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LETTERS TO THE EDITOR

COMMENTS ON "RAYLEIGH ESTIMATES OF THE FUNDAMENTAL FREQUENCIES OF CIRCULAR PLATES"

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The writers found the study by Pavlović and Mbakogu [1] quite interesting from the point of view of developing an approximate determination of the fundamental frequency of clamped and simply supported circular plates with a central support. This approach will undoubtedly be of great value in the case of plates of non-uniform thickness.

It is important to point out that similar polynomial expressions to the ones used in reference [1] were employed in reference [2] when dealing with circular plates elastically restrained against rotation and subjected to in-plane hydrostatic loading. Actually the eigenvalues presented in Table 1 of reference [1] practically coincide with those shown in Table 1 of reference [2] where the Galerkin method was used which for the problem under consideration is equivalent to the Rayleigh–Ritz approach.

The polynomial approximation has been used extensively in the case of circular plates of non-uniform thickness, orthotropic and aeolotropic plates [3–8]. Use of an undetermined exponent in the co-ordinate functions, following Rayleigh's suggestion, has allowed for optimization of the results [9].

It is also interesting to point out that the polynomial approach has also been used in the case of circular plates with a central support [10].

In the case of a simply supported plate and for Poisson's ratio equal to 0·3 one obtains $\sqrt{\rho h/D}\omega_1 a^2 = 14{\cdot}82$ and when the plate is clamped the methodology yields 22·78 which compares admirably well with the exact results: 14·80 and 22·78, respectively [11].

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We are grateful to the writers for the interest shown in our recent note on fundamental frequency estimates of circular plates with central support [1]. In particular, drawing attention to the two earlier works of Laura *et al*. [2, 3] in similar areas was useful as we were unaware of them despite a copious survey of works on the vibration of circular plates. This is not surprising in view of the vast—and still rapidly increasing—data on the natural vibration response of solids, and one often finds closely related articles (and results) by different authors working independently as, for instance, the approximate analytical approach for a vibrating beam partially embedded in Winkler-type foundations [4] which was obviously developed without knowing about the existence of the exact solution to this problem published earlier [5, 6].

Having now studied references [2] and [3], we still find several important features in which these works differ from our own, especially as regards the original aim and the approach adopted. Although some of these features should be self-evident, it is, in the present circumstances, clearly necessary to list them below.

First, the writers mention the use of the Galerkin method and the so-called Rayleigh–Ritz (strictly speaking, Ritz) technique. Now, both these approaches are based on approximations of deflected shapes in the form of series and, hence, lead to a system of simultaneous equations, the size of which depends on the number of terms retained in the (truncated) series. On the other hand, our article was based on the much simpler quotient of Rayleigh, obviating the need to solve simultaneous equations.

Secondly, unlike the writers' more involved techniques (which, of course, can also cater for higher modes), our simple closed-form expressions were consistent with the aim to provide basic formulae that would be of use to structural engineers for quick fundamental frequency estimates at the preliminary design stage. This was prompted by predominantly civil engineering needs in flat slabs with optional intermediate support and, possibly, the need to consider also orthotropic materials such as timber. (The latter case was not mentioned in our paper, which served as a preliminary test case for the more complex orthotropic problem studied subsequently and which is to be reported elsewhere [7].)

Thirdly, instead of the approximating polynomials adopted by the writers, only (static) deflection shapes—readily available (or derivable) and satisfying all boundary conditions—were used in our work. Admittedly, the results listed in reference [2] for the case of isotropic circular plates either simply-supported or clamped at their boundaries coincide in accuracy with our own because the one-term approximation chosen in Galerkin's method consists of a fourth order polynomial which, when the orthogonality requirement is imposed [2], leads essentially to the expressions for the relevant deflected shapes of the slabs under static loading [1]. On the other hand, the more involved problem of (additional) central support does not lend itself to a similarly straightforward equivalence; instead, the more rigorous static deflection used in our article can only be mimicked by the one-term polynomial solution after additional calculations aimed at optimizing the polynomial's form through a minimization process with respect to its power(s). It is only then that accuracy similar to that of reference [1] can be attained judging by the two isolated cases considered in reference [3], as the latter work is concerned primarily with vibrating membranes (rather than plates) and with two- (as well as one-) term approximations which further increases the complexity of the polynomial formulation.

Though limited in scope, our approach has—besides its inherent simplicity—the signal advantage of being amenable to almost trivial, but quite general, parametric studies. In the case of isotropy, this is evident in the closed-form expressions that are explicitly functions of Poisson's ratio ν. This advantage becomes even more evident in the instance of material orthotropy, especially for some apparently singular problems in which such singularities are eliminated in the final expression for the non-dimensionalized natural frequency through the use of symbolic computation [7]. In contrast, the approach espoused in reference [3] requires that, even for isotropy, a problem be solved anew for each value of ν; this is certainly true when more than a single term is used for the approximations, but it also seems to be the case when only one term is retained because of the need to proceed with the minimization process mentioned above (unless it can be shown formally that the functional relation Ω_1 (γ) for the one-term approximation is always minimized around $\gamma = 2{\cdot}40$, as for the two problems considered in reference [3]).

The importance of quick and relatively economic parametric studies for preliminary design purposes is obvious. In addition, they sometimes lead to the detection of unexpected phenomena or, at least, regimes of behaviour, which, though apparently trivial, are, in fact, quite complex. An example of this is the (incorrect) statement in reference [4] that one is dealing with a "rather basic structural system" (p. 143), whereas the earlier parametric studies of that very system [5, 6] had exposed some startling characteristics in its natural response.

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