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DYNAMIC DISPLACEMENTS AND STRESS RESULTANTS IN BEAMS AND SLABS SUDDENLY SUBJECTED TO THEIR OWN WEIGHT

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1. INTRODUCTION

The present study deals with the dynamic behavior of simply supported beams and rectangular plates, Figures 1(a) and (b), when suddenly subjected to their own weight. The problem arose from questions from civil engineering undergraduates taking regular reinforced concrete courses.

No claim of originality is made but it is hoped that the present, classical treatment of the problems under consideration will be useful from both educational and professional viewpoints.

2. ANALYTICAL SOLUTIONS

It may serve a useful purpose to recall that in the case of a one-degree-of-freedom vibrating element the solution of the governing differential system (see Figure 1(c))

$$(W/g)(\mathrm{d}^2x/\mathrm{d}t^2) + kx = W, \qquad x(0) = \dot{x}(0) = 0 \tag{1, 2}$$

is given by the functional relation

$$x(t) = (W/k)(1 - \cos\omega_n t) \tag{3}$$

and is plotted in Figure 1(d). Accordingly the maximum value of the amplification coefficient is 2 [1].

In the case of the simply supported beam one has

$$EI\,\partial^4 w/\partial x^4 + \rho A_0\,\partial^2 w/\partial t^2 = \rho g A_0, \tag{4}$$

$$w(0, t) = w(L, t) = (\partial^2 w/\partial x^2)(0, t) = (\partial^2 w/\partial x^2)(L, t) = 0, \qquad w(x, 0) = (\partial w/\partial t)(x, 0) = 0, \tag{5, 6}$$

Expressing ($\rho\ g\ A_0$) in terms of a Fourier series of the type

$$\rho g A_0 = \sum_1^\infty b_n \sin n\pi x/L \tag{7}$$

and assuming

$$w(x, t) = \sum_1^\infty \sin(n\pi x/L)T_n(t) \tag{8}$$

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one obtains, after substitution of equations (7) and (8) in equation (4) and straightforward manipulations:

$$w(x,t) = \frac{4}{\pi^5}\frac{\rho g A_0 L^4}{EI}\sum_{1,3}^{\infty}\frac{1}{n^5}\sin\frac{n\pi x}{L}\,(1-\cos\omega_n t), \tag{9}$$

where

$$\omega_n = \sqrt{EI/\rho A_0}(\pi^2/L^2)n^2.$$

Once $w(x, t)$ is known, one determines the dynamic bending moment using the well known expression

$$M(x,t) = -EI\,\partial^2 w/\partial x^2 \tag{10}$$

and the shear force

$$Q(x,t) = -EI\,\partial^3 w/\partial x^3. \tag{11}$$

The procedure is similar in the case of a simply supported rectangular plate subjected to similar boundary and initial conditions and governed by the partial differential equation

$$D\nabla^4 w = \rho g h - \rho h\,\partial^2 w/\partial t^2. \tag{12}$$

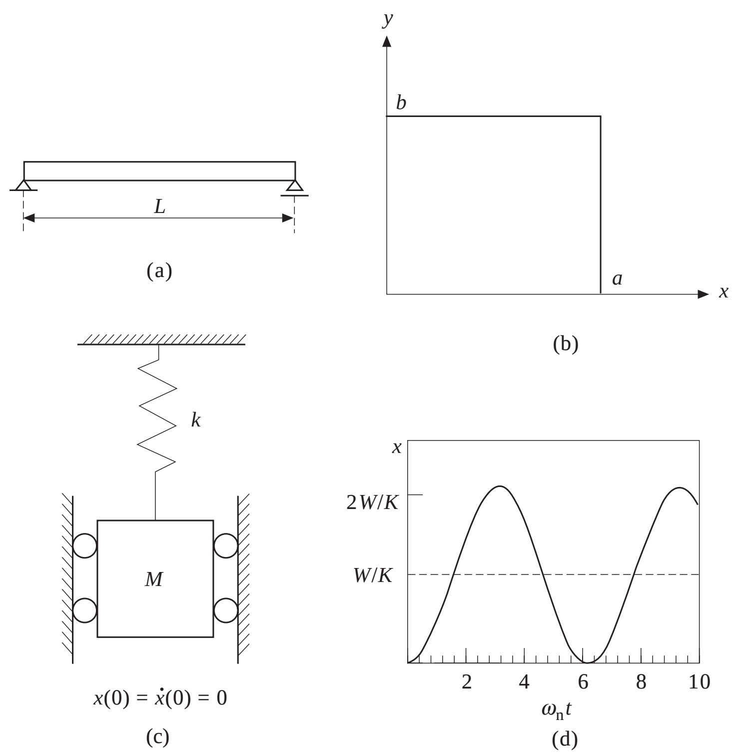

Figure 1. Vibrating structural systems under study: (a) simply supported beam, (b) simply supported rectangular plate, (c) one-degree-of-freedom system and (d) its response when suddenly subjected to its own weight.

