Journal of Sound and Vibration (1997) **206**(1), 109–113

VIBRATIONS OF AN ORTHOTROPIC RECTANGULAR PLATE WITH A FREE EDGE IN THE CASE OF DISCONTINUOUSLY VARYING THICKNESS

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(*Received 7 April* 1997)

1. INTRODUCTION

Anisotropic structural elements play a fundamental role in several fields of applied science and technology: geophysics, mining, solid state physics, etc. A particular case of anisotropy is orthotropic material, which when dealing with a plate structural element, requires the knowledge of four mechanical parameters. An important chapter of structural mechanics is that of artificially induced orthotropic characteristics and this situation takes place with corrugated plates and membranes, ribbed shells and plates, etc. [1].

The present study deals with the determination of the fundamental frequency of transverse vibration of the system shown in Figure 1 and which presents two complicating features: the presence of the free edge at $x = a$, and a discontinuous variation of the thickness at $x = c$.

The fundamental frequency of vibration is determined approximating the mode shape by means of sinusoidal terms in the x-direction which contain optimization parameters in their argument which allow for minimization of the eigenvalue when employing the Rayleigh–Ritz method [2]. An independent solution is obtained using the finite element method [3, 4]. The agreement, in the case of the fundamental frequency coefficient, is excellent when comparing the F.E. results with those obtained analytically.

2. APPROXIMATE ANALYTICAL SOLUTION

Based open previous studies [2, 5] it was considered convenient to employ the following co-ordinate functions as approximations for the fundamental mode shape: three simply supported edges, Fig. 1(a),

$$W \simeq W_a = \sum_{j=1}^{J} A_j \sin \frac{\pi x}{\gamma_j a} \sin \frac{\pi y}{b} \qquad \gamma_1 > 1, \tag{1}$$

and three clamped edges, Fig. 1(b),

$$W \simeq W_a = \sum_{j=1}^{J} A_j \sin^2 \frac{\pi x}{\gamma_j a} \left(\frac{y^2}{b^2} - 2\frac{y^3}{b^3} + \frac{y^4}{b^4} \right) \qquad \gamma_1 > 1. \tag{2}$$

Following reference [6] one expresses the governing function in the form

$$J[W] = \frac{1}{2} \iint \left[D_1 \left(\frac{\partial^2 W}{\partial x^2} \right)^2 + 2D_1 \nu_2 \frac{\partial^2 W}{\partial x^2} \frac{\partial^2 W}{\partial y^2} + D_2 \left(\frac{\partial^2 W}{\partial y^2} \right)^2 \right.$$

0022–460X/97/360109 + 05 $25.00/0/sv971042

TABLE 1

Fundamental frequency coefficients $\sqrt{\rho h/D}\omega_1 a^2$ *of the isotropic plate shown in Figure* 1(*a*).

		h_2/h_1: 4/5			3/5			
		c/a						
a/b		4/5	1/2	1/5	4/5	1/2	1/5	Uniform thickness
1	FE	11·037	10·185	9·634	10·698	8·824	7·609	11·685
	[7]	11·127	10·283	9·711	10·875	9·268	7·846	11·79
	Eq. (1)	11·040	10·202	9·650	10·701	8·854	7·630	11·687
$a/b = 2/5$	FE	2·948	2·786	2·571	2·966	2·685	2·204	3·008
	Eq. (1)	2·948	2·787	2·572	2·966	2·688	2·209	3·008

$$+4D_k\left(\frac{\partial^2 W}{\partial x \partial y}\right)^2\Bigg]\,\mathrm{d}x\,\mathrm{d}y - \frac{\rho\omega^2}{2}\iint hW^2\,\mathrm{d}x\,\mathrm{d}y. \tag{3}$$

Substituting the approximating function in equation (3) and minimizing $J[W]$ with respect to the A_j's results in a homogeneous, linear system of equations in the A_j's. The non-triviality condition yields a secular determinant whose lowest root constitutes the fundamental frequency coefficient $\Omega_1 = \sqrt{\rho h/D_1}\omega_1 a^2$. Since

$$\Omega_1 = \Omega_1(\gamma_1, \gamma_2, \ldots, \gamma_j), \tag{4}$$

minimizing Ω_1 with respect to the γ_j's one obtains an optimized value of Ω_1. In the present study $J = 3$.

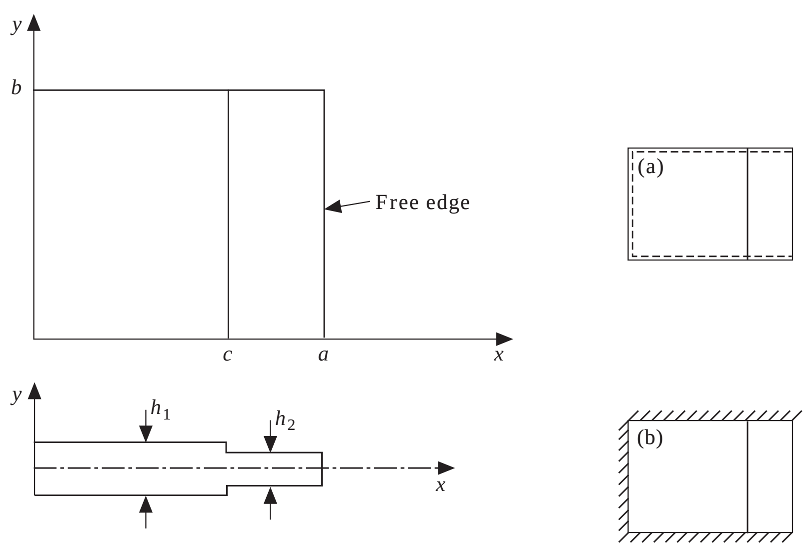

Figure 1. Rectangular plates of discontinuously varying thickness executing transverse vibrations. Case (a): three edges are simply supported, the fourth is free. Case (b): three edges are clamped.

