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EFFECT OF HUMAN RESPONSE TIME ON ROCKING INSTABILITY OF A TWO-WHEELED SUITCASE

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A two-wheeled suitcase being pulled in a straight line on a horizontal ground is considered. Rocking motion of the suitcase from one wheel to the other is analyzed. The action of the puller to resist the rocking of the suitcase involves a time delay, due to the human response time. This time delay may increase the likelihood that the suitcase will become unstable and fall onto one of its sides. The effects of the following quantities are investigated: the magnitude of the time delay, the coefficient of the puller's restoring moment, and the amplitude and frequency of the excitation moment applied by the puller during walking. The results exhibit a fractal behavior in certain cases due to the sensitivity of the response to variations in the excitation.

1. INTRODUCTION

A two-wheeled suitcase may begin to rock from side to side as it is pulled, and the oscillations may grow until the suitcase falls on its side. An analysis of this dynamic problem was presented in reference [1]. As the suitcase rocks, the puller attempts to return the suitcase to its vertical state by applying a restoring moment to the handle. One factor that is important in the instability of the system, but was not considered in reference [1], is the response time of the puller. Even a small time delay in the application of the restoring moment can cause a significant change in the motion of the suitcase. The effect of this time delay is the subject of the present investigation.

Differential equations which include one or more terms with a time delay (or lag) are often called delay equations, retarded equations, differential–difference equations, functional differential equations, or equations with retarded or deviating argument [2–17]. A time delay may change a stable system into an unstable one (e.g., reference [18]), and

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the possibility of chaotic behavior in the solutions of delay equations has been demonstrated (e.g., references [19, 20]).

In the problem formulated here, the governing equation for the rocking angle has a constant time delay in a linear term. In addition, the equation contains non-linear terms and is discontinuous. One of the terms in the equation changes sign whenever the rocking angle changes its sign (i.e., when one wheel impacts the ground and the other one lifts off the ground), and at impact the magnitude of the rocking velocity is suddenly decreased. Due to these complicating effects, as well as the sensitivity of the response, it is not an easy task to obtain an accurate numerical solution [21].

In section 2, the problem is formulated and the solution procedure is described. The puller is assumed to walk at a constant speed and to naturally apply a periodic excitation moment to the handle of the suitcase. When the suitcase rocks from side to side, the puller also applies a restoring moment proportional to the rocking angle, but after constant time delay due to the human response time. The resulting motion depends on the magnitude and frequency of the periodic moment, the coefficient of the restoring term, and the time delay. The effects of these parameters are described in section 3, with attention focused on the critical magnitude of the periodic moment (i.e., the lowest magnitude for which the suitcase falls on its side). Some typical time histories and a phase portrait are shown. The number of impacts prior to overturning is computed for a range of excitation amplitudes and frequencies, and the fractal behavior and sensitivity of the response is illustrated. Concluding remarks are given in section 4.

2. ANALYSIS

The suitcase is pulled in a straight line on a rigid, horizontal surface, and the wheels roll without slipping. It is assumed that the suitcase may rock but does not have any motion in pitch or yaw. The equation of motion for the rocking angle $\theta(t)$ is assumed to be [1]

$$I\,\mathrm{d}^2\theta(t)/\mathrm{d}t^2 + \operatorname{sign}(\theta(t))M_b\cos\theta(t) - M_h\sin\theta(t) + k_0\,\theta(t-\Delta) = q_0\sin(\omega t+\eta), \quad (1)$$

where I is the effective moment of inertia of the suitcase for rocking about either wheel, M_b is the product of the weight and half the effective width of the suitcase between its wheels, M_h is the product of the weight and half the effective height of the suitcase, k_0 is the coefficient of the restoring moment, Δ is the time delay (i.e., the human response time), q_0, ω, and η are the amplitude, frequency, and phase of the excitation moment, respectively, and

$$\operatorname{sign}(\theta) = \begin{cases} +1, & \theta > 0, \\ 0, & \theta = 0, \\ -1, & \theta < 0. \end{cases} \quad (2)$$

