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FREE VIBRATION OF AN ANNULAR PLATE WITH PERIODIC RADIAL CRACKS

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1. INTRODUCTION

The increasing application of fracture mechanics concepts in design has prompted researchers to gain an understanding of the dynamic behaviour of structures with cracks or crack-like defects. Extensive studies have been reported on cracked rotors [1]. The study of the dynamic behaviour of annular plates with periodic radial cracks is important, as several machine components, such as flywheels, clutch plates, etc., with cracks can be considered as annular plates for the purpose of analysis. In this note, an experimental investigation on the effects of the number and length of periodic radial cracks on the natural frequencies of an annular plate is reported. The inner edge of the plate is clamped, while the outer edge is kept free.

2. EXPERIMENTAL DETAILS

In Figure 1 are shown the details of the annular plate, along with the specialized mounting to simulate the clamped end conditions at the inner boundary. The annular plate is cut from a rolled aluminium sheet of 3·18 mm thickness. The effective inner diameter of the plate is 60 mm and the outer diameter is 260 mm. Annular plates with cracks emanating from the inner boundary as well as the outer boundary are investigated for various crack lengths (25, 40, 60 and 75 mm). The cracks are simulated by milling thin slots of 0·5 mm width and the slot is finally finished to 0·35 mm width at the crack tip using a saw. For cracks emanating from the inner boundary, experiments are carried out with four, six, eight and twelve cracks, whereas for cracks emanating from the outer boundary the results are presented for only six cracks.

Natural frequencies are detected in two ways. In the first, a rap test is performed by hitting the plate with a hammer (B&K 8202). The response at a point of the plate is measured using an accelerometer (B&K 4374). The output of the accelerometer is amplified by a charge amplifier (B&K 2635) and is analyzed using a spectrum analyzer (HP 3582A). In the second method, the clamped disk is mounted on an IMV electrodynamic shaker and different resonances are detected by varying the exciting frequency. As before, the response is measured by an accelerometer.

The fixed end condition at the inner boundary is effected by an annular ring pressed by a set of six clamping bolts (see Figure 1). Sufficient care is exercised to tighten these bolts uniformly so that the fixed end condition is closely simulated. The cracked plates are experimented with at different orientations of the cracks with respect to the clamping bolts to see whether the relative orientation of the cracks with the clamping bolts affects the results. The change in resonance frequencies is found to be of the order of 1–2 Hz, which is very small.

Apart from recording the natural frequencies, nodal patterns are also recorded by sprinkling sand particles uniformly over the specimen. Initially, experiments are conducted

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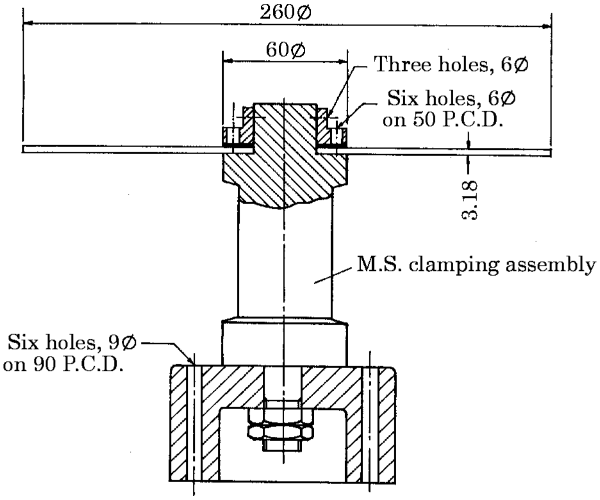


Figure 1. The clamping details of the annular plate.

for an annular plate without cracks. In Table 1 is shown a comparison of the experimental results obtained for an uncracked annular plate with the analytical results reported by Vogal and Skinner [2]. Since a perfectly clamped inner edge, assumed in the theoretical analysis, provides a higher stiffness than can be achieved in practice, the experimental frequencies have turned out to be lower than the theoretical values.