TABLE 2

Fundamental frequency coefficients $\sqrt{\rho h/D}\omega_1 a^2$ *of the isotropic plate shown in Figure* 1(*b*)

		h_2/h_1						
		4/5			3/5			
		c/a						
a/b		4/5	1/2	1/5	4/5	1/2	1/5	Uniform thickness
1	FE	21·873	19·966	19·368	20·405	16·014	14·740	23·922
	[7]	22·069	20·103	19·525	20·931	16·583	14·978	24·20
	Eq. (2)	21·963	20·052	19·465	20·506	16·136	14·859	24·035
$a/b = 2/5$	FE	5·903	5·647	5·227	5·983	5·399	4·257	6·010
	Eq. (2)	5·921	5·665	5·269	6·004	5·484	4·399	6·030

3. FINITE ELEMENT SOLUTION

The present paper makes use of the orthotropic plate element developed in reference [4] which is an extension of the well known isotropic plate element due to Bogner *et al*. [3]. In the case of a square plate ($a/b = 1$) one half of the plate has been subdivided into 200 square elements resulting in 231 nodes. For the plate simply supported along three sides it turns out that when half of the plate is considered, 820 degrees of freedom are generated. On the other hand, when the plate is clamped along three sides, a similar modelling yields 760 degrees of freedom. Another configuration studied was a rectangular plate of aspect ratio $a/b = 2/5$. One half of the structure was subdivided into 160 rectangular elements resulting in 672 degrees of freedom when the plate was simply supported at three edges and 620 degrees of freedom when three edges were clamped.

4. NUMERICAL RESULTS

The previously described analytical and numerical techniques were applied first to an isotropic plate taking Poisson's ratio equal to 0·3. The results are shown in Table 1 in the

TABLE 3

Fundamental frequency coefficients $\sqrt{\rho h/D_1}\omega_1 a^2$ *of the orthotropic plate shown in Figure* 1(*a*). ($D_2/D_1 = 1/2,\ D_k D_1 = 1/3,\ \nu_2 = 0{\cdot}30$)

		h_2/h_1						
		4/5			3/5			
		c/a						
a/b		4/5	1/2	1/5	4/5	1/2	1/5	Uniform thickness
1	FE	8·881	8·266	7·743	8·721	7·401	6·249	9·267
	Eq. (1)	8·884	8·288	7·765	8·725	7·427	6·272	9·270
2/5	Fe	2·705	2·570	2·359	2·741	2·516	2·053	2·736
	Eq. (1)	2·706	2·572	2·361	2·741	2·520	2·057	2·736

Table 4

Fundamental frequency coefficients $\sqrt{\rho h/D_1}\omega_1 a^2$ *of the orthotropic plate shown in Figure* 1(*b*). ($D_2/D_1 = 1/2$, $D_k D_1 = 1/3$, $\nu_2 = 0{\cdot}30$)

		h_2/h_1						
		4/5			3/5			
		c/a						
a/b		4/5	1/2	1/5	4/5	1/2	1/5	Uniform thickness
1	FE	16·541	15·143	14·571	15·717	12·504	11·200	17·849
	Eq. (2)	16·632	15·230	14·667	15·806	12·618	11·336	17·963
2/5	FE	5·384	5·201	4·765	5·531	5·107	3·934	5·387
	Eq. (2)	5·413	5·229	4·819	5·562	5·204	4·093	5·419

case of the configuration shown in Figure 1(a) and in Table 2 when three edges are clamped.

The agreement between the finite element results and the eigenvalues determined using the analytical approximations (1) and (2) is excellent from an applied engineering viewpoint. On the other hand these analytical predictions are better than those obtained using the polynomial approximations which have been presented in reference [7]. In all cases the fundamental eigenvalues have been truncated after the third decimal figure.

Tables 3 and 4 deal with orthotropic plates subjected to the boundary conditions depicted in Figures 1(a) and 1(b), respectively. A hypothetical material has been assumed such that $D_2/D_1 = 1/2$, $D_k/D_1 = 1/3$ and $\nu_2 = 0{\cdot}30$. The agreement between the results obtained by means of the finite element algorithmic procedure and those determined using the optimized Rayleigh–Ritz method and the "Pseudo" Fourier expansions (1) and (2) is excellent for all the situations considered. In summary, the simple analytical approach presented in this study seems quite convenient and accurate for investigating the rather complex elastodynamics problem defined in this paper.

ACKNOWLEDGMENTS

The present study has been sponsored by CONICET Research and Development Program (PIA 6002/96) and by Comisión de Investigaciones Cientificas (Buenos Aires Province).

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