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# ON THE APPROXIMATE DETERMINATION OF THE FUNDAMENTAL FREQUENCY OF VIBRATION OF RECTANGULAR, ANISOTROPIC PLATES CARRYING A CONCENTRATED MASS

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## 1. Introduction

The determination of natural frequencies in transverse vibration of isotropic rectangular plates is a problem that has been extensively studied by several researchers. Leissa's works [1, 2] constitute excellent compilations of the pertinent literature. Also, the problem of vibration of rectangular plates with complicating effects, such as variable thickness, presence of concentrated masses, elastically restrained edges, etc., has received considerable treatment [3, 13]. Reference [14] constitutes an excellent survey of the literature concerning dynamics of plate-type structural elements of composite material. Laura and co-workers [15–18] have supplied much of the information regarding the use of polynomial expressions as approximating functions. There is comparatively limited amount of information on the vibration of anisotropic plates.

The present paper deals with the application of the Rayleigh–Schmidt method for the determination of the fundamental frequency coefficient, for rectangular anisotropic plates, [19–22]. The algorithm developed allows the inclusion of analysis of anisotropic, orthotropic and isotropic materials, presence of a concentrated mass and elastically restrained edges. The values obtained are accurate from an engineering viewpoint and the entire algorithm can be implemented on a personal computer. The software constitutes a useful tool in design work, since vibrating anisotropic plate problems which involve several complicating effects can be solved.

## 2. Statement of the problem and its approximate solution by the Rayleigh–Schmidt procedure

The Rayleigh–Schmidt method requires the minimization of the Rayleigh quotient which for the fundamental natural frequency is given by [17, 18]

$$\omega^2 = U_{max}/T_{max} \tag{1}$$

where $U_{max} = U_{p,max} + U_{r,max}$, and where $U_{p,max}$ is the maximum strain energy of the plate, $U_{r,max}$ is the maximum strain energy stored in rotational restraints at the plate edges, and $T_{max}$ is the maximum kinetic energy of the mechanical system under study. The assumed shape functions for using the Rayleigh–Schmidt procedure are given by

$$W(x, y) = A_1 X_1(x) Y_1(y) + A_2 X_2(x) Y_2(y), \tag{2}$$

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where

$$X_1(x) = \sum_{i=0}^{4} a_i x^{ni}, \qquad Y_1(y) = \sum_{i=0}^{4} a_i' y^{ni}; \qquad X_2(x) = \sum_{i=0}^{5} b_i x^{mi}, \qquad Y_2(y) = \sum_{i=0}^{5} b_i' y^{mi}, \tag{3, 4}$$

$a_4 = a_4' = 1$, $n_0 = 0$, $n_1 = 1$, $n_2 = 2$, $n_3 = 3$, $b_5 = b_5' = 1$, $m_0 = 0$, $m_1 = 1$, $m_2 = 2$, $m_3 = 3$, and the exponents $n_4$, $m_4$ and $m_5$ are the adjustable parameters. The coefficients $a_i$, $b_i$, $a_i'$ and $b_i'$ are obtained from the corresponding boundary conditions. The fact that the natural boundary conditions need not be satisfied by the chosen co-ordinate functions is a very important feature of the Rayleigh–Schmidt method, specially when dealing with problems for which such satisfaction is difficult to achieve. In this case it is possible to replace the natural boundary conditions by more easily applied conditions. This procedure has been successfully used in several previous works [15–18] and is also used in the present work.

Minimization of the Rayleigh quotient (1) with respect to each parameter $A_i$ leads to the necessary conditions