An uncracked annular plate is an axisymmetric structure. However, when periodic radial cracks are introduced, it becomes a cyclically symmetric structure. With axisymmetric structures, it is known that most modes of vibration occur in degenerate orthogonal pairs. The possible mode shapes in a cyclically symmetric structure fall into three classes, depending on the relationship between the shapes for individual substructures [3]. These are as follows: (a) that each substructure has the same mode shape as its neighbours; (b) that each substructure has the same mode shape as its neighbours, but is vibrating in antiphase with them; and (c) all other possible mode shapes. Modes of class (a) and (b) do not exhibit degeneracy, except for any "accidental" degeneracy that may occur if an unrelated mode shape is associated with the same natural frequency. All of the mode shapes that fall in category (c) exhibit degeneracy. Mallik and Mead [4] observed that even for a one-dimensional closed structure such as a ring that is supported periodically, the degenerate modes exist. They verified their analytical development by conducting

TABLE 1

A comparison of theoretical and experimental resonance frequencies for an annular plate

Mode (nodal diameter, nodal circle)	Resonance frequencies (Hz)		
	Analytical (reference [2])	Experimental, ω_u	% difference
(1, 0)	231·8	225	2·85
(0, 0)	262·6	256	2·51
(2, 0)	320·6	307	4·24
(3, 0)	594·9	590	0·83
(0, 1)	1730·8	1567	9·46

TABLE 2

Resonance frequencies for an annular plate with cracks emanating from the inner boundary. The table summarises the results from both the shaker table test and the rap test. The mode shapes observed are shown in Figures 2(*a*)–2(*d*), *and the figure sequence* (*i*), (*ii*), (*iii*) *etc. represents the sequence of frequencies observed in the shaker table test*

Number of cracks	Mode number	Mode type	Crack length (mm)			
			25	40	60	75
4	1	(1, 0)	209	198	199	195
	2	—	225*	209*	212*	208*
	3	(0, 0)	256*	241*	234*	225*
	4	(2, 0)	300	294	293	292
	5	(3, 0)	590	583	577	567
	6	—	—	—	577	—
	7	—	—	—	995†	956†
	8	(0, 1)	1560	1549	1543	1525
	9	—	—	—	1559†	—
6	1	(1, 0)	210*	205	190	188
	2	—	—	—	208*	—
	3	(0, 0)	245	234*	223	203
	4	(2, 0)	301	293	292	300
	5	—	—	—	303*	—
	6	(3, 0)	586*	582*	582*	580
	7	(0, 1)	1552	1544	1534	1506
	8	—	—	1560†	1550†	1540†
8	1	(1, 0)	210	204	187	184
	2	(0, 0)	239	224	208	201
	3	(2, 0)	301	291	287	286
	4	—	—	—	298*	—
	5	(3, 0)	586*	579*	566	539*
	6	(0, 1)	1540	1513	1509	1487
12	1	(1, 0)	208	200	193	180
	2	(0, 0)	234	220	202	190
	3	(2, 0)	293	299	286	281
	4	—	—	—	—	290*
	5	(3, 0)	585	578	556	525
	6	—	—	—	—	996†
	7	(0, 1)	1525	1529	1507	1456

* Observed only in a rap test.
† Observed only in the shaker table test.

experiments on a ring supported by three radial supports. For a degenerate mode, although theoretically one has to observe two different modes for the same value of frequency, experimentally Mallik and Mead observed two modes (one symmetric and the other antisymmetric) at slightly different frequencies. This is referred to as a split in the resonance frequency and is attributed to the fact that, in an experimental situation, the structure may not be perfectly cyclically symmetric.

The natural frequencies for an annular plate with cracks emanating from the inner boundary are shown in Table 2. The table shows the possible mode classification in terms of nodal diameters and nodal circles as in the case of an annular plate without cracks. Although this classification is strictly not correct, nevertheless it offers a convenient standpoint for discussion of the results.