The loss of energy when one of the wheels impacts the ground is modelled with the use of a coefficient of restitution e $(0 < \mathrm{e} < 1)$, so that the angular velocities $(\mathrm{d}\theta/\mathrm{d}t)^-$ and $(\mathrm{d}\theta/\mathrm{d}t)^+$ just before and just after impact, respectively, are related by the equation

$$(\mathrm{d}\theta/\mathrm{d}t)^+ = \mathrm{e}(\mathrm{d}\theta/\mathrm{d}t)^-. \quad (3)$$

For the particular suitcase under consideration, the fixed parameters are chosen as $I = 3{\cdot}84$ kgm^2, $M_b = 20{\cdot}2$ kgm^2/sec^2, $M_h = 81{\cdot}3$ kgm^2/sec^2, and $\mathrm{e} = 0{\cdot}913$ [1].

The following non-dimensional quantities are introduced:

$$\tau = t(M_h/I)^{1/2}, \quad \gamma = M_b/M_h = 0{\cdot}248, \quad A = q_0/M_h, \quad \Omega = \omega(I/M_h)^{1/2},$$
$$\beta = k_0/M_h, \quad \delta = \Delta(M_h/I)^{1/2}. \tag{4}$$

Then equation (1) becomes

$$\mathrm{d}^2\theta(\tau)/\mathrm{d}\tau^2 + \mathrm{sign}\,(\theta(\tau))\gamma\cos\theta(\tau) - \sin\theta(\tau) + \beta\theta(\tau - \delta) = A\sin(\Omega\tau + \eta). \tag{5}$$

The non-dimensional excitation moment and restoring moment are depicted in Figure 1. It is assumed that the suitcase is vertical ($\theta = 0$) before $\tau = 0$.

In non-dimensional terms, the weight of the suitcase furnishes an initial restoring moment γ about each wheel of the suitcase (i.e., when $\theta = 0$). Therefore rocking will only occur if the periodic excitation moment exceeds this value at some times, i.e., if $A > \gamma$, and only this range needs to be considered. The phase η will be chosen such that the initial applied moment $A \sin\eta$ is equal to γ, so that motion will begin at $\tau = 0$. For this value of η, for the values of γ and e listed earlier, and for specified values of β, δ, A, and Ω, equation (5) is integrated numerically for 20 cycles of the excitation moment, i.e., from $\tau = 0$ till $\tau = 40\pi/\Omega$. The impact condition (3), with t replaced by τ, is applied whenever $\theta = 0$. The suitcase is said to have overturned if θ reaches $\pi/2$ or $-\pi/2$ during the computed time history.

Numerical integration of equation (5) is carried out using the DKLAG5 delay differential equation solver described in reference [21]. This solver implements continuously imbedded explicit Runge–Kutta–Sarafyan methods. Continuously imbedded means that associated with the basic Runge–Kutta method are (1) a second imbedded Runge–Kutta method used for the purpose of error estimation, and (2) an imbedded Runge–Kutta polynomial interpolant used to approximate the solution at non-integration grid points.

During any integration step from τ_n to $\tau_{n+1} = \tau_n + h$ (where h is the integration stepsize), the polynomial interpolant in (2) is used to obtain approximate delayed solution values $\theta(\tau - \delta)$ for $\tau_n \leqslant \tau \leqslant \tau_{n+1}$. Following any integration step, the imbedded method in (1) is used in conjunction with the basic method to estimate the integration error. This estimate is used in the calculation of the stepsize with which to repeat the step if the error estimate is too large or with which to perform the next step if the error estimate meets the error requirements prescribed for the problem. In addition, the solver automatically locates the times at which delay-induced derivative discontinuities may exist and includes these times as integration grid points in order to preserve the integrity of the numerical solution.

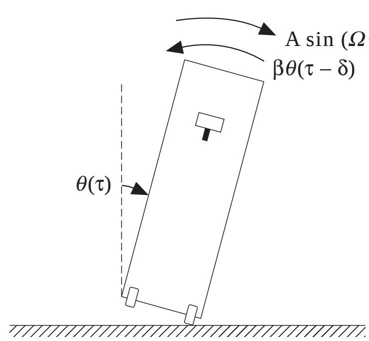

Figure 1. Front view of suitcase with non-dimensional excitation moment and restoring moment.

