i) and imaginary (a, ii and b, ii) parts of the complex displacement (W) along the x-axis. In Figure 2 we have set $I = J = 8$ and are plotting the solution for various values of n, $n = 100$ (solid line), $n = 225$ (dashed line), $n = 400$ (chained line) and $n = 625$ (dotted line). It is clear that the chained and dotted lines overlie, and that the solution is converging and we may conclude that 400 points is sufficient for this geometry. In Figure 3 we are plotting the solution for fixed n ($n = 400$) and are varying I and J, $I = J = 4$ (solid line),

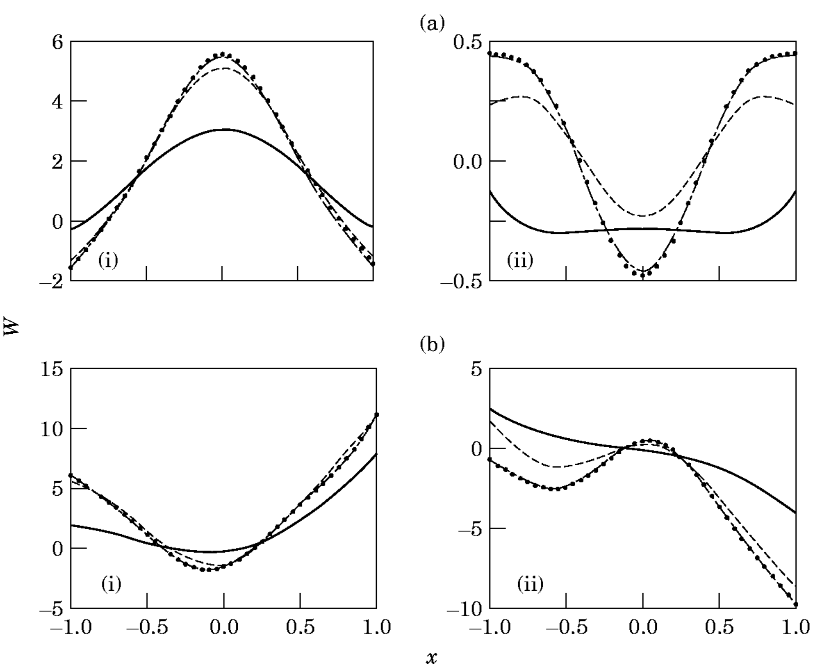

Figure 3. The real (a, i and b, i) and imaginary (a, ii and b, ii) parts of the displacement function (W) along the x-axis for a 2×2 square thin plate subject to a pressure $p_0 = \delta(\mathbf{P} - \mathbf{P}_0)$, where $\mathbf{P}_0 = (0, 0)$ (a, i and a, ii) and $\mathbf{P}_0 = (0, 1)$ (b, i and b, ii). $n = 400$ and $I = J = 4$ (——), $I = J = 6$ (- - - -), $I = J = 8$ (– - – - –) and $I = J = 10$ (. . . . . .). $\alpha = \pi$, $\beta = 0{\cdot}005$ and $\gamma = 0{\cdot}01$.