TABLE 3

A table of parameters to evaluate ω_λ *for the modes* (1, 0), (0, 0) *and* (2, 0) *for various number of cracks*

Number of cracks	(1, 0)		(0, 0)		(2, 0)	
	ω_s (Hz)	m	ω_s (Hz)	m	ω_s (Hz)	m
4	193·796	3·019	200·758	0·584	291·355	2·612
6	179·623	1·336	174·289	0·571	287·779	1·852
8	171·371	1·129	185·040	1·134	282·378	1·608
12	170·332	1·137	169·650	1·053	281·575	2·531

Most of the modes are observed in both the rap test and the shaker table test. However, a few modes are observed only in a rap test or in the shaker table test, and these are appropriately indicated in the tables. As expected, in Table 2 it is shown that the value of the natural frequency decreases when the number of cracks or the length of the crack is increased. This decrease is more prominent for the umbrella mode (0, 0).

The results reported in Table 2 are meant for specific crack lengths. It is desirable that an empirical relation is obtained from the experimental data, so that the natural frequencies for any arbitrary crack length can be obtained. For any arbitrary non-dimensional crack length λ ($\lambda = l/(b-a)$, where l is the crack length, and b and a are the outer and inner radii of the annular ring), the natural frequency ω_λ, can be represented as,

$$\omega_\lambda = \omega_s + (\omega_u - \omega_s)(1-\lambda)^m, \tag{1}$$

TABLE 4

Resonance frequencies for an annular plate with six periodic radial cracks emanating from the outer boundary. The table summarizes the results from both the shaker table test and the rap test. The mode shapes observed are shown in Figures 3(*a*)–3(*d*) *and the figure sequence* (*i*), (*ii*), (*iii*) *etc. represents the sequence of frequencies observed in the shaker table test*

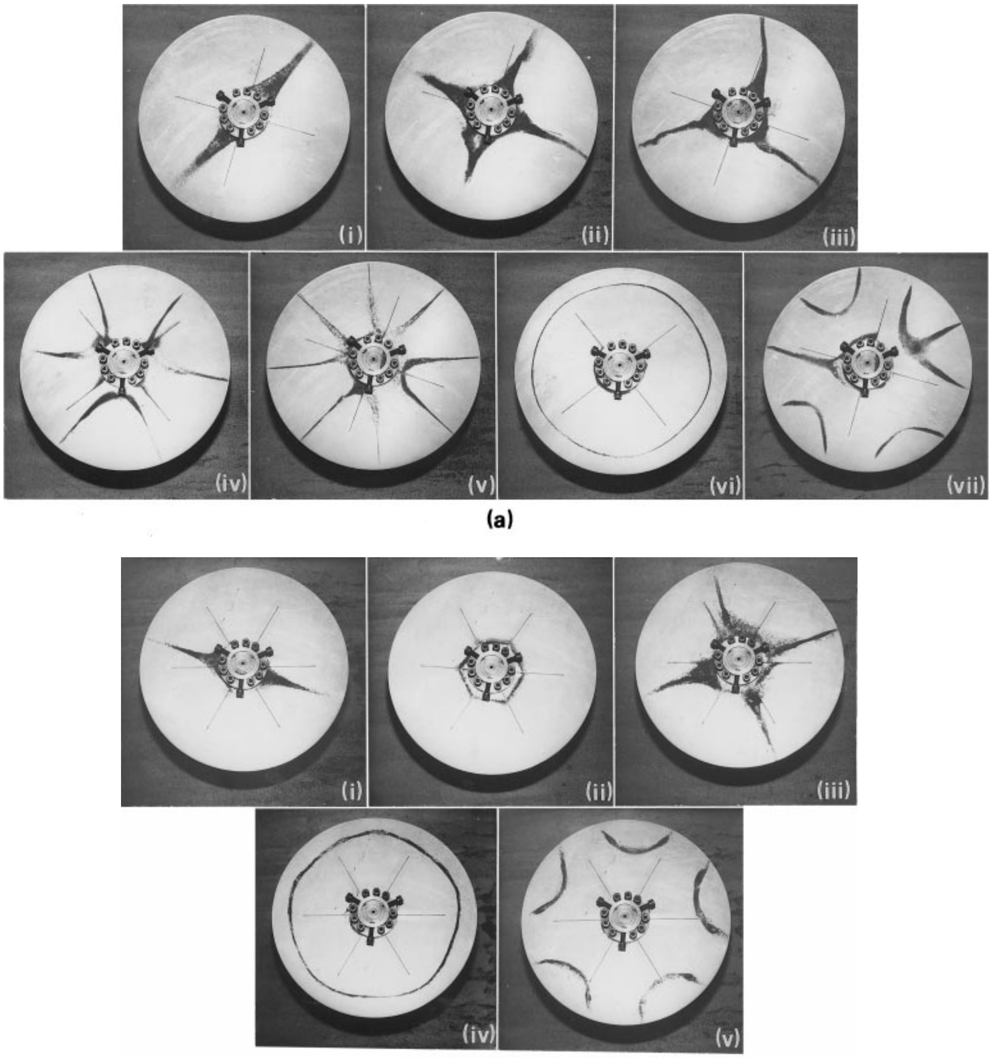
Mode number	Mode type	Crack length (mm)			
		25	40	60	75
1	(1, 0)	205	180	200	201
2	—	226	—	214	—
3	(0, 0)	254	249	230	233
4	(2, 0)	289	—	—	—
5	—	300*	—	—	—
6	(3, 0)	539	491	—	516
7	—	860*	729	1333	1296
8	—	895	993	—	—
9	—	1305	1470	1457	1472
10	—	1330	—	—	1515
11	—	1554	1763	1619	1551
12	—	—	—	1654	—
13	—	1662	—	1699	1706

