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LETTERS TO THE EDITOR

A BOUNDARY ELEMENT APPROACH TO SOUND TRANSMISSION/RADIATION PROBLEMS

Z.-L. Ji and J.-Z. Sha

Institute of Acoustics, Nanjing University, Nanjing 210093, *People's Republic of China*

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1. INTRODUCTION

In a complete acoustic analysis of sound transmission/radiation problems, it is common to use the radiation impedance of a piston as the boundary condition at the opening; then the sound field in the interior and exterior space may be calculated. However, the above approach is valid at sufficiently low frequencies only, and quite restrictive, because one must know a suitable impedance to represent the opening, but the radiation impedance is unknown in most cases of practical interest.

It has been demonstrated that the coupled boundary element method (BEM) is applicable to solution of sound transmission/radiation problems [1]. However, this approach has the disadvantage that too much computation time is consumed on solving the final set of simultaneous algebraic equation. In this letter, a new approach is proposed for solution of sound transmission/radiation problems, and it can save computation time in comparison with the coupled BEM.

2. APPROACH TO SOUND TRANSMISSION/RADIATION PROBLEM

For a sound transmission/radiation system, the sound field can be divided into interior and exterior acoustic domains separated by the opening, as depicted in Figure 1.

By using the BEM, the following algebraic equation can be obtained for the boundary points of the interior acoustic domain:

$$[H_S \quad H_W \quad H_O]\begin{bmatrix} P_S \\ P_W \\ P_O \end{bmatrix} = [G_S \quad G_W \quad G_O]\begin{bmatrix} V_S \\ V_W \\ V_O \end{bmatrix}. \tag{1}$$

Here P and V are the sound pressure and the normal particle velocity vectors, H and G are the coefficient matrices, and the subscripts S, W and O denote corresponding quantities on the source, wall and opening boundaries, respectively.

The acoustic boundary condition on the wall can be expressed as

$$[V_W] = [A_W][P_W], \tag{2}$$

where A_W is a diagonal matrix, the element of which is the acoustic admittance on the wall.

Substituting equation (2) into equation (1), yields

$$\begin{bmatrix} P_S \\ P_W \\ P_O \end{bmatrix} = [H_S \quad H_W - A_W G_W \quad H_O]^{-1}[G_S \quad G_O]\begin{bmatrix} V_S \\ \\ V_O \end{bmatrix}. \tag{3}$$

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Thereby, the following equation can be obtained:

$$\begin{bmatrix} P_S \\ P_O \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} V_S \\ V_O \end{bmatrix}. \tag{4}$$

Here T is called the transmission impedance matrix between the source and opening. For an acoustic system with complicated geometry, the substructure boundary element/transfer impedance matrix technique [2] can be used to obtain the transmission impedance matrix for the entire acoustic system.

For the exterior acoustic domain shown in Figure 1, the following equation can be obtained by the BEM:

$$[H'_O \quad H'_W] \begin{bmatrix} P'_O \\ P'_W \end{bmatrix} = [G'_O \quad G'_W] \begin{bmatrix} V'_O \\ V'_W \end{bmatrix}. \tag{5}$$

Here P' and V' are the sound pressure and the normal particle velocity vectors, H' and G' are the coefficient matrices, and the subscripts O and W denote the contributions from the opening and wall boundaires.

If wall is rigid, one can obtain

$$\begin{bmatrix} \mathrm{P}'_O \\ P'_W \end{bmatrix} = [H'_O \quad H'_W]^{-1} [G'_O][V'_O]. \tag{6}$$

and hence

$$[\mathrm{P}'_O] = [Z_O][\mathrm{V}'_O], \tag{7}$$

where Z_O is called the sound radiation impedance matrix at the opening.

At the opening, the solution should satisfy the continuity conditions of sound pressure and particle velocity:

$$[P_O] = [P'_O], \qquad [V_O] = -[V'_O], \tag{8, 9}$$

From equations (4) and (7), as well as the continuity conditions (8) and (9), the following equations are derived:

$$[P_S] = [T_{11} - T_{12}(T_{22} + Z_O)^{-1} T_{21}][V_S], \tag{10}$$

$$[P_O] = [Z_O][T_{22} + Z_O]^{-1}[T_{21}][V_S], \tag{11}$$

$$[V_O] = -[T_{22} + Z_O]^{-1}[T_{21}][V_S]. \tag{12}$$

Evidenently, all variables on the source and opening boundaries can be found if the source velocity is known.

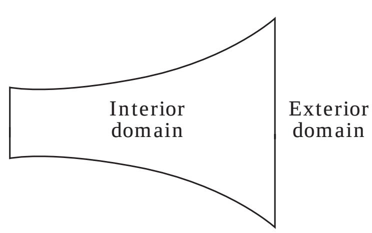

Figure 1. A sound transmission/radiation system opening into infinite space.

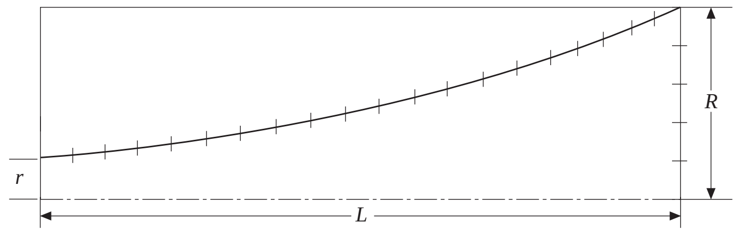

Figure 2. The exponential duct and discretization of the generator. $r = 0{\cdot}10$, $R = 0{\cdot}488$, $L = 1{\cdot}50$ (dimensions in m).