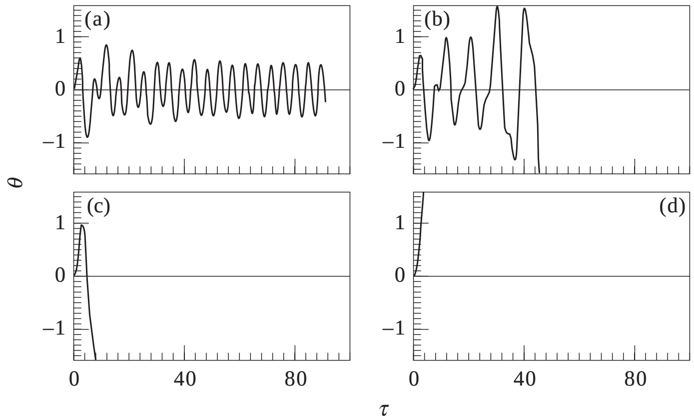

Figure 2. Time history of rocking angle θ for $A = 0{\cdot}6$, $\Omega = 1{\cdot}37$, $\beta = 1$. (a) $\delta = 0$; (b) $\delta = 0{\cdot}1$; (c) $\delta = 0{\cdot}5$; (d) $\delta = 1{\cdot}0$.

DKLAG5 contains a rootfinding (or "gstop") feature which is used in the numerical computations described in this paper. This feature allows the user to specify event functions $g(\tau, \theta(\tau), \theta'(\tau))$ which depend on the solution. Zeroes of these functions are approximated using the interpolant in (2) and a rootfinder. At any such zero, the user is allowed to perform any necessary parameter changes before the integration is continued. In the present computations, this feature is used to locate the times at which the wheels impact the ground in order to allow the replacements required by equation (3).

3. RESULTS

The effects of the non-dimensional excitation amplitude A, excitation frequency Ω, restoring moment coefficient β, and time delay δ are of interest. If Ω, β, and δ are fixed and A is increased from γ, the lowest value at which overturning occurs is called the critical amplitude A_{cr}. The average side-to-side frequency of a person walking is approximately 1 Hz [22], which corresponds to $\Omega = 1{\cdot}37$, and the average human response time is about 0·1 sec [12], which corresponds to $\delta = 0{\cdot}46$.

Time histories of the rocking angle are shown in Figure 2 for $A = 0{\cdot}6$, $\Omega = 1{\cdot}37$, $\beta = 1$, and four values of δ (0, 0·1, 0·5, and 1·0). The non-dimensional period, $2\pi/\Omega$, of the excitation moment is 4·59. In Figure 2(a), with no time delay, the suitcase does not overturn during 20 cycles of excitation. If $\delta = 0{\cdot}1$ (Figure 2(b)), the suitcase falls down after 11 impacts. With $\delta = 0{\cdot}5$ in Figure 2(c), overturning occurs after one impact, and in Figure 2(d) with $\delta = 1{\cdot}0$ the suitcase exhibits "immediate overturning" in one direction with no rocking back and forth.

Figure 3 depicts the time history and phase plane portrait for the case of $A = 0{\cdot}75$, $\Omega = 1{\cdot}37$, $\beta = 1$, and $\delta = 0{\cdot}1$. There are two impacts before the suitcase overturns. In Figure 3(b), the trajectory begins at the origin and the sudden decrease in velocity at impact is seen when it reaches the vertical axis ($\theta = 0$). The local maximum and minimum in the time history in Figure 3(a) between $\tau = 8$ and $\tau = 10$ cause the small loop in Figure 3(b). For cases involving more cycles of rocking, the phase plane portraits often tend to repeat a heart shape similar to that seen in Figure 3(b).

