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ACTIVE CONTROL OF SOUND RADIATION FROM VIBRATING SURFACES USING ARRAYS OF DISCRETE ACTUATORS

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This paper reports an investigation of the active control of sound radiation from vibrating surfaces by using arrays of discrete actuators, or tiles, which cancel local volume velocity. The radiation of sound can be controlled by reducing the vibration levels on the structure and by changing its radiation efficiency. Reductions in vibration level are shown to be purely a function of the number of tiles per structural wavelength. Reductions in radiation efficiency are shown to be dependent on the relationship between the acoustic wavenumber in the fluid, the structural wavenumber on the vibrating surface, and the size of the tiles. It is also shown that there are three distinct regions of control. In the first region of control the acoustic wavelength is larger than the structural wavelength and large reductions in radiation efficiency are possible as long as there are at least two tiles per structural wavelength. In the second region of control the acoustic wavelength is smaller than the structural wavelength but is still more than twice as large as an individual tile. Control in this region is greatly improved if the tile size is reduced. In the third region half an acoustic wavelength is smaller than a tile and no reduction in radiation efficiency is possible. In the third region, attenuation is possible only by reducing the overall vibration level. The cancellation of local volume velocity by using small acoustic sources placed on or close to the vibrating surface is also considered and is shown to achieve even higher levels of attenuation in the radiated sound power than for tiles which cover the entire surface.

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1. INTRODUCTION

The active control of sound radiation from vibrating surfaces has attracted great academic interest in recent years [1–6]. Much of the published work has been concentrated on radiation from simple structures such as beams or plates and in general the frequencies of interest have been relatively low. However, if the size of the structure is large compared to the size of an acoustic wavelength many secondary sources and many error sensors will be required to achieve good attenuation. In such circumstances it is unlikely that fully coupled multichannel control systems would be used since the computational load would be excessive.

One method of controlling the sound radiation from a structure which is large, compared to an acoustic wavelength, is to cover the radiating surface with arrays of active components or "tiles" (see Figure 1) which alter the vibration of the surface. Each tile acts on *local* information such as local velocity or pressure and produces a purely *local* reaction. For this control method one assumes that the control systems operating on each tile are sufficiently uncoupled from one another to allow the control systems to remain stable. This paper presents a general analysis of the mechanisms involved in this control strategy and outlines the physical limitations of such a method of control. Specific tile design will not

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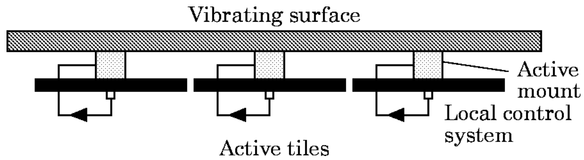

Figure 1. Controlling the radiation from a vibrating surface by using a number of locally acting tiles.

be taken into account and only a few basic assumptions about the behaviour of the tiles will be made. The tiles are assumed to be light compared with the equivalent surface area of the structure and small enough to be rigid. It does not seem sensible to design tiles that are stiff and heavy as compared to the structure since in that case it would probably be more effective simply to stiffen the structure.

There are two mechanisms by which a reduction in the sound power radiated from a surface can be achieved: (i) vibration reduction and (ii) radiation efficiency reduction. If a vibrating surface is covered with rigid tiles which are driven so as to cancel their local volume velocity, then the sound power radiation can be reduced by both of these mechanisms. This paper is concerned with the control of sound radiation from harmonically excited baffled beams and plates. For this analysis the radiating surface of each structure is assumed to be covered with active tiles which cancel local volume velocity (surface integrated normal velocity). In the majority of the analysis presented in this paper tiles which cover the entire radiating surface will be considered but the case of small acoustic control sources placed close to the radiating surface will also be investigated. If the tiles are rigid and cover the entire radiating surface then cancellation of volume velocity is equivalent to cancellation of the velocity at the centre of each tile. If the tiles are rigid then it is possible that there will be discontinuities in the displacement of the radiating surface at the edges of the tiles. The tiles are effectively assumed to be sealed together with a flexible membrane to prevent sound transmission from the cracks, but this is omitted from Figures 1 to 3 for clarity.

2. REDUCTIONS IN VIBRATION LEVEL

There are three possible modes of vibration for a rigid tile. These are (i) whole body motion (volume velocity mode) and (ii, iii) rocking (dipole type) modes along either axis (see Figure 2). If the tiles are placed along a vibrating beam then only two modes of vibration are possible. In this section the amplitude of the rocking modes after cancellation of the volume velocity mode will be investigated. It is assumed that each active mount is able to counteract the volumetric motion due to the structure (i.e., cancel the first mode) without affecting the amplitudes of the other modes of vibration. This will be achieved only if the actuator excites the tile symmetrically and the tiles are light enough such that

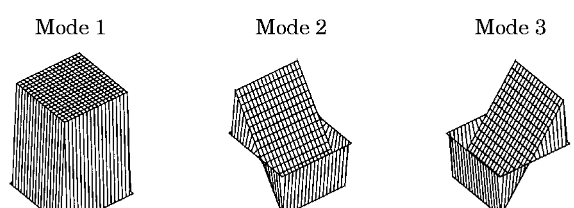

Figure 2. The three modes of vibration of a rigid tile.

the back reaction onto the structure is small (i.e., the vibration of the structure is unchanged after control).

2.1. A DEFINITION OF VIBRATION LEVEL

The kinetic energy of the structure, due to flexural waves, will be taken as a measure of the overall vibration level and is proportional to the integral of the time averaged squared *normal* velocity (V). For a single frequency of excitation, V will be given by

$$V = \tfrac{1}{2}\int_S |v(x, y)|^2 \, \mathrm{d}S, \tag{1}$$

where the integral is over the entire surface S of the structure and $v(x, y)$ is the complex normal surface velocity. If the overall surface is discretized such that each tile is divided into several elements, each of area ΔS, then the velocity can be represented as

$$V = (\Delta S/2)\mathbf{v}^{\mathrm{H}}\mathbf{v}. \tag{2}$$

The vibration levels and the radiated sound power will be calculated by approximating the vibration of the surface by a number of elements, (eight elements per tile for the beam and sixteen elements per tile for the plate). The levels of V before and after control will be investigated to determine the circumstances in which good vibration reduction can be achieved.