* Observed only in a rap test.

where ω_u is the natural frequency of the uncracked plate. It can be seen that when $\lambda = 0$, equation (1) gives the frequency corresponding to an uncracked plate (ω_u), and when $\lambda = 1$ it gives the natural frequency of a sector of the plate (ω_s). The parameters, ω_s, representing the natural frequency of the sector of a cyclically cracked plate and m, the exponent of $(1 - \lambda)$, are to be determined from experimental data reported in Table 2.

The parameters ω_s, and m of equation (1), are determined in a least squares sense from the results reported in Table 2 for the modes (1, 0),(0, 0) and (2, 0). The values of these two parameters so obtained are given in Table 3. For all these cases, the experimental value of ω_u is used, and is taken from Table 1 for the respective modes.

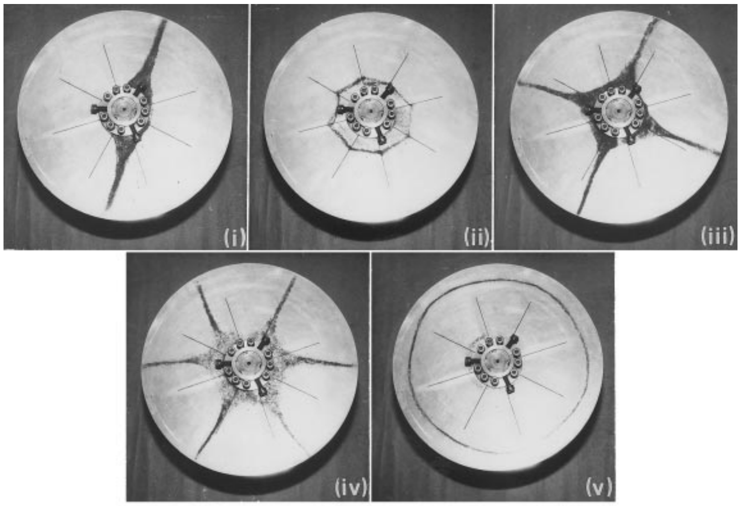
In Figures 2(a)–2(d) are shown the typical nodal patterns observed for a crack of 60 mm length emanating from the inner boundary for four, six, eight and twelve cracks respectively. The frequencies for the figure sequence (i), (ii), (iii) etc. are the ones mentioned in Table 2 corresponding to the shaker table test. Figures 2(a), iii and 2(a), iv represent