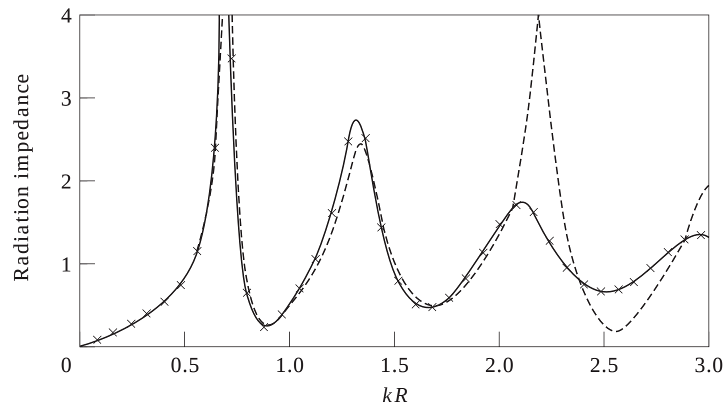

Figure 3. The calculated results for the radiation impedance of the piston. ——, Present approach; xx, coupled BEM; ---, 1-D theory.

3. COMPUTATIONAL EFFICIENCY

With the BEM, the computer time is consumed mainly on matrix formation and equation solution. The present approach and the coupled BEM consume the same time on the matrix formation, but the present approach can save computation time on the equation solution. The detailed analysis is as given below.

It is known that Gauss elimination and Gauss–Jordan elimination are the most effective procedures for solving the system of linear equations and evaluating the inverse matrix, and the computation times for them depend directly on $n^3/3$ and n^3, approximately (n is the dimension of the equation system or matrix), respectively, if n is greater. Consider a situation in which the numbers of interior and exterior nodes are N and M, respectively. When the coupled BEM is used, the dimension of the final simultaneous equation system is $(N + M)$, and the computation time consumed on the equation solution depends directly on $(N + M)^3/3$, approximately. When the present approach is used, the computation time will be consumed mainly on evaluating the inverse matrices in equations (3) and (6), and depends directly on $(N^3 + M^3)$, approximately, if the number of nodes on the opening boundary is much smaller than N and M. Clearly, the present approach can save computational time on the equation solution in comparison with the coupled BEM when $N/2 < M < 2N$. The optimum case occurs when $N = M$, in which case the equation solution time will be reduced by a factor of approximately 4/3.

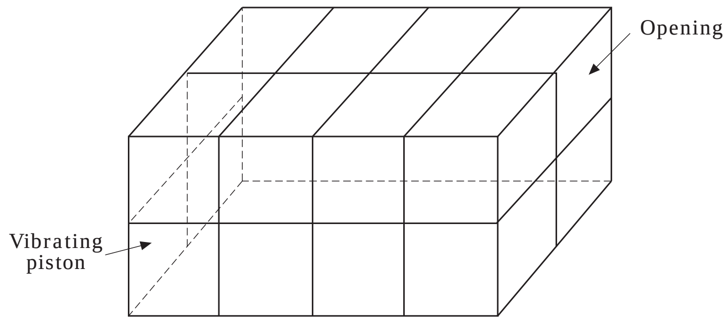

Figure 4. The rectangular chamber and discretization.

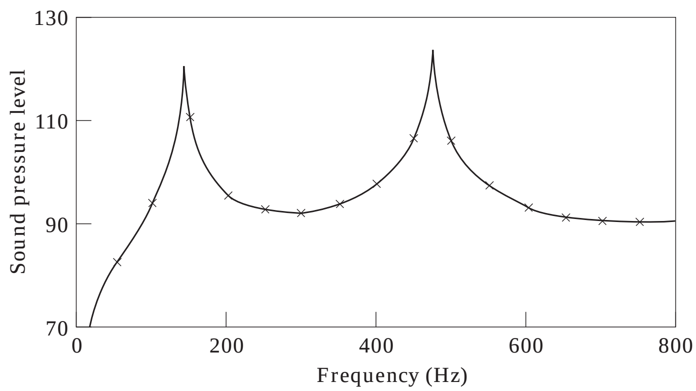

Figure 5. The calculated results for the sound pressure at the opening. ——, Present approach; xx, coupled BEM.

4. NUMERICAL EXAMPLES

In Figure 2 is shown an exponential duct that is excited by a plane piston at the left interior end and open to infinite space at the right end. The present approach and the coupled BEM were used to calculate the radiation impedance of the vibrating piston, and the calculated results are shown in Figure 3. Excellent agreement between the results obtained by the two methods is observed. This agreement confirms that the present approach and formulation are valid. To examine the effect of a non-planar wave, the results calculated from the plane wave theory by using the radiation impedance of a piston as the boundary condition at the opening is also given [3]. From Figure 3, it may be seen that the effect of non-planar wave is observable.

To verify the computational efficiency of the present approach, the example for calculating the sound pressure level at the center of the opening for the rectangular

TABLE 1

The computation time(*s*) *for the modal in Figure* 4

Computation time	On matrix formation	On equation solution
Present method	195	190
Coupled method	195	284

chamber in Figure 4 is considered. The sizes of the chamber are 0·4 m × 0·2 × 0·2 m, the vibrating piston is square with the edges 0·1 m × 0·1 m, and the opening is also square and 0·1 m × 0·1 m. The calculated results are shown in Figure 5, and the computation times are given in Table 1. Comparisons of calculated results and computation times show that the present approach has good accuracy and is more efficient than the coupled BEM.

5. CONCLUSIONS

The present approach is suitable for acoustic systems with any complicated geometry, and overcomes excess computation time problems in the solution of the large set of simultaneous equations when the coupled BEM is applied to this class problem. The agreement between the present approach and coupled BEM results is excellent for the examples used; this demonstrates that the present approach can be used to obtain accurate results for sound transmission/radiation system analysis.

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