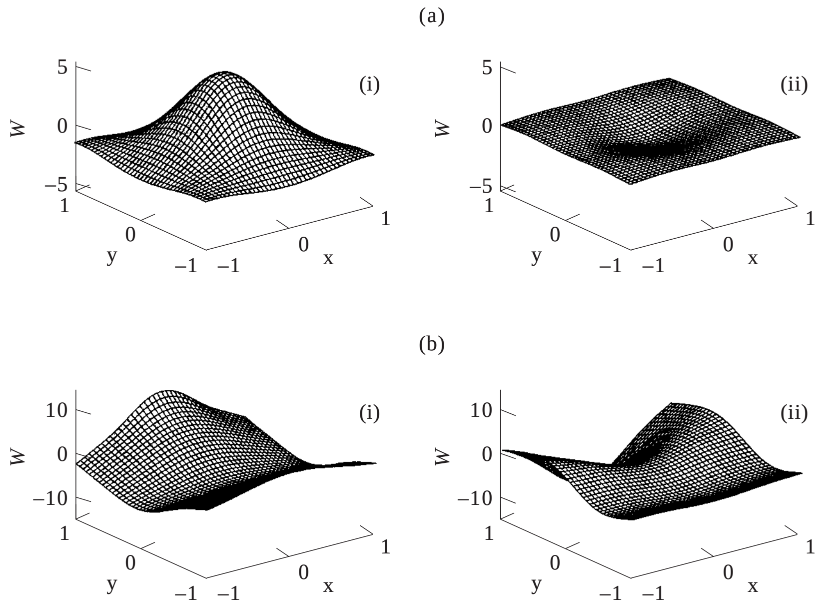

Figure 4. The real (a, i and b, i) and imaginary (a, ii and b, ii) parts of the displacement function (W) for a 2×2 square thin plate subject to a pressure $p_0 = \delta(\mathbf{P} - \mathbf{P}_0)$, where $\mathbf{P}_0 = (0, 0)$ (a, i and a, ii) and $\mathbf{P}_0 = (0, 1)$ (b, i and b, ii). $\alpha = \pi$, $\beta = 0{\cdot}005$ and $\gamma = 0{\cdot}01$.

$I = J = 6$ (dashed line), $I = J = 8$ (chained line) and $I = J = 10$ (dotted line). Again, the solution shows convergence, and we can conclude that $I = J = 8$ are sufficient basis functions for this geometry.

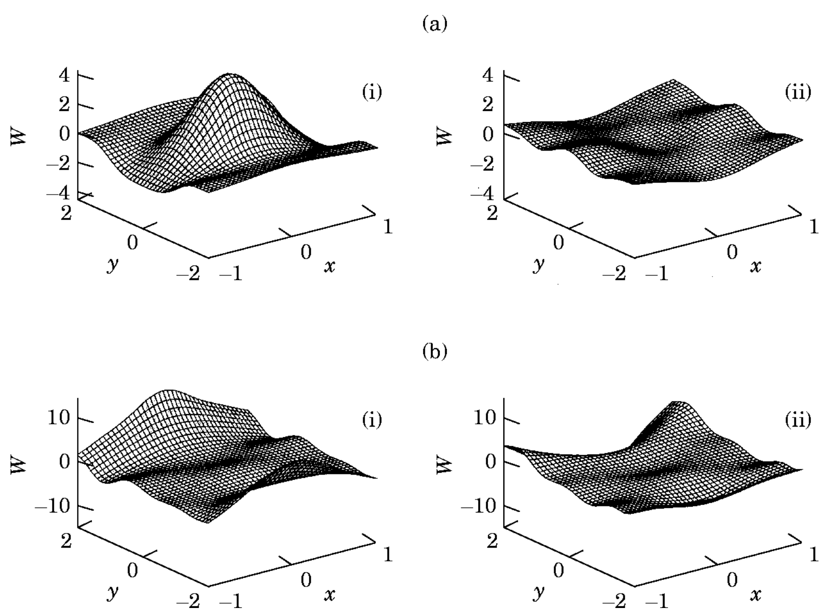

Figure 5. The real (a, i and b, i) and imaginary (a, ii and b, ii) parts of the displacement function (W) for a 2×4 rectangular thin plate subject to a pressure $p_0 = \delta(\mathbf{P} - \mathbf{P}_0)$, where $\mathbf{P}_0 = (0, 0)$ (a, i and a, ii) and $\mathbf{P}_0 = (0, 2)$ (b, i and b, ii). $\alpha = \pi$, $\beta = 0{\cdot}005$ and $\gamma = 0{\cdot}01$.