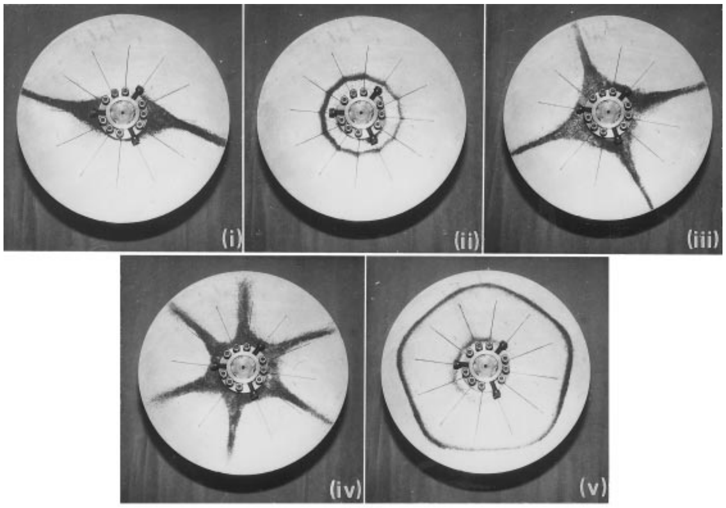
(a)

(b)

Fig. 2. (*Continued—overleaf*)



(c)



(d)

Figure 2. Nodal patterns corresponding to radial cracks of 60 mm length for various number of cracks emanating from the inner boundary: (a) Four cracks; (b) Six cracks; (c) Eight cracks; (d) Twelve cracks.

the nodal patterns for the degenerate mode at the frequency 577 Hz. The result is significant in that, even experimentally, two different nodal patterns could be observed for a single resonance frequency (within the margin of measuremental accuracy). However, the result is more of an exception, as for most of the other degenerate modes a split in the natural frequency is observed, as reported by Mallik and Mead. The split is more readily observed in a rap test than while exciting the plate by a shaker. In most of the experiments, a split is observed for the modes (1, 0) and (2, 0), as these happen to be the degenerate modes. It is to be noted that the mode (2, 0) is a degenerate mode only for plates with cracks above four.

A limited experimental study is also performed for cracks emanating from the outer boundary. In Table 4 are shown the natural frequencies for the case of six cracks emanating from the outer boundary for various crack lengths. The table shows that several new modes are excited and the split in resonance frequencies is observed for a large number of modes. In Figures 3(a)–3(d) are shown typical nodal patterns observed for an annular plate with six periodic cracks for crack lengths of 25, 40, 60 and 75 mm, respectively. In contrast to the case of cracks emanating from the inner boundary, the individual sectors