2.2. TILES ON A BEAM

If a vibrating beam is covered by a number of rigid tiles, as shown in Figure 3(a), and the *normal* velocity component at the centre of each tile is cancelled, then the tiles will continue to vibrate in a rocking mode, as shown in Figure 3(b). For a given tile the relative amplitudes of the first (volumetric) and second (rocking) modes will determine the level of vibration reduction that will be achieved by cancelling the amplitude of the first mode. The velocity of the ith tile at the position x along the beam can be represented by

$$v_i(x) = a_{i_1}\psi_1(x) + a_{i_2}\psi_2(x), \tag{3}$$

where the velocity of the tile $v_i(x)$ is due to the amplitudes of the tile mode a_{i_1} and a_{i_2}, whose mode shapes are given by $\psi_1(x)$ and $\psi_2(x)$.

The shape for the first (piston) tile mode is given by

$$\psi_1(x) = \sqrt{2/l}, \tag{4}$$

and the shape for the second (rocking) tile mode, for the ith tile, is given by

$$\psi_2(x) = \sqrt{24/l^3}(x - x_i - l/2). \tag{5}$$

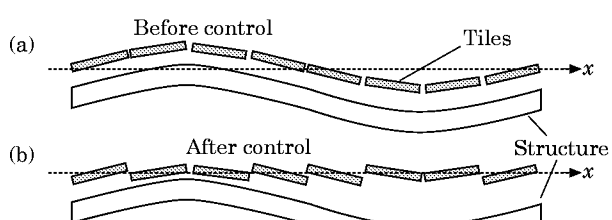

Figure 3. Rigid tiles placed on a vibrating beam before (a) and after (b) the volume velocity of each tile has been cancelled.

These mode shapes have been normalized such that the integrated time averaged squared surface velocity V_i due to the ith tile is given by $V_i = a_{i_1}^2 + a_{i_2}^2$. It is assumed that the amplitude of the modes for the ith tile, a_{i_1} and a_{i_2} are due to the integral of the velocity of the beam under the tile multiplied by their respective mode shapes $\psi_1(x)$ and $\psi_2(x)$. This implies a distributed mounting system where the motion of every point of the structure under the tile affects its motion. The amplitudes of the modes of the ith tile, due to the motion of the underlying structure only, are therefore given by

$$a_{i_1} = \int_{x_i}^{x_i + l} \psi_1 v_b(x)\,\mathrm{d}x \qquad \text{and} \qquad a_{i_2} = \int_{x_i}^{x_i + l} \psi_2 v_b(x)\,\mathrm{d}x. \tag{6, 7}$$

where $v_b(x)$ is the complex velocity distribution along the beam and the tile of length l is positioned between x_i and $x_i + l$.

It is assumed that if a control system is employed to reduce or cancel the amplitude of the first tile mode it will not affect the amplitude of the second tile mode. This will be the case if the active mount actuates the tile symmetrically. Under these conditions, the level of vibration reduction achieved will be dependent on the relative amplitudes of the first and second modes. The total reduction in the vibration of the entire beam will be given by the summation of the vibration level before and after control on all of the I tiles,

$$V_a / V_b = \sum_{i=1}^{I} a_{i_2}^2 \bigg/ \sum_{i=1}^{I} a_{i_1}^2 + a_{i_2}^2. \tag{8}$$

where V_b and V_a are the levels of vibration before and after control.

Figure 4 shows the vibration reductions achieved by active tiles covering a simply supported beam for different numbers of tiles per wavelength. The larger the number of tiles per structural wavelength the larger the vibration reduction and about 5·5 tiles per structural wavelength are required to reduce the overall vibration level by 10 dB. It should be noted that this graph is applicable to all the structural modes of a simply supported beam. The nth-mode will have $n/2$ wavelengths along the beam and will thus require $5{\cdot}5n/2 \approx 3n$ tiles if a 10 dB reduction in overall vibration level is required.

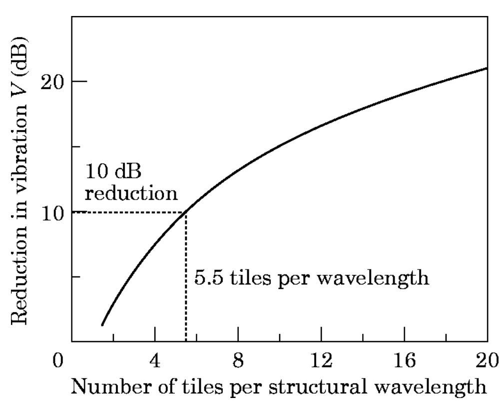

Figure 4. The reduction in the level of vibration as a function of the number of tiles per structural wavelength. Approximately 5·5 tiles per structural wavelength are required to achieve 10 dB of reduction (dotted lines).

2.3. TILES ON A RECTANGULAR PANEL

When covering a panel a rigid tile will have three modes of vibration and therefore the velocity of the ith tile can be described by

$$v_i(x) = a_{i_1}\psi_1(x) + a_{i_2}\psi_2(x) + a_{i_3}\psi_3(x), \tag{9}$$

where the three normalized tile mode shapes for the ith tile are given by

$$\psi_1 = 2/\sqrt{l_x l_y}, \qquad \psi_1 = \sqrt{24/l_x^3 l_y}(x - x_i - l_x/2), \qquad \psi_3 = \sqrt{24/l_x l_y^3}(y - y_i - l_y/2). \tag{10}$$

The dimensions of the tile are given by l_x and l_y and the tile is positioned from $x = x_i$ to $x_i + l_x$ and $y = y_i$ to $y_i + l_y$. The total reduction in the vibration level after the cancellation of the amplitude of the first mode is given by

$$V_a/V_b = \sum_{i=1}^{I} a_{i_2}^2 + a_{i_3}^2 \Bigg/ \sum_{i=1}^{I} a_{i_1}^2 + a_{i_2}^2 + a_{i_3}^2. \tag{11}$$

Figure 5 is a contour plot of the vibration reduction achieved by cancelling the local volume velocity of rigid tiles on a simply supported plate for different numbers of tiles per wavelength in both the x and y directions. The larger the number of tiles per wavelength the larger the vibration reduction, and 7·7 tiles per structural wavelength are required in each direction for 10 dB reduction in vibration level. It is interesting to note that $7{\cdot}7 \approx \sqrt{2} \times 5{\cdot}5$. Once again this graph applies to all modes once normalized by the number of half wavelengths along each direction. The (n, m) mode of a simply supported plate would thus require about $7{\cdot}7n/2 \times 7{\cdot}7m/2 \approx 15nm$ tiles for a 10 dB reduction in vibration.