In Figures 4–6 $\alpha = \pi$, $\beta = 0{\cdot}005$ and $\gamma = 0{\cdot}01$. For the 2×2 square plate $I = J = 8$ and the plate is divided into 400 panels. For the 2×4 rectangular plate $I = J = 10$ and the plate is divided into 900 panels. The displacement W is a complex function and it is the real part of $W\,e^{-it\sqrt{\alpha}}$, which represents the physical displacement as a function of time and therefore we plot the real and imaginary parts of W. In Figures 4 and 5 p_0 is chosen to be a delta function pressure at a point $\mathbf{P}_0$; i.e., $p_0 = \delta(\mathbf{P} - \mathbf{P}_0)$. Figure 4 is a plot of the real (a, i and b, i) and imaginary (a, ii and b, ii) parts of W for 2×2 square thin plate with $\mathbf{P}_0 = (0, 0)$ (a, i and a, ii) and $\mathbf{P}_0 = (0, 1)$ (b, i and b, ii). Figure 5 is the same as Figure 4, except the plate is a 2×4 rectangular plate, and $\mathbf{P}_0 = (0, 0)$ (a, i and a, ii) and $\mathbf{P}_0 = (0, 2)$ (b, i and b, ii). As would be expected, the induced motion is complicated, especially for the larger plate, and the position of the forcing pressure has a significant effect on the plate motion. Figure 6 is a plot of the real (a, i and b, i) and imaginary (a, ii and b, ii) parts of W for a uniform pressure $p_0 = 1$. Again, a complicated motion is induced in the plate.

Having solved for a harmonic forcing pressure it is straightforward to solve for a more complicated pressure using a spectral method. If we denote the time dependent forcing pressure by,

$$p_f(\mathbf{P}, t) = p_0(\mathbf{P}) f(t),$$

then the plate displacement in Fourier space is given by

$$W(\mathbf{P}, \sqrt{\alpha}) = W_0(\mathbf{P}, \sqrt{\alpha}) F(\sqrt{\alpha}), \tag{16}$$

where $W_0(\mathbf{P}, \sqrt{\alpha})$ is the plate displacement due to the forcing pressure $p_0(\mathbf{P})$ at radial frequency $\sqrt{\alpha}$ and $F(\sqrt{\alpha})$ is the Fourier transform of $f(t)$. The time dependent motion is then calculated by taking the inverse Fourier transform of equation (16). In Figure 7 is shown the time dependent motion for a 2×4 plate due to an impulsive unit forcing

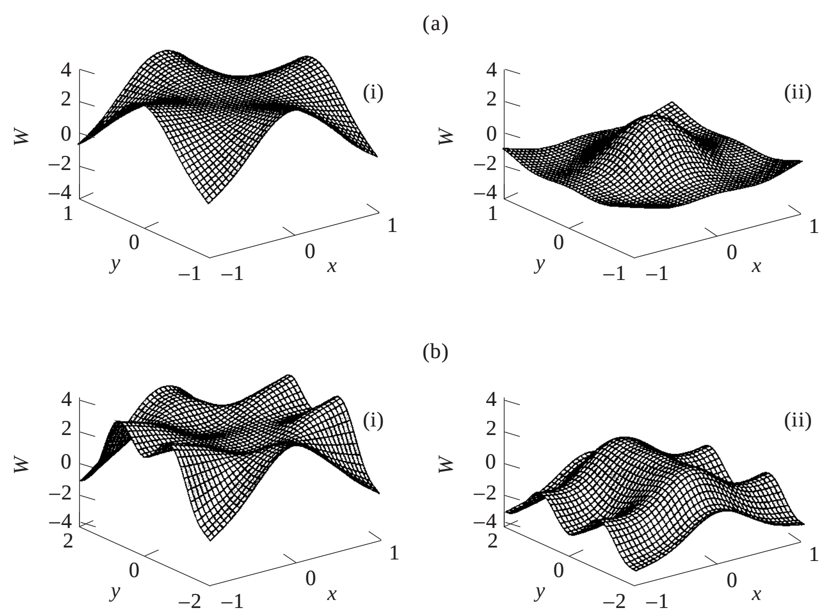

Figure 6. The real (a, i and b, i) and imaginary (a, ii and b, ii) parts of the displacement function (W) for a 2×2 (a, i and a, ii) and a 2×4 (b, i and b, ii) thin plate subject to a pressure $p_0 = 1$. $\alpha = \pi$, $\beta = 0{\cdot}005$ and $\gamma = 0{\cdot}01$.