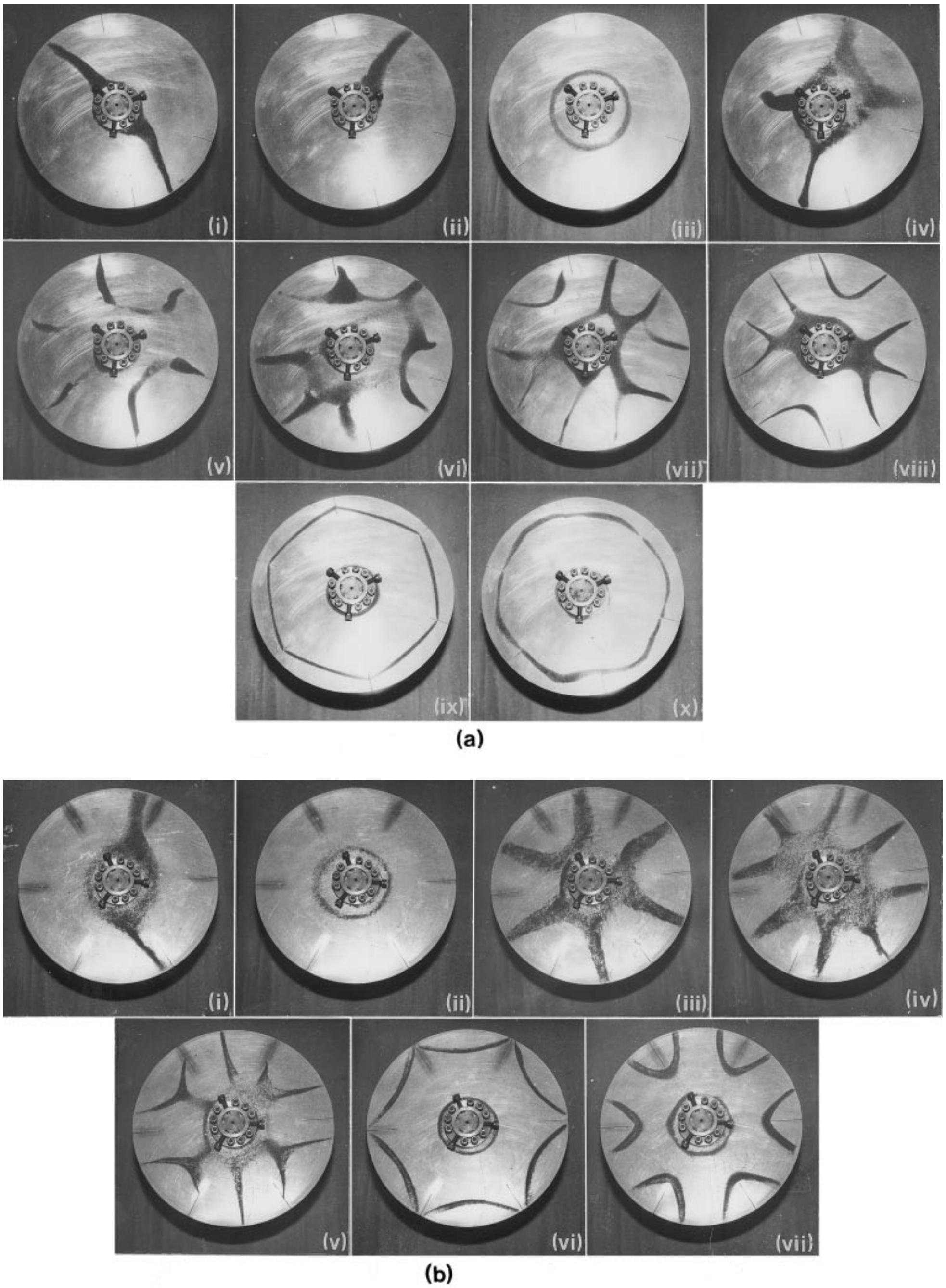


Fig. 3. (*Continued overleaf*)