In Figure 4, the critical excitation amplitude A_{cr} is plotted as a function of the excitation frequency Ω for a restoring moment coefficient $\beta = 1$ and for several values of the time delay δ. In order to compute these results, Ω was fixed at values 0·70, 0·75, 0·80, . . . , 2·00

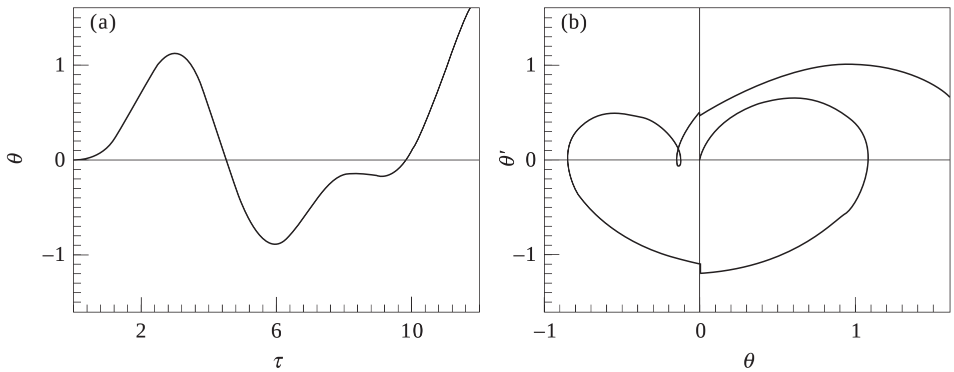

Figure 3. (a) Time history and (b) phase plane portrait for $A = 0{\cdot}75$, $\Omega = 1{\cdot}37$, $\beta = 1$, $\delta = 0{\cdot}1$.

and, for each of these values, A was increased from 0·248 in increments of 0·001 until overturning occurred. The resulting points are connected with straight line segments. The curve for $\delta = 0$ was plotted in reference [1] for the range $1{\cdot}0 < \Omega < 2{\cdot}0$.

For a given value of the time delay in Figure 4, the critical amplitude is essentially constant with $A_{cr} = 0{\cdot}259$ or 0·260 for small excitation frequencies. If the time delay is sufficiently small, A_{cr} then increases for a while as Ω increases, and exhibits a dip for $\delta = 0$, 0·01, 0·05, and 0·075. For $\delta = 0.25$ and higher in the range shown, the excitation frequency has almost no influence on the critical amplitude.

As seen in Figure 4, an increase in the human response time tends to decrease the critical amplitude or keep it the same. This property is examined in Figure 5, where A_{cr} is plotted as a function of δ for $\beta = 1$ and for six values of Ω. With one exception ($\Omega = 1{\cdot}8$ and δ near 0·07), A_{cr} decreases with increasing δ until it reaches a value of approximately 0·259, and then remains essentially constant. The rate of decrease of A_{cr} varies greatly along some of the curves.

Figure 6 illustrates the influence of the restoring moment coefficient β on the critical excitation amplitude A_{cr}. The excitation frequency is fixed at $\Omega = 1{\cdot}37$, and results for $\delta = 0$, 0·050, 0·075, and 0·100 are plotted. The curve for no time delay was also presented in reference [1]. One might expect that A_{cr} would always increase as β increases, but this does not occur. For small values of β, A_{cr} is in the range 0·258–0·261. If $\delta = 0$, 0·050, or 0·075, A_{cr} then increases sharply for a while after β passes a certain value, and later

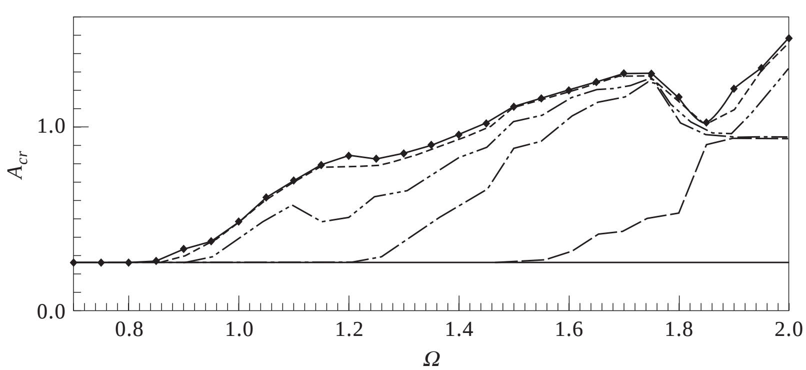

Figure 4. Critical excitation amplitude versus excitation frequency for $\beta = 1$: —◆—, $\delta = 0$; - - - - - , $\delta = 0{\cdot}01$; — - - —, $\delta = 0{\cdot}05$; — - —, $\delta = 0{\cdot}075$; – – – –, $\delta = 0{\cdot}10$; ——, $\delta = 0{\cdot}25$.