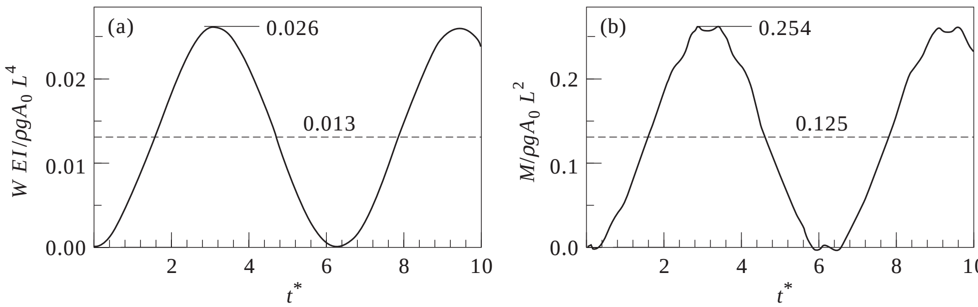

Figure 2. Simply supported beam suddenly subjected to its own weight: (a) dynamic dimensionless displacement at $x = L/2$ as a function of $t^* = \sqrt{EI/\rho A_0}\pi^2/L^2 t$ and (b) dimensionless bending moment at $x = L/2$ as a function of $t^* = \sqrt{EI/\rho A_0}\pi^2/L^2 t$.

The final solution, in terms of a double Fourier series, is

$$w(x, y, t) = \frac{16\rho g h}{a^4 \pi^6 D} \sum_{1,3}^{\infty} \sum_{1,3}^{\infty} \frac{1}{nm[n^2 + [m/b/a]^2]^2} \sin\frac{n\pi x}{a} \sin\frac{m\pi y}{b} (1 - \cos\omega_{nm} t), \qquad (13)$$

where

$$\omega_{nm} = \sqrt{D/\rho h}(\pi^2/a^2)[n^2 + (m/b/a)^2].$$

The determination of stress resultants is achieved following well established procedures, for instance the dynamic bending moments are given by

$$M_x = -D(\partial^2 w/\partial x^2 + \mu\, \partial^2 w/\partial y^2), \qquad M_y = -D(\partial^2 w/\partial y + \mu\, \partial^2 w/\partial x^2). \qquad (14)$$

3. NUMERICAL RESULTS AND CONCLUSIONS

Figure 2 depicts dimensionless dynamic displacements and bending moments at the center of the beams as a function of $t^* = \sqrt{(EI/\rho A_0)}(\pi^2/L^2)t$. Considering the static case (where the weight of the beam is released in a very slow fashion) one observes that the maximum amplification coefficients are higher than 2, which corresponds to the one-degree-of-freedom system.

Similarly Figure 3 shows dimensionless dynamic displacements and bending moments at the center of a simply supported square plate (obviously, for this case $M_x = M_y$) as a function of $t^* = \sqrt{(D/\rho h)}(\pi^2/a^2)t$. It is again observed that the corresponding maximum amplification coefficients are higher than 2.

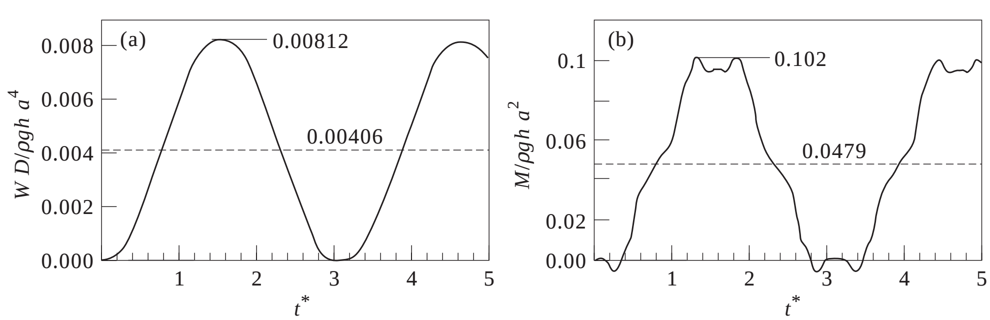

Figure 3. Simply supported square plate suddenly subjected to its own weight: (a) dynamic dimensionless displacement at $x = y = a/2$ as a function of $t^* = \sqrt{D/\rho h}(\pi^2/a^2)t$, (b) dimensionless bending moment M_x at $x = y = a/2$ as a function of $t^* = \sqrt{D/\rho h}(\pi^2/a^2)t$.

Certainly, internal damping of the beam and plate material† and shear and rotatory inertia effects in the vibrating structural elements will lower, considerably, the amplification coefficients. Furthermore, in the case of a reinforced concrete structural element, the forms are seldom removed simultaneously from the entire structure. But the designer and also the construction engineer must be aware that under certain conditions, e.g., imperfections in the structure, the curing process not entirely accomplished, etc., sudden application of the structure self weight may lead to rather severe damages of the structural element.

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† Internal damping will be significant in the case of a reinforced concrete structure.