$$\partial\omega^2/\partial A_i = 0, \qquad i = 1, 2. \tag{5}$$

TABLE 1

*Values of fundamental frequency coefficient* $\Omega_{00} = \sqrt{(\rho h/D_{11})}\omega_{00}a^2$ *for a rectangular plate of generalized orthotropy which supports a concentrated mass at the center. Edges* 1 *and* 3 *are simply supported*, 4 *is rigidly clamped*, *and edge* 2 *is elastically restrained against rotation* $R_1 = R_3 = 0$, $R_4 = \infty$, $R_2 = r_2 a/D_{11}$, *rm is the ratio concentrated mass/plate mass. The generalized orthotropy is characterized by the following values*: $D_{12}/D_{11} = 0{\cdot}3245569$, $D_{22}/D_{11} = 0{\cdot}2130195$, $D_{16}/D_{11} = 0{\cdot}5120546$, $D_{26}/D_{11} = 0{\cdot}1694905$, $D_{66}/D_{11} = 0{\cdot}3387559$, *where the notations follow those of reference* [18]. *The rotational coefficient used is given by* $R_2 = r_2 a/D_{11}$ *and the aspect ratio is* $rs = a/b$.

| | | $\Omega_{00}(rs = a/b)$ | | |
|---|---|---|---|---|
| $R_2$ | $rm$ | 0·5 | 1·0 | 1·5 |
| $\infty$ | 0 | 16·69 | 21·67 | 30·48 |
| | 0·1 | 13·94 | 18·11 | 25·47 |
| | 0·5 | 9·37 | 12·19 | 17·14 |
| 10 | 0 | 14·74 | 19·85 | 28·77 |
| | 0·1 | 12·33 | 16·61 | 24·07 |
| | 0·5 | 8·31 | 11·20 | 16·23 |
| 1 | 0 | 12·27 | 17·94 | 27·30 |
| | 0·1 | 10·30 | 15·07 | 22·92 |
| | 0·5 | 6·99 | 10·20 | 15·53 |
| 0 | 0 | 11·55 | 17·45 | 26·98 |
| | 0·1 | 9·71 | 14·68 | 22·68 |
| | 0·5 | 6·58 | 9·96 | 15·39 |

When applying equations (5) in order to minimize Rayleigh quotient (1), one obtains a homogeneous system of two equations for the two constants $A_1$ and $A_2$. For a non-trivial solution, the determinant of the coefficient must be zero. One thus obtains a frequency equation of the type

$$A\Omega^4 + B\Omega^2 + C = 0. \tag{6}$$

The frequency coefficients are functions of the parameters $n_i$ and $m_j$ of the assumed shape functions. Therefore, it can be written

$$\Omega_k = \Omega_k(n_i, m_j), \qquad k = 1, 2. \tag{7}$$

The Rayleigh–Schmidt method requires the minimization of the frequency coefficients with respect to the exponential parameters $n_i$ and $m_j$. This procedure has been performed numerically.

Table 1 shows results of the fundamental frequency coefficient

$$\Omega_{00} = \sqrt{(\rho h/D_{11})}\omega_{00}a^2 \tag{8}$$

for a rectangular plate of generalized orthotropy which supports a concentrated mass at the center. Edges 1 and 3 are simply supported, 4 is rigidly clamped and edge 2 is elastically restrained against rotation. The rotational coefficient used is given by $R_2 = r_2 a/D_{11}$ and the plate sides relation $rs$ is $rs = a/b$.

## 3. CONCLUSIONS

A general algorithm has been presented to deal with free transverse vibration of rectangular plates. The Rayleigh–Schmidt method was applied to the problem with a polynomial expression with adjustable exponents as an approximating function. A frequency equation was thus derived in a very simple form. The operations of differentiation and integration needed when substituting expression of $W(x, y)$ in Rayleigh quotient (1) are quite simple. This feature allows the inclusion of analysis of several geometric and mechanical characteristics of the system which can be varied without any difficulty. Thus a close form approximate solution is obtained with all the governing geometrical and mechanical parameters included. In addition, a great advantage of the present approach is the fact that the entire algorithm obtained can be easily implemented in a personal computer. To sum up, it appears that the procedure used yields a convenient and adequate analytical approach, which allows rapid and inexpensive estimates of the natural fundamental frequency of rectangular anisotropic plates, which is important information for design.