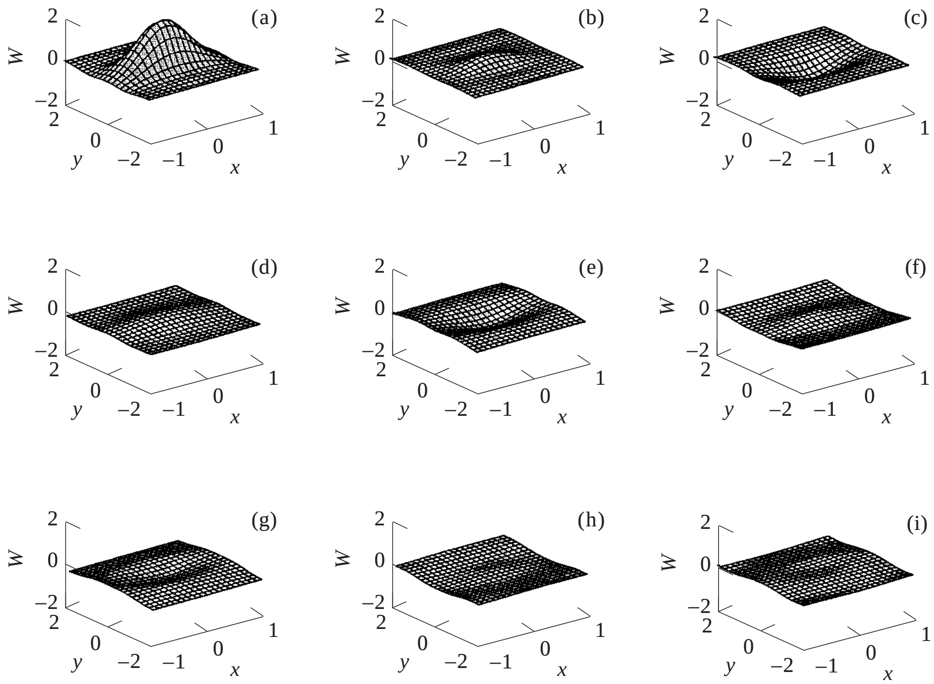

Figure 7. The time evolution of the displacement of 2 × 4 thin plate with $\beta = 0{\cdot}005$ and $\gamma = 0{\cdot}01$ due to an impulsive forcing pressure $p_f(\mathbf{P}, t) = \delta(\mathbf{P} - \mathbf{P}_0)\delta(t)$, $\mathbf{P}_0 = (0, 0)$. (a) $t = 0$; (b) $t = 0{\cdot}44$; (c) $t = 1{\cdot}33$; (d) $t = 3{\cdot}10$; (e) $t = 4{\cdot}87$; (f) $t = 6{\cdot}65$; (g) $t = 8{\cdot}42$; (h) $t = 10{\cdot}2$; (i) $t = 12{\cdot}0$.

pressure at $\mathbf{P}_0 = (0, 0)$; i.e., $p_f = \delta(\mathbf{P})\delta(t)$. The spectrum is windowed with a Blackman window to reduce the effect of truncating the frequency at some finite maximum. The displacement function W_0 is calculated at 65 frequencies linearly space from $\sqrt{\alpha} = 0$ to $\sqrt{4\pi}$, $I = 6$, $J = 10$ and the plate is divided into 400 panels. The decay and consequent oscillation of the plate displacement is clearly apparent.

7. CONCLUSIONS

We have presented a solution for the forced vibration of an arbitrary thin plate floating on the surface of an infinite liquid without making any assumptions about the plate or liquid motion beyond linearizing the equations. A variational equation which governs the problem was derived for the case of spatially arbitrary but temporally harmonic forcing pressure. A solution of this variational equation using the Rayleigh–Ritz method was then derived. Solutions for a square and rectangular plate geometry have been presented as an example of the calculations which are possible.

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