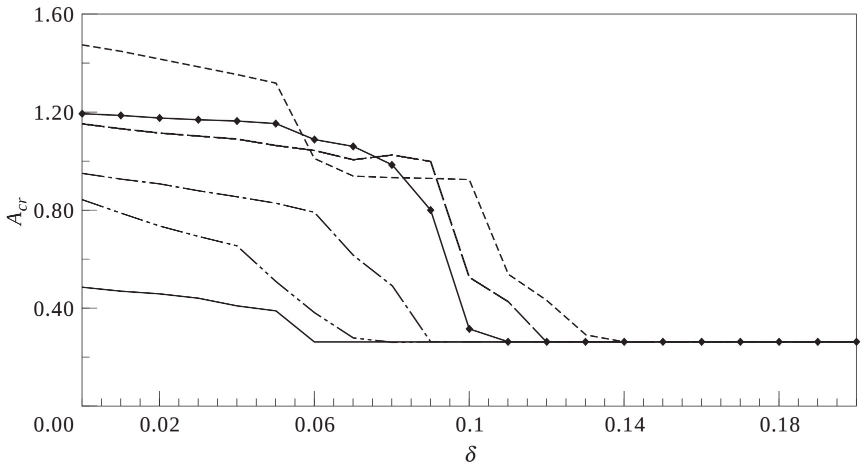

Figure 5. Critical excitation amplitude versus time delay for $\beta = 1$: ——, $\Omega = 1{\cdot}0$; — - - —, $\Omega = 1{\cdot}2$; — - —, $\Omega = 1{\cdot}4$; —◆—, $\Omega = 1{\cdot}6$; – – – –, $\Omega = 1{\cdot}8$; - - - - -, $\Omega = 2{\cdot}0$.

decreases back to the minimum critical amplitude (or close to it). For $\delta = 0$, the curve subsequently rises steadily as β increases, while the curve for $\delta = 0{\cdot}050$ rises slightly and the one for $\delta = 0{\cdot}075$ has an insignificant increase. For $\delta = 0{\cdot}100$ and higher, the value of β has almost no effect on A_{cr} in the range shown.

An interesting feature of this problem is that the system may be stable (i.e., not overturn) for values of applied amplitude A above A_{cr}. This property was demonstrated in reference [1] for $\delta = 0$ with $\beta = 0$ and $\beta = 0{\cdot}5$, and is illustrated in Figure 7 for the case $\delta = 0{\cdot}05$ and $\beta = 1$. The range for the abscissa is $1{\cdot}0 < \Omega < 2.0$ and the four parts of Figure 7 cover an ordinate range of $0{\cdot}4 < A < 1{\cdot}40$. For a grid with increments of 0·005 in Ω and 0·001 in A (i.e., for $201 \times 1001 = 201\ 201$ combinations of Ω and A), the governing equation is integrated. If overturning occurs, the number of impacts is recorded. This number ranges from zero to 37.