3. REDUCTIONS IN RADIATION EFFICIENCY

The radiation efficiency can be defined as the ratio of the sound power radiation to the integral of the time averaged surface velocity squared (equation (1)) multiplied by the characteristic acoustic impedance [7]: i.e.,

$$\sigma = W/\rho c V, \tag{12}$$

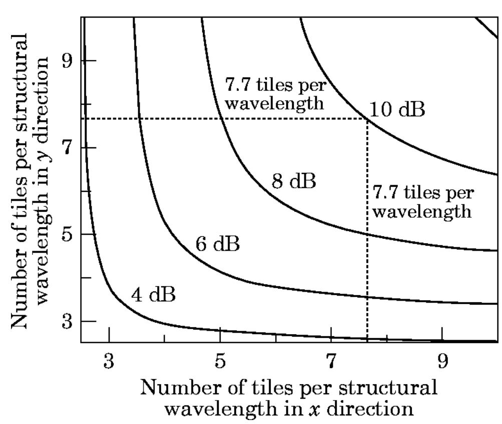

Figure 5. A contour plot showing the reduction in the level of vibration as a function of the number of tiles per wavelength in both the x and y-directions. Approximately 7·7 tiles per structural wavelength in both directions are required to achieve 10 dB of reduction (dotted lines).

where W is the total acoustic power output, ρ is the density of the fluid and c is the speed of sound in the fluid. If the behaviour of a surface is approximated by using L finite elements then the total sound power radiation can be calculated as

$$W = \frac{\Delta S}{2} \sum_{i=1}^{L} \mathscr{R}(v_i^* \, p_i) \tag{13}$$

where ΔS is the area of an element, $\mathscr{R}$ denotes the real part, * denotes the complex conjugate, p_i is the pressure at the ith element and v_i is the velocity of the ith element. This can also be expressed in vector form as [5],

$$W = (\Delta S/2)\mathscr{R}(\mathbf{v}^{\mathrm{H}}\mathbf{p}), \tag{14}$$

where H denotes the Hermitian transpose, $\mathbf{v} = (v_1 \ldots v_L)^{\mathrm{T}}$ and $\mathbf{p} = (p_1 \ldots p_{\mathrm{L}})^{\mathrm{T}}$. If the pressure at the surface is due purely to the velocity of the *tiled* surface then the vector of pressure $\mathbf{p}$ is equal to $\mathbf{Zv}$ where $\mathbf{Z}$ is an L by L matrix of self and transfer impedances and the power can then be expressed as

$$W = (\Delta S/2)\mathscr{R}(\mathbf{v}^{\mathrm{H}}\mathbf{Z}\mathbf{v}) = (\Delta S/2)\mathbf{v}^{\mathrm{H}}\mathbf{R}\mathbf{v} \tag{15}$$

where $\mathbf{R}$ is the real part of the impedance matrix relating the pressure at every element due to the velocity of every element. If the structures are baffled and radiating into a free field then the elements of $\mathbf{R}$ can be calculated analytically as [8]

$$R_{ij} = \frac{\omega^2 \rho \Delta S^2}{4\pi c}\left[\frac{\sin(k_a r_{ij})}{k_a r_{ij}}\right], \tag{16}$$

where k_a is the acoustic wavenumber, ω is the frequency and r_{ij} is the distance between ith and jth elements.

3.1. RADIATION EFFICIENCY REDUCTIONS ON A BEAM

To gain an understanding of the effects of cancelling the local volume velocity (first mode) of a number of tiles which cover a vibrating surface a signal processing analogy will be used. Figure 6 shows a continuous "signal" (a) in the spatial domain, x, which represents the out-of-plane vibration of a section of an infinite beam. This signal is first sampled (b) and then passed through a "zeroth order hold" to give (c). The resulting signal

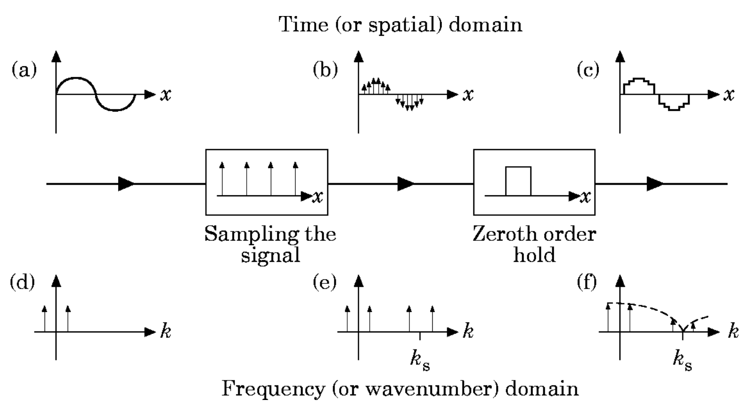

Figure 6. A signal processing analogy to the measurement of volume velocity of a number of tiles on a continuous surface.

is an estimate of the volume velocity of the tiles and the difference (or error) between the continuous signal and the sampled signal ((a) and (c)) will be equivalent to the residual motion of the tiles after volume velocity cancellation. In this analogy the tiles are not assumed to be rigid but if the tiles are small compared to a structural wavelength then the residual motion will be mainly due to rocking and this analogy will be reasonably good.

This process can also be viewed in the "frequency" or strictly the wavenumber domain k. An infinite sine wave in the spatial domain (see Figure 6(a)) can be represented as a pair of delta functions in the wavenumber domain (see Figure 6(d)) [9]. Sampling the signal in the spatial domain is equivalent to aliasing [9] in the wavenumber domain (e). The aliasing wavenumber k_s is the wavenumber interval at which the spectrum is copied and is due to the number of tiles per wavelength. If the tiles are small then the sample rate will be large (i.e., k_s will be large). The zeroth order hold acts as a rectangular window which, in the wavenumber domain, is equivalent to multiplication by a sinc function (f) whose period is inversely proportional to the size of the window [9] which in this case is the size of the tile.

Figure 7(a) shows the wavenumber spectrum of the motion of a narrow simply supported beam vibrating in the third structural mode. The wavenumber k_x is normalized by the beam length L. On an infinite beam the wavenumber spectrum would appear as a delta function at $k_x L = 3\pi$ but because the beam is of finite length the signal is effectively windowed and the normalized wavenumber spectrum is convolved with a sinc function of period 2π. The period of this sinc function is therefore dependent on the length of the beam. A case in which the beam is covered with eight equally sized tiles is considered here. The wavenumber spectrum of the volume velocity component of the tiles is computed as is shown in Figure 7(b). This wavenumber spectrum is equal to the original one multiplied by the sinc function due to the tile spacing, which has a period of 16π, causing the signal in the wavenumber domain to vary as the product of two sinc functions with periods depending on the size of the beam and the size of the tiles. The cancellation of volume velocity sets the wavenumber response to zero at $k_x = 0$. Figure 7(c) shows the wavenumber spectrum of the residual vibration of the beam after the volume velocity of each tile has been cancelled. The main components in the wavenumber spectrum of the

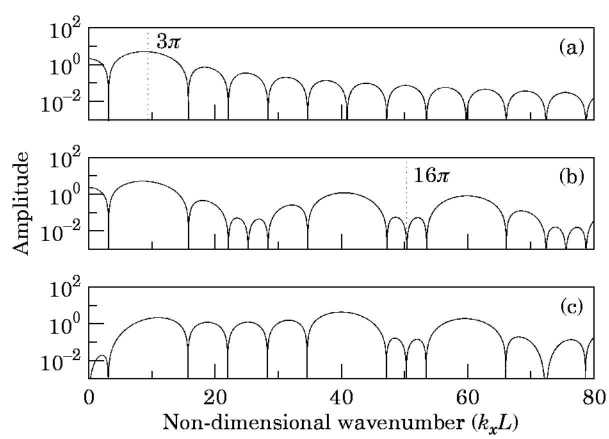

Figure 7. (a) The wavenumber spectrum of the waves on a simply supported beam vibrating in the third structural mode; maximum at 3π (dotted line); (b) the wavenumber spectrum of the volume velocity of the eight tiles; (c) the residual spectrum after volume velocity on each tile is cancelled. Sample wavenumber at 16π (dotted line in (b)).

residual signal are due to differences in the amplitudes of the wavenumber spectrums of the vibration before control and the volume velocity of the tiles (i.e., Figures 7(a) and 7(b)).

The components of the wavenumber spectrum which radiate sound will be those which are supersonic: i.e., those with wavenumber components below $k_a = \omega/c$. As the frequency increases, if a fixed velocity distribution on the beam is assumed an increasing range of wavenumber components will thus radiate sound [10].

Figure 8 shows a second example where the simply supported beam is vibrating in the fourth structural mode. The maximum peak in the wavenumber domain is now at $k_x L = 4\pi$. The wavenumber spectrum of the volume velocity components of the eight tiles and the residual vibration of the beam after subtraction of this component is shown in the lower graphs of Figure 8. Since the mode considered here is even, the wavenumber component before control (see Figure 8(a)) tends to zero at low frequencies. After control (see Figure 7(c)) the wavenumber component still tends to zero at low frequencies but does so at a faster rate.

The radiation efficiency of the third structural mode on the beam before and after control is displayed in Figure 9 (note that the wavenumber axes now have a logarithmic scale). It should be noted that the radiation efficiency after control is proportional to the sound power radiation after control divided by the integrated time averaged mean squared velocity *after* control and hence compensates for the reduction in vibration levels due to control. The beam is considered to be very narrow (the width is 1/1000 the length) such that the radiation characteristics are that of a one-dimensional structure. This accounts for the relatively low levels of radiation efficiency shown in Figure 9.

The behaviour shown in Figure 9(a) can be considered separately in three regions: (i) below $k_a L = 3\pi$ where the acoustic wavelength is larger than the structural wavelength and large reductions in the radiation efficiency are achieved; (ii) the region between $k_a L = 3\pi$ and 13π (i.e., $(16 - 3)\pi$) where the acoustic wavelength is smaller than a structural wavelength but still larger than an individual tile and small reductions in radiation efficiency are achieved; (iii) above $k_a L = 13\pi$ where the acoustic wavelength is comparable to, or smaller than a tile and no significant reductions in radiation efficiency are possible. The first region is below the main peak in the wavenumber spectrum and is strongly

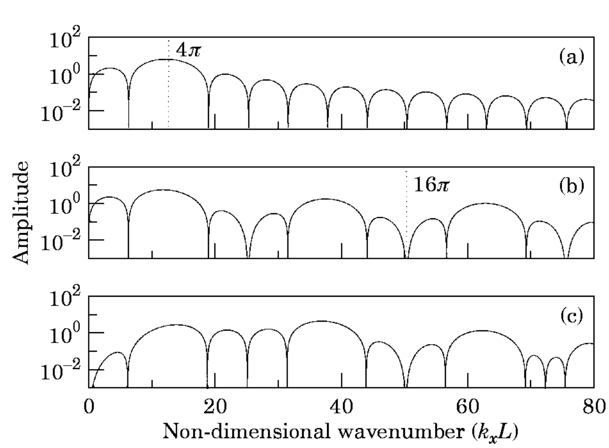

Figure 8. (a) The wavenumber spectrum of the waves on a simply supported beam vibrating in the fourth structural mode; maximum at 4π (dotted line); (b) the wavenumber spectrum of the volume velocity of the eight tiles; (c) the residual spectrum after volume velocity on each tile is cancelled. Sample wavenumber at 16π (dotted line in (b)).

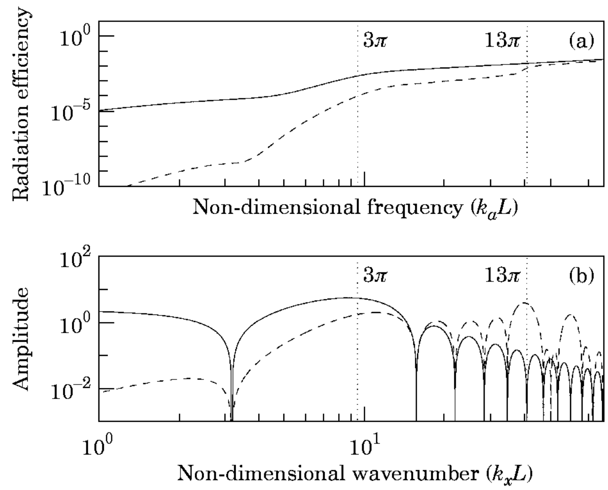

Figure 9. (a) The radiation efficiency of the third structural mode of the beam before (solid line) and after (dashed line) control by using eight tiles which cancel load volume velocity; (b) also repeated for reference are the wavenumber spectra of the beam's velocity before (solid line) and after (dashed line) control (bottom graph). The three control regions are separated by dotted lines at 3π and 13π.

affected by the additional zero placed at the origin by the cancellation of the volume velocity of the tiles. The second region is between the main peak and the reflection of the main peak about the folding frequency due to the tile size (sampling) at $k_x L = 16\pi$. This division into three sections is also demonstrated by calculating the radiation efficiency before and after control when the beam is vibrating in the fourth structural mode (see Figure 10). For this case the first section is below $k_a L = 4\pi$ and the third section is above $k_a L = 12\pi$.

It follows from the above analysis that the size of the tiles mainly affects the size of the second region but does not greatly affect the first region. This can be demonstrated by comparing the radiation efficiency of a simply supported beam vibrating in the third

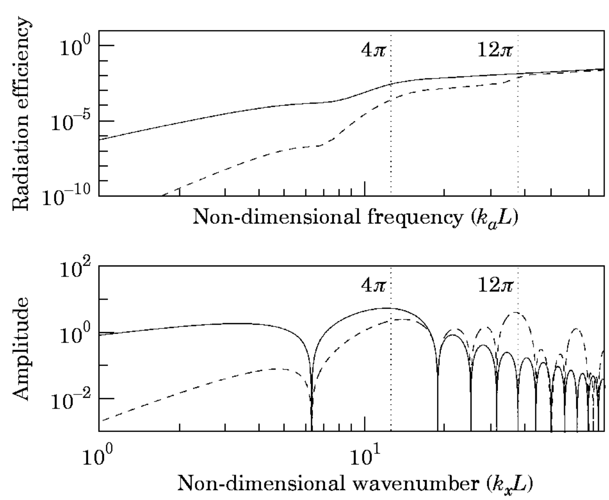

Figure 10. (a) The radiation efficiency of the fourth structural mode of the beam before (solid line) and after (dashed line) control when using eight tiles which cancel load volume velocity; (b) also repeated for reference are the wavenumber spectra of the beam's velocity before (solid line) and after (dashed line) control. The three control regions are separated by dotted lines at 4π and 12π.

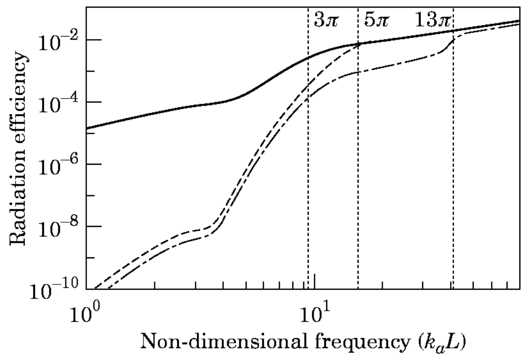

Figure 11. The radiation efficiency of the third structural mode of the beam before (solid line) and after control when using four tiles (dashed line) and eight tiles (dash–dot line). The three dotted lines at 3π and 5π show the separation between control regions for the case using four tiles and the dotted lines at 3π and 13π for the case with eight tiles.

structural mode when it is covered with four and then eight tiles. Figure 11 shows that only small reductions in the radiation efficiencies are achieved at low frequencies by doubling the number of tiles used (approximately 2·5 dB). The second region of control is, however, increased from $k_a L = 5\pi$ to $k_a L = 13\pi$ (i.e., $(8-3)\pi$ to $(16-3)\pi$). Therefore at low frequencies, as long as there are at least two tiles per structural wavelength, increasing the numbers of tiles will be effective only in reducing the vibration levels but will not strongly affect radiation efficiency. The reduction in vibration level will be increased from 4·5 dB to 9·8 dB by increasing the number of tiles from four to eight. If the acoustic wavenumber is such that the system is operating in the second region, i.e., $k_a L > n\pi$ and $k_a L < (2N - n)\pi$ where N is the total number of tiles and n is the number of half wavelengths in the structural mode, then an increased number of tiles per structural wavelength will be useful in reducing the radiation efficiency.

It is interesting to note that the size of the beam does not strongly affect the nature of the radiation from the structure before and after control. This can be demonstrated by considering a beam operating in the ninth structural mode when covered with twenty four tiles. This is the same number of tiles per structural wavelength as in the case of eight tiles on a beam excited at the third mode, the results of which were shown in Figure 9. Figure 12 shows the radiation efficiency before and after control for the two cases of eight tiles

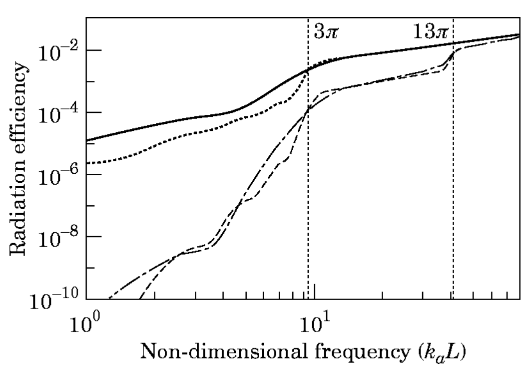

Figure 12. The radiation efficiency of the third structural mode of the beam before (solid line) and after control when using eight tiles (dash–dot line) and the efficiency of the ninth structural mode before (dotted line) and after control (dashed line) when using twenty four tiles. The length of the beam in the second case is taken to be $3L$ for comparison. The dotted lines at 3π and 13π show the separation betwen control regions.

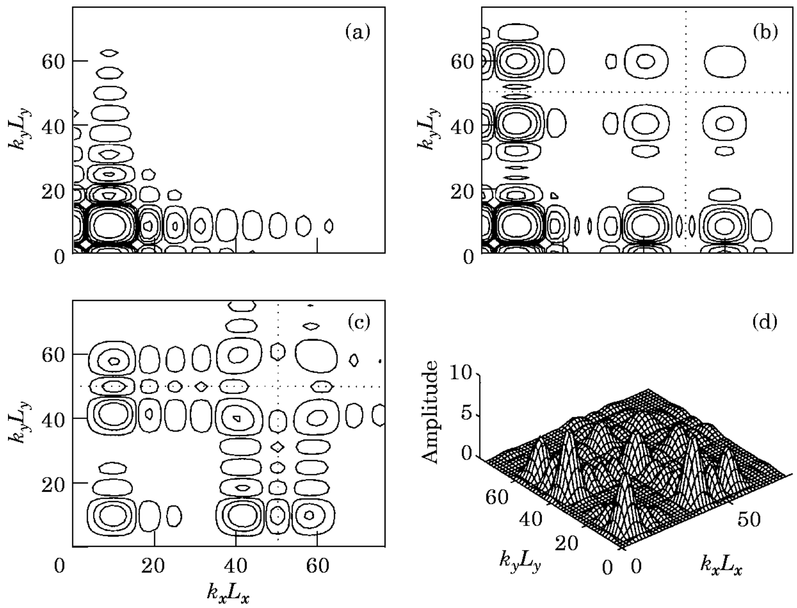

Figure 13. (a) A contour plot of the wavenumber components of a plate vibration in the (3,3) structural mode; (b) the wavenumber components of the volume velocity of an 8×8 array of tiles; (c), (d) the wavenumber spectrum of the residual vibration after control. The dotted lines show the sample wavenumbers in both directions.

controlling a third mode and twenty four tiles controlling a ninth mode. The length of the beam which is operating in the ninth mode is considered to be $3L$ so that the wavenumber spectra match at high frequencies. This example demonstrates that it is the relationship between the structural wavenumber, the acoustic wavenumber and the size of the *tiles* which determines the transition points between regions of control and *not* the length of the beam.

3.2. RADIATION EFFICIENCY REDUCTIONS ON A PLATE

The radiation efficiency of a plate before and after control when using a number of tiles which cancel their local volume velocity can be viewed in a similar manner to the reductions due to the tiling of a beam. The signal processing analogy will hold for this case if the sampling and windowing are considered as two dimensional functions, i.e., in the x and y directions, and hence produce wavenumber spectra which are a function of both k_x and k_y. The aliasing due to sampling will occur in both the k_x and k_y directions in the wavenumber domain and the radiating or supersonic components will be given by $k \leqslant k_a$ where $k = \sqrt{k_x^2 + k_y^2}$ [10].

Figure 13(a) shows a contour plot of the wavenumber components for the (3,3) mode of a simply supported plate of dimensions L_x, L_y. If the plate is covered by an 8×8 array of tiles the wavenumber spectrum of the volume velocity of the tiles is shown in Figure 13(b). The original peak in the spectrum, which is at $k_x L_x = 3\pi$ and $k_y L_y = 3\pi$, is aliased about the aliasing frequency which is shown as the dotted lines in Figure 13. Cancellation of volume velocity again places a zero at the origin of the wavenumber spectrum. The residual spectrum is therefore composed of a peak at $k_x L_x = 3\pi$ and $k_y L_y = 3\pi$ with the largest peaks at $k_x L_x = 3\pi$, $k_y L_y = 13\pi$ and $k_x L_x = 13\pi$, $k_y L_y = 3\pi$ as shown in Figures 13(c) and 13(d).

The radiation efficiency before and after control can again be split into three sections which will again be determined by the wavenumber of the main peak in the wavenumber spectrum of the vibration before control, i.e., $k = \sqrt{(n\pi/L_x)^2 + (m\pi/L_y)^2}$, and the wavenumbers of the reflected peaks. Some of the reflected peaks have different wavenumbers so the one with the lowest wavenumber is taken as separating the second and third sections in the radiation efficiency plots. This is given by $k = \sqrt{(n\pi/L_x)^2 + ((2N - m)\pi/L_y)^2}$, where $m \geqslant n$ and N is the number of tiles across the plate (here it is assumed that there is the same number of tiles in the x and y directions). The radiation efficiency plots can therefore be separated into the three sections already identified above for the case of a beam.

Figure 14 shows the radiation efficiency before and after control for a simply supported plate vibrating in the (3,3) mode and controlled by an 8×8 array of tiles (dash–dot line) which cancel their local volume velocity. For comparison the radiation efficiency after control using a 4×4 array of tiles is also shown (dashed line). As for the case of a beam the major effect of increasing the number of tiles is to extend the second region of control. The line which divides the first and second regions of control is at $k_a L = \pi\sqrt{3^2 + 3^2} \approx 4{\cdot}2\pi$. For the case of a 4×4 array of tiles the second and third regions of control are divided by $k_a L = \pi\sqrt{3^2 + 5^2} \approx 5{\cdot}8\pi$. For the case of a 8×8 array of tiles the second and third regions of control are divided by $k_a L = \pi\sqrt{3^2 + 13^2} \approx 13{\cdot}3\pi$.

Figure 15 shows the *sound power radiation* by a plate vibrating in the (3,3) mode before and after control using a 4×4 and then an 8×8 array of tiles. This plot includes both the radiation efficiency term and the vibration reduction term. Up to $k_a = 5{\cdot}8\pi$ there is an increase in attenuation of approximately 8 dB when using an 8×8 array instead of a 4×4 array which is due to both a reduction in the vibration and a reduction in radiation efficiency. This extra attenuation is obtained, however, at the expense of a four fold increase in the required number of tiles. It is between $k_a = 5{\cdot}8\pi$ and $k_a = 13{\cdot}3\pi$ that an 8×8 array of tiles exhibits the most significant increase in performance over a 4×4 array of tiles, and the 13 dB difference in performance in this frequency range is mainly due to decreases in radiation efficiency.

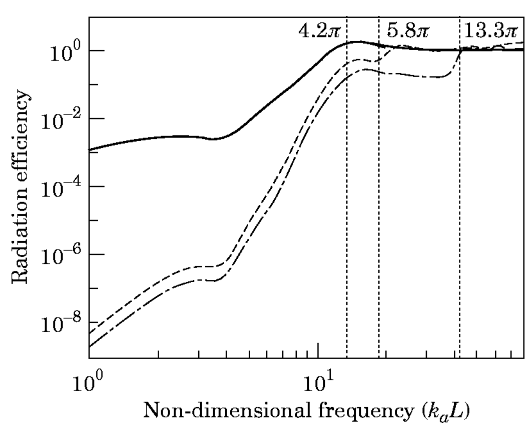

Figure 14. The radiation efficiency of the (3,3) structural mode of a simply supported plate before (solid line), after control when using a 4×4 array of tiles (dashed line) and after control when using an 8×8 array of tiles (dash–dot line) which cancel local volume velocity. The dotted lines at $4{\cdot}2\pi$ and $5{\cdot}8\pi$ show the separation between control regions for the case using four tiles and the dotted lines at $4{\cdot}2\pi$ and $13{\cdot}3\pi$ for the case with eight tiles.

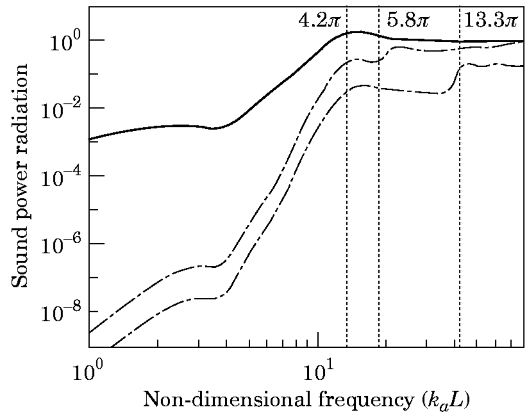

Figure 15. The second power radiation from the (3,3) structural mode of a simply supported plate before (solid line), after control when using a 4×4 array of tiles (dashed line) and after control when using an 8×8 array of tiles (dash–dot line) which cancel local volume velocity. The dotted lines at $4{\cdot}2\pi$ and $5{\cdot}8\pi$ show the separation between control regions for the case when using four tiles and the dotted lines at $4{\cdot}2\pi$ and $13{\cdot}3\pi$ for the case with eight tiles.

3.3. RADIATION EFFICIENCY REDUCTIONS WHEN USING ACOUSTIC SOURCES PLACED CLOSE TO THE RADIATING SURFACE

In this section the possibility is examined of using a set of small acoustic sources placed on or near a vibrating surface to control its sound radiation. A surface mounted acoustic source can be considered to be a small section of every "tile" which is driven such that it compensates for the volume velocity of the other parts of the tile: i.e., when the vibrating surface is flexing outwards the acoustic sources are moving inwards such that the total volume velocity is zero. Figure 16(a) shows three small acoustic secondary sources positioned on the vibrating surface. The velocity of the radiating surface before control,

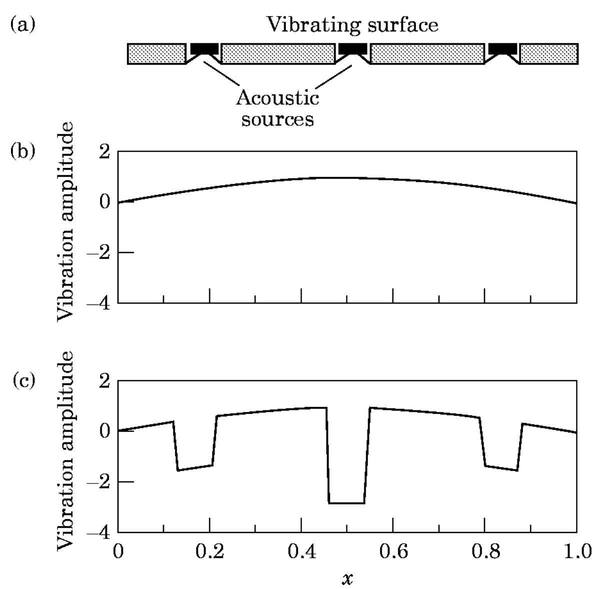

Figure 16. Three acoustic sources which compensate for the volume velocity of a surface (a), the vibration amplitude of the radiating surface, which includes the three acoustic sources, before control (b) and after control (c).

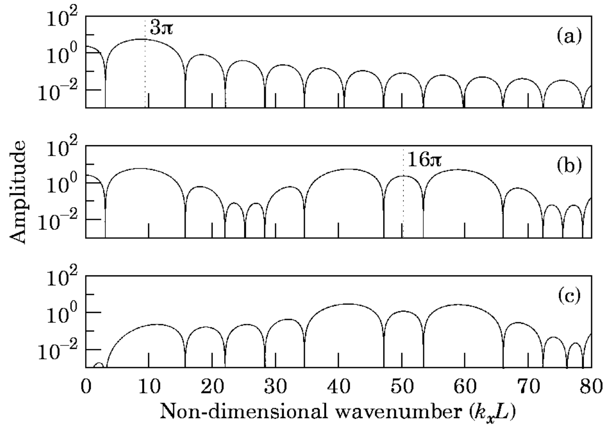

Figure 17. The wavenumber spectrum of the waves on a simply supported beam vibrating in the third structural mode; maximum at 3π (dotted line); (b) the wavenumber spectrum of the eight finite secondary sources; (c) the residual spectrum after control. Sample wavenumber at 16π (dotted line in (b)).

which is vibrating as the first structural mode of a simply supported beam, is shown in Figure 16(b). Figure 16(c) shows the velocity of the radiating surface after control and includes the three acoustic surfaces which have finite length and negative volume velocity. The acoustic sources in this example do not modify the plate vibration but compensate for its volume velocity. These velocity distributions can also be analysed by looking at their wavenumber spectra. A beam vibrating in the third structural mode, which is controlled by using eight sources, is taken as an example for comparison with the results presented in Figures 7 and 9. The wavenumber spectrum of the beam before control, the wavenumber spectrum of the compensating acoustic sources and the wavenumber spectrum after control are all shown in Figure 17. In this case each source is taken to be a quarter of the size of each "tile" where a tile is considered to be the section of beam that each source compensates for. By reducing the size of the secondary source the velocity required by the source to compensate for the total volume velocity of the tile increases and therefore the total sum of the squared velocities (V) also increases. In the case shown in Figure 17 the vibration after control is *increased* by 4·3 dB (note that the wavenumber axes have a logarithmic scale). The radiation efficiency as a function of frequency is shown in Figure 18(a). Figure 18(b) shows the wavenumber spectrum of the velocities before and after control.

Compensation for the total volume velocity of the tile by using only a section of the tile will tend to produce higher order wavenumber components but these components are generally in a region where they do not radiate sound. The overall reductions in radiation efficiency are actually larger than those achieved by cancelling the amplitude of the volumetric mode of a rigid tile. Although this method will tend to increase the vibration level, the resulting reductions in radiation efficiency more than compensate for this increase. The sound power radiation from a surface vibrating in the third structural mode of a simply supported beam before and after control using both tiles and acoustic sources is shown in Figure 19. The method of volume velocity compensation by using compact sources achieves better attenuation in both the first and second regions of control. For the results shown here the secondary sources are a quarter the length of each tile. If they are made even smaller then slight improvements in attenuation are possible but these are not significant. Small sources tend to produce higher levels of high wavenumbers but also tend

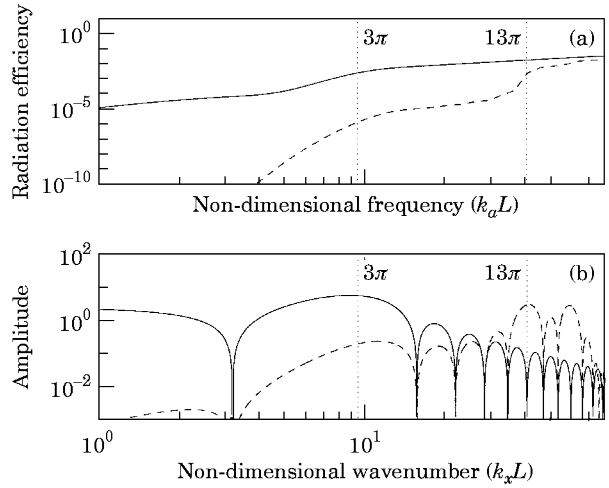

Figure 18. (a) The radiation efficiency of the third structural mode of the beam before (solid line) and after (dashed line) control when using eight secondary sources which compensate for the local volume velocity; (b) also repeated for reference are the wavenumber spectra of the beam's velocity before (solid line) and after (dashed line) control. The dotted lines at 3π and 13π show the separation between control regions.

to match the original wavenumber spectrum more accurately in the low wavenumber region.

4. CONCLUSIONS

The sound power radiation from a vibrating structure is the product of the integrated time averaged surface velocity squared (vibration level) and the radiation efficiency times the characteristic impedance of the fluid. By covering a surface with a number of rigid tiles which actively cancel their local volume velocity (which effectively means cancelling the velocity at the centre of the tile), both the vibration level and the radiation efficiency can be reduced.

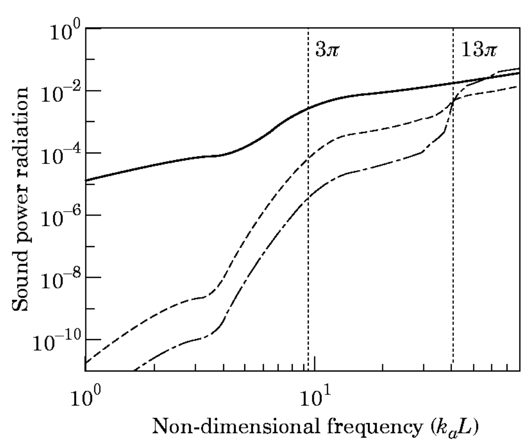

Figure 19. The second power radiation from the third structural mode of a simply supported beam before (solid line), after control when using eight tiles (dashed line) and after control when using eight small acoustic sources (dash–dot line) which compensate for local volume velocity. The dotted lines at 3π and 13π show the separation between control regions.

Reductions in vibration will be dependent solely on the number of tiles per structural wavelength. If the tiles act to cancel their own volume velocity without affecting the other modes of vibration, then approximately $15nm$ tiles will be required to reduce the vibration by a 10 dB on a simply supported plate, vibrating in the (n, m) mode.

A frequency domain approach to aliasing, normally due to temporal sampling, has been shown to be very useful in a wavenumber analysis of the radiation of sound from a spatially sampled structure such as a tiled panel. Reductions in radiation efficiency have been shown to fall into three regions in the wavenumber domain. For the (n, m) mode on a plate of dimensions L_x by L_y, the first region is below the main peak in the wavenumber domain at $k = \sqrt{(n\pi/L_x)^2 + (m\pi/L_y)^2}$. If the acoustic wavenumber k_a lies below this wavenumber then large reductions in efficiency can be achieved as long as there are at least two tiles per structural wavelength. In the second region, where the acoustic wavenumber lies between the main peak and the first aliased peak in the wavenumber domain, smaller reductions in radiation efficiency are possible. The wavenumber about which aliasing occurs is inversely proportional to the size of the tiles so that smaller tiles equate to higher aliasing wavenumbers. Therefore, by making the tiles smaller, the second region of control will be extended and increased reductions in radiation efficiency will be achieved. The third region starts when the acoustic wavenumber is larger than the wavenumber of the first main aliased peak and in this section no reductions in radiation efficiency can be expected.

It has also been shown that if small acoustic sources, which compensate for the local volume velocity, are used instead of tiles which cover the entire surface, even larger attenuations in radiation efficiency can be achieved. Compensation for the net volume velocity will tend to increase the vibration level, but overall the strongest effect is due to reduced radiation efficiency. The three regions of control are the same as for the above case and hence the conditions for good attenuation are the same.

From these results it can be concluded that if the acoustic wavelength is much larger than the structural wavelength then only two tiles per structural wavelength are necessary to achieve good control. This control is obtained by placing an extra zero in the wavenumber spectrum at $k = 0$. If the acoustic wavelength is smaller than the structural wavelength but still large compared to the size of the tiles, i.e., in the second region of control, then reductions in sound power radiation, due to a reduction in squared velocity and reductions in radiation efficiency, can be achieved. Therefore, if the acoustic wavelength is much smaller than the structural wavelength then the tiles must be significantly smaller than the acoustic wavelength to achieve a reduction in radiation efficiency (i.e., $k_a < 2\pi/L$). The main mechanism of control when the acoustic wavelength is much smaller than the dimensions of the tiles is likely to be vibration reduction.

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