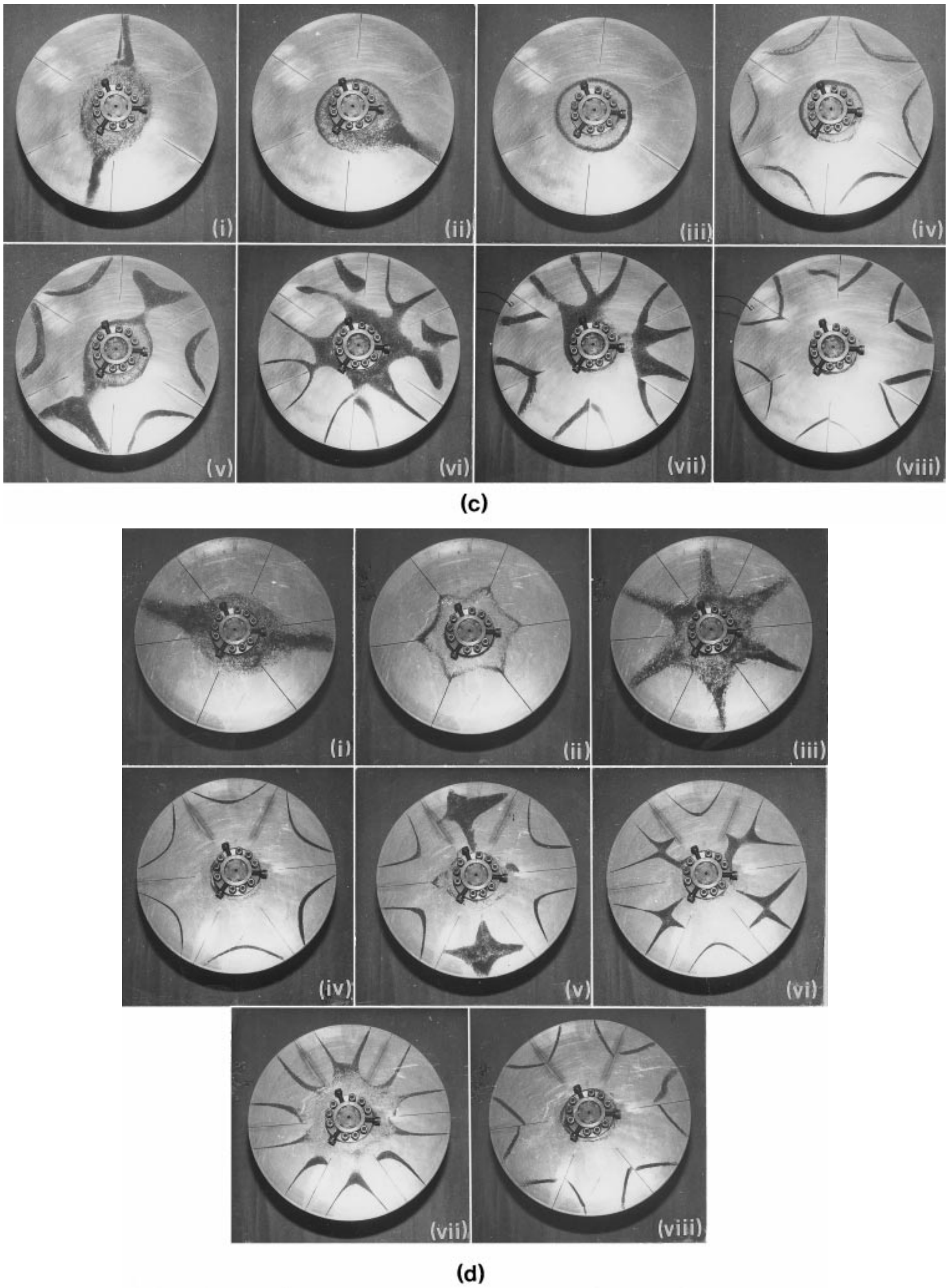


Figure 3. Nodal patterns corresponding to six radial cracks of various crack lengths emanating from the outer boundary: (a) 25 mm; (b) 40 mm; (c) 60 mm; (d) 75 mm.

of the plate (inner boundary is clamped) with radial cracks emanating from the outer boundary are strongly coupled. This has resulted in the excitation of a large number of new modes, and the degenerate modes are also observed more frequently.

3. CONCLUSIONS

In general, the presence of periodic radial cracks in an annular plate introduces additional modes, and these are very significant for the case of cracks emanating from the outer boundary, as the individual sectors are strongly coupled. A split in the resonance frequencies is observed for degenerate modes and these are readily observed for cracks emanating from the outer boundary. Normally, the change in the resonance frequencies is more for circumferential modes than for the diametral modes. The trend is well defined when either the number or the length of the cracks is increased. However, the change in resonance frequencies due to the presence of cracks is rather too small to develop any condition monitoring technique based on this premise. Nevertheless, the result presented in this paper supports the wave propagation concept [4, 5] for analyzing the dynamic behaviour of cyclically symmetric structures.

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