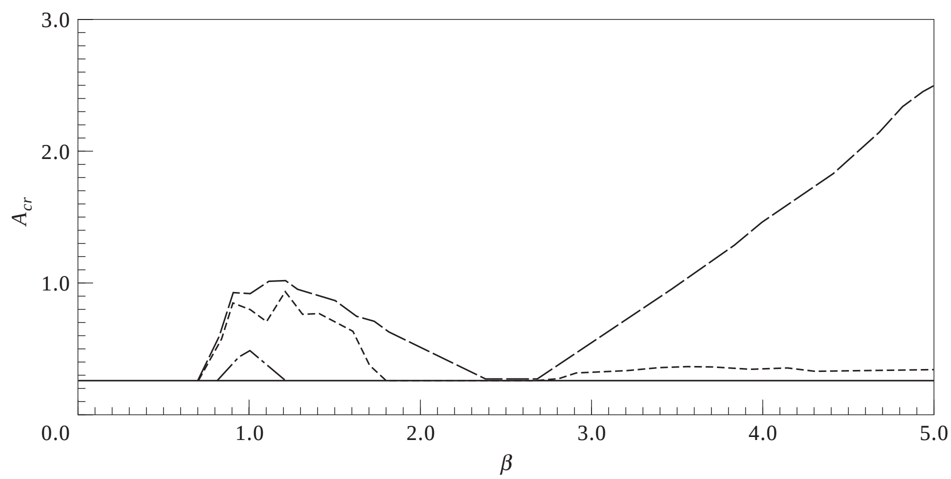

Figure 6. Critical excitation amplitude versus restoring moment coefficient for $\Omega = 1{\cdot}37$. – – – –, $\delta = 0$; - - - - -, $\delta = 0{\cdot}05$; — - —, $\delta = 0{\cdot}075$; ——, $\delta = 0{\cdot}1$.

In Figure 7, points that are white indicate that the suitcase does not overturn. Grey points indicate combinations of amplitude and frequency for which the suitcase overturns, and the shade of grey corresponds to the number of impacts, with the darkest shade corresponding to a large number of impacts and the lightest shade indicating immediate overturning (no impacts). At a given frequency, the critical value of A is the lowest grey point, and these values correspond to the curve for $\delta = 0{\cdot}05$ in Figure 4. However, the increments in Ω are ten times smaller in Figure 7 than in Figure 4, and some values of A_{cr} in Figure 7 lie below the line segments connecting the computed values in Figure 4.