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## REFERENCES

1. A. W. LEISSA 1969 *NASA SP* 160, Vibration of plates.
2. A. W. LEISSA 1973 *Journal of Sound and Vibration* **31**, 257–293. The free vibration of rectangular plates.
3. E. M. NASSAR 1979 *American Institute of Aeronautics and Astronautics Journal* **17**, 6–11. Rapid calculation of the resonance frequency for rotational restrained rectangular plates.

4. T. E. Carmichael 1959 *Quarterly Journal of Mechanics and Applied Mathematics* **12**, 29–42. The vibration of rectangular plates with edges elastically restrained against rotation.
5. M. Mukhopadhyay 1987 *Journal of Sound and Vibration* **113**, 547–558. Vibration analysis of elastically restrained rectangular plates with concentrated masses.
6. S. F. Bassily and S. M. Dickinson 1975 *Journal of Applied Mechanics* **42**, 858–864. On the use of beam functions for problems of plates involving free edges.
7. S. F. Bassily and S. M. Dickinson 1978 *Journal of Sound and Vibration* **59**, 1–14. Buckling and vibration of in-plane loaded plates treated by a unified Ritz approach.
8. K. Vijayakumar and G. K. Ramaiah 1978 *Journal of Sound and Vibration* **56**, 127–135. Analysis of vibration of clamped square plates by the Rayleigh–Ritz method with asymptotic solution from a modified Bolotin method.
9. S. M. Dickinson and A. Di Blasio 1986 *Journal of Sound and Vibration* **108**, 51–62. On the use of orthogonal polynomials in the Rayleigh–Ritz method for the study of the flexural vibration and buckling of isotropic and orthotropic rectangular plates.
10. G. B. Warburton and S. Edney 1984 *Journal of Sound and Vibration* **95**, 537–552. Vibrations of rectangular plates with elastically restrained edges.
11. P. A. A. Laura and R. O. Grossi 1981 *Journal of Sound and Vibration* **75**, 101–107. Transverse vibrations of rectangular plates with edges elastically restrained against translation and rotation.
12. P. A. A. Laura, R. O. Grossi and S. R. Soni 1979 *Journal of Sound and Vibration* **62**, 493–503. Free vibrations of a rectangular plate of variable thickness elastically restrained against rotation along three edges and free on the fourth edge.
13. P. A. A. Laura and R. O. Grossi 1978 *Journal of Sound and Vibration* **59**, 355–368. Transverse vibrations of a rectangular plate elastically restrained against rotation along three edges and free on the fourth edge.
14. C. W. Bert 1982 The Shock and Vibration Digest 17–34. Research on dynamics of composite and sandwich plates, 1979–81.
15. R. O. Grossi and P. A. A. Laura 1981 *Fibre Science and Technology* **14**, 311–317. Transverse vibrations of orthotropic rectangular plates with thickness varying in two directions and with edges elastically restrained against rotation.
16. R. O. Grossi and P. A. A. Laura 1979 Ocean Engineering **6**, 527–539. Transverse vibrations of rectangular orthotropic plates with one or two free edges while the remaining are elastically restrained against rotation.
17. R. O. Grossi, P. A. A. Laura and M. Mukhopadhyay. *Journal of Sound and Vibration* **103**, 443–445. Fundamental frequency of vibration of orthotropic rectangular plates with three edges elastically restrained against rotation while the fourth is free.
18. P. A. A. Laura and R. O. Grossi 1979 *Journal of Sound and Vibration* **64**, 257–267. Transverse vibrations of rectangular anisotropic plates with edges elastically restrained against rotation.
19. R. Schmidt 1981 *The Journal of the Industrial Mathematics Society*, **31**, 37–46. A Variant of the Rayleigh–Ritz Method.
20. R. O. Grossi 1987 *The Journal of the Industrial Mathematics Society* **37**, 29–35. A note on the Rayleigh–Schmidt Method.
21. C. W. Bert 1987 *Journal of Sound and Vibration* **119**, 317–326. Application of a version of the Rayleigh technique to problems of bars, beams, columns, membranes and plates.
22. R. O. Grossi 1990 *The Journal of the Industrial Mathematics Society*, **40**, 115–122. On the use of the Rayleigh–Schmidt approach.