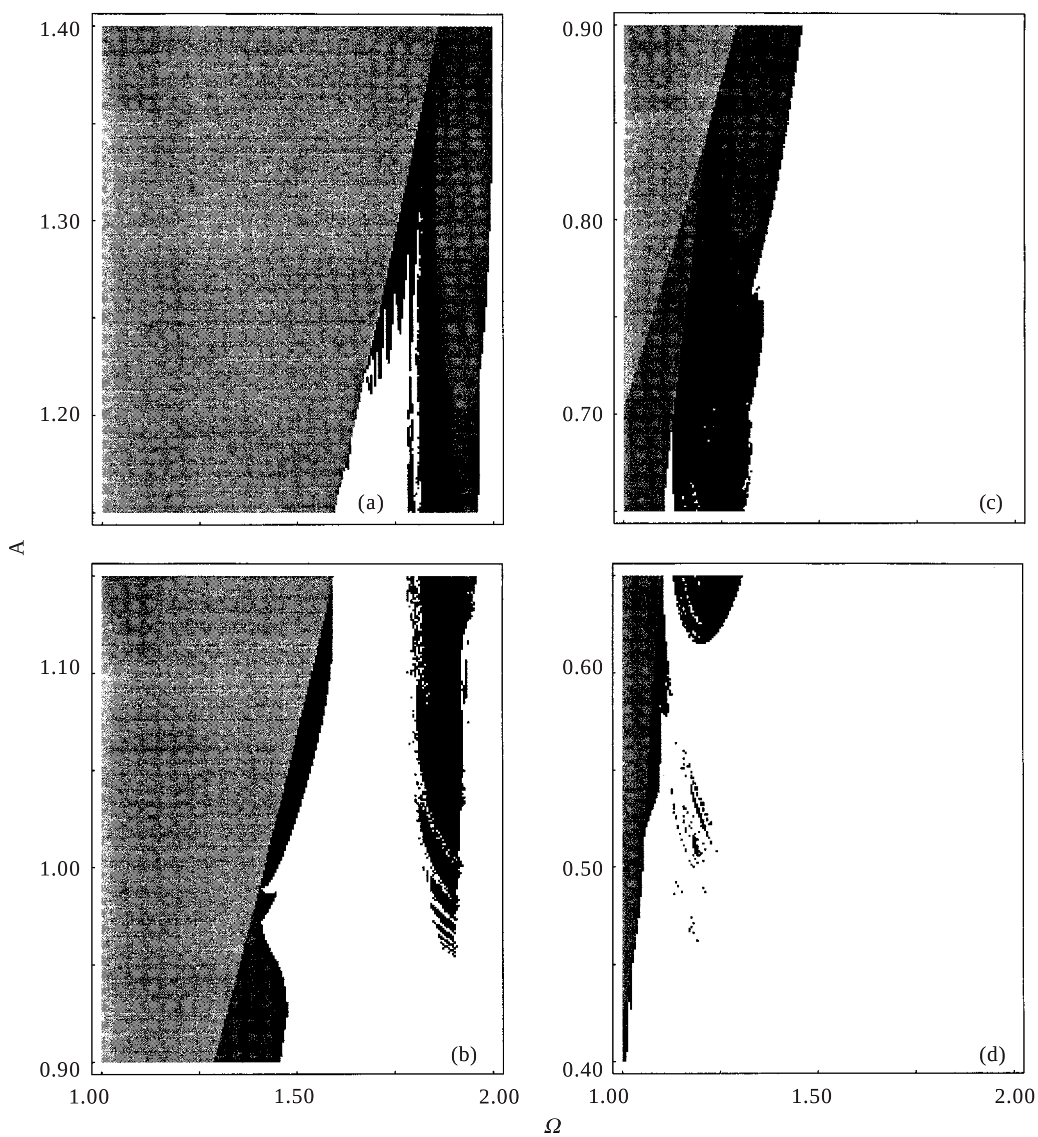

Figure 7. Excitation amplitude versus excitation frequency for $\delta = 0{\cdot}05$, $\beta = 1$. White areas indicate no overturning; grey shading indicates overturning, with a darker shade denoting more impacts; (a)–(d) are four parts of one figure, each for a different range of A.

White points located above grey points in all four parts of Figure 7 indicate that the suitcase can be stable for some values of A larger than A_{cr}. Also, there are some isolated points of overturning inside the large white region associated with no overturning. In the related problem of a rigid block that rocks on an oscillating foundation, the boundary between overturning and no overturning exhibits a fractal behaviour [23, 24], and here this boundary also appears to be fractal in some regions.

Immediate overturning occurs in the upper left portions of parts (b)–(d) of Figure 7. The boundary of this region is not fractal. In some cases it touches the large region of no overturning. For example, if $\Omega = 1{\cdot}600$ in Figure 7(d), the suitcase does not overturn if $A = 1{\cdot}153$, but overturns with no impacts if $A = 1{\cdot}154$.

4. CONCLUDING REMARKS

The numerical solution of the equation of motion (1) is challenging. The equation not only involves a time delay, but also exhibits a discontinuity whenever the suitcase passes through the vertical (i.e., the rocking angle passes through zero). The times of occurrence of these discontinuities must be computed accurately, and then the initial conditions for the subsequent time interval change suddenly and the equation of motion changes. At the beginning of this new time interval, however, the term involving the time delay is governed by the result from the previous interval, involving the previous equation.

Twenty cycles of excitation moment are applied to the suitcase. When the suitcase rocks, energy is dissipated whenever the point of contact with the ground changes from one wheel to the other (i.e., an impact occurs). In some cases the suitcase does not overturn. In the other cases, overturning may occur after many cycles of rocking (i.e., many impacts), or after a few cycles, or "immediately" with no impacts. In the plane of the excitation amplitude and excitation frequency (for fixed values of the time delay and restoring moment coefficient), the boundary between overturning and no overturning may exhibit a fractal character (Figure 7).

The critical excitation amplitude was examined previously under the assumption of no time delay [1]. The time delay associated with the human response time in reaction to the rocking motion tends to reduce the critical excitation amplitude and hence to be detrimental to the stability of the system. If the time delay is greater than a threshold value, any further increase in it has little effect on the critical amplitude.

For the particular geometry of the suitcase considered here, with the non-dimensional parameter γ equal to 0·248, no rocking occurs if the non-dimensional excitation amplitude A is less than 0·248. Also, no overturning occurs if A is less than 0·258, independent of the values of the non-dimensional excitation frequency Ω, non-dimensional time delay δ, and non-dimensional restoring moment coefficient β. The effects of Ω, δ, and β on the critical value A_{cr} are interesting if the human response time is sufficiently small, as seen in Figures 4–6. However, the average response time in the case under investigation corresponds to $\delta = 0{\cdot}46$, for which A_{cr} is close to its minimum value of 0·258 for any of the values of Ω and β treated in this